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AIR–WATER INTERFACE AREA IN SELF-AERATED FLOWS

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Abstract—In high velocity open channel flows, air is entrained at the free surface. This process is called self-aeration. In air–water flows the presence of air bubbles within the flow increases substantially the amount of air–water interface area and enhances the air–water transfer of atmospheric gases (e.g. nitrogen and oxygen). This paper presents a method to estimate the air–water interface area in uniform self-aerated flows on chutes and spillways. It is shown that the interface area is a function of the channel slope, discharge and roughness height only. The results provide a simple estimation of the air–water interface area and hence enable more accurate calculations of air–water gas transfer in hydraulic structures.

Key words—air–water interface area, air entrainment, self-aeration, air–water gas transfer, spillways

NOMENCLATURE

a = specific air–water interface area (m^{-1})
a_{mean} = mean specific air–water interface area (m^{-1})
B' = integration constant of the air concentration distribution
C = air concentration defined as the volume of air per unit volume
C_b = air concentration at the outer edge of the air concentration boundary layer
C_e = uniform equilibrium air concentration defined as: $(1 - Y_{90}) * C_e = d$
C_{gas} = concentration of dissolved gas in water (kg/m^3)
C_s = saturation concentration of dissolved gas in water (kg/m^3)
d = equivalent clear water depth (m) defined as:

$$d = \int_0^{Y_{90}} (1 - C) * \mathrm{d}y$$

d_b = maximum air bubble diameter (m)
f = non-aerated flow friction factor
f_e = self-aerated flow friction factor
G' = integration constant of the air concentration distribution
g = gravity constant (m/s^2)
K_L = liquid film coefficient (m/s)
n = exponent of the power law velocity distribution
q = discharge per unit width (m^2/s)
t = time (s)
V = velocity (m/s)
V_{90} = characteristic velocity (m/s) at which $y = Y_{90}$
$(We)_e$ = self-aerated flow Weber number defined as:

$$(\mathrm{We})_e = \rho_w * \frac{V_{90}^2 * Y_{90}}{\sigma}$$

Y_{90} = characteristic depth (m) where the air concentration is 90%
y = distance measured perpendicular to the spillway surface (m)
y' = dimensionless distance measured perpendicular to the spillway surface: $y' = y/Y_{90}$;
α = spillway slope
δ_{ab} = air concentration boundary layer thickness (m)
ρ = density (kg/m^3)
σ = surface tension between air and water (N/m)

Subscripts

air = air flow
w = water flow

INTRODUCTION

In high velocity down a steep chute air is entrained at the free surface (Fig. 1). This process, called self-aeration, is caused by the turbulent velocity acting next to the air–water interface. Along a spillway, the upstream flow region is smooth and glassy. However turbulence is generated next to the boundary, and when the outer edge of the growing boundary layer reaches the free surface, turbulence can initiate free-surface aeration (Falvey, 1980; Wood, 1985). Downstream of the point of inception of air entrainment, a layer of air–water mixture extends gradually through the fluid. Far downstream the flow becomes uniform, and for a given discharge any measure of flow depth, air concentration and velocity distributions do not vary along the chute. This region is defined as the uniform equilibrium flow region. The hydraulic characteristics of uniform self-aerated flows were developed initially by Wood (1983), and later re-analysed by Chanson (1989) and Hager (1991).

Air entrainment on spillways and chutes has been recognized recently for its contribution to the air–water transfer of atmospheric gases such as oxygen and nitrogen (Wilhelms and Gulliver, 1989; Gulliver *et al.*, 1990). This process must be taken into account to explain the high fish mortality downstream of large hydraulic structures (Smith, 1973), but also for the reoxygenation of polluted streams and rivers (Gulliver *et al.*, 1990).

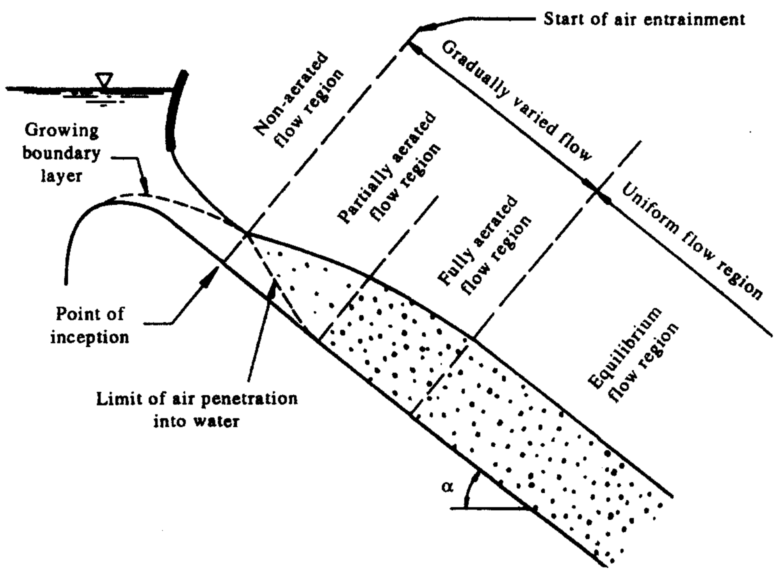


Fig. 1. Air entrainment region above a chute spillway.

The presence of air bubbles entrained within the flow enhances the air–water transfer of atmospheric gases (e.g. nitrogen, oxygen, carbon dioxide). Fick's law states that the mass transfer rate of a chemical across an interface normal to the x-direction and in a quiescent fluid varies directly as the coefficient of molecular diffusion D_{gas} and the negative gradient of gas concentration in the fluid (Streeter and Wylie, 1981). For atmospheric gases and using Henry's law, it is usual to write Fick's law as:

$$\frac{d}{dt} C_{gas} = K_L * a * (C_s - C_{gas}) \qquad (1)$$

where C_{gas} is the concentration of dissolved gas in water, C_s is the dissolved gas concentration at saturation, K_L is the coefficient of transfer and a is the interface area per unit volume. The rate of air–water gas transfer is directly proportional to the air–water interface area within the flow. This area is evaluated from the total quantity of air entrained and the bubble size distribution across the flow. This paper develops a method to estimate the air–water interface area in uniform self-aerated flows.

FLOW PARAMETERS IN SELF-AERATED FLOWS

In uniform flows, the re-analysis of data obtained in the model (Straub and Anderson, 1958) and prototype (Aivazyan, 1986) indicate that the mean air concentration is independent of the discharge, roughness height and flow depth, and is a function of the

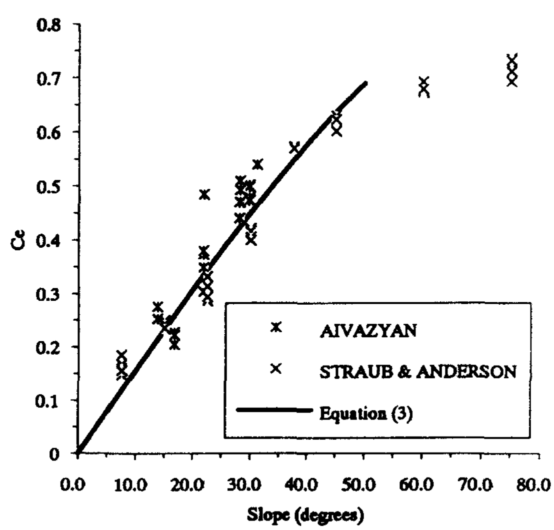


Fig. 2. Uniform mean air concentration as a function of the channel slope.

Table 1. Uniform self-aerated flow parameters

Slope α (degrees)	Average air concentration C_e†	f_e/f equation (4)	$\frac{V_{90} * Y_{90}}{q_w}$	$G' * \cos\alpha$‡	B'‡
0.0	0.0	1.0	$\frac{n+1}{n}$§	+ infinite	0.0
7.5	0.161	0.968	1.453	7.99952	0.003021
15.0	0.241	0.870	1.641	5.74469	0.028798
22.5	0.310	0.765	1.805	4.83428	0.071572
30.0	0.410	0.613	2.141	3.82506	0.196353
37.5	0.569	0.389	2.985	2.67484	0.620262
45.0	0.622	0.313	3.319	2.40096	0.815675
60.0	0.680	0.228	4.151	1.89421	1.353931
75.0	0.721	0.167	4.859	1.57440	1.864181

†Data from Straub and Anderson (1958).
‡Computed from Straub and Anderson's (1958) data.
§Analytical formula for a velocity distribution of the form: $\frac{V}{V_{90}} = \left(\frac{y}{Y_{90}}\right)^{1/n}$.

slope only (Wood 1983, 1985; Chanson, 1992a). For these data the depth averaged mean air concentration in uniform flows C_e is plotted as a function of the channel slope α on Fig. 2, where C_e is defined as

$$C_e = \frac{1}{Y_{90}} * \int_{y=0}^{y=Y_{90}} C * \mathrm{d}y \qquad (2)$$

C is the local air concentration defined as the volume of air per unit volume of air and water, y is the distance measured perpendicular to the channel surface and Y_{90} is the depth where the local air concentration is 90%. For slopes flatter than 50°, the mean air concentration can be correlated as (Chanson, 1992b):

$$C_e = 0.9 * \sin\alpha \qquad (3)$$

where α is the channel slope (Fig. 2). For $\alpha > 50°$, the values of C_e can be interpolated from Table 1, column 2.

In self-aerated flows Jevdjevich and Levin (1953) and Wood (1983) showed that the presence of air within the flow layers reduces the friction losses along the spillway. The author re-analysed prototype data (Jevdjevich and Levin, 1953; Aivazyan 1986) and model data (Straub and Anderson, 1958) using the same method as Wood (1983). The results are presented in Fig. 3. For these data the reduction of friction losses can be estimated as:

$$\frac{f_e}{f} = 0.5 * \left(1 + \tanh\left(0.70 * \frac{0.490 - C_e}{C_e * (1 - C_e)}\right)\right) \qquad (4)$$

where f is the non-aerated flow friction factor, f_e is the aerated flow friction factor and: $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$. Values of f_e/f, computed using equation (4), are shown in Fig. 3 and in Table 1, column 3.

In uniform flows, the equivalent clear water flow depth d can be deduced from the momentum equation. For a wide channel it yields (Wood, 1983):

$$d = \sqrt[3]{\frac{q_w^2 * f}{8 * g * \sin\alpha} * \frac{f_e}{f}} \qquad (5)$$

where g is the gravity constant and q_w is the discharge per unit width.

The characteristic depth Y_{90} is obtained from equation (1): $Y_{90} = d/(1 - C_e)$. The characteristic

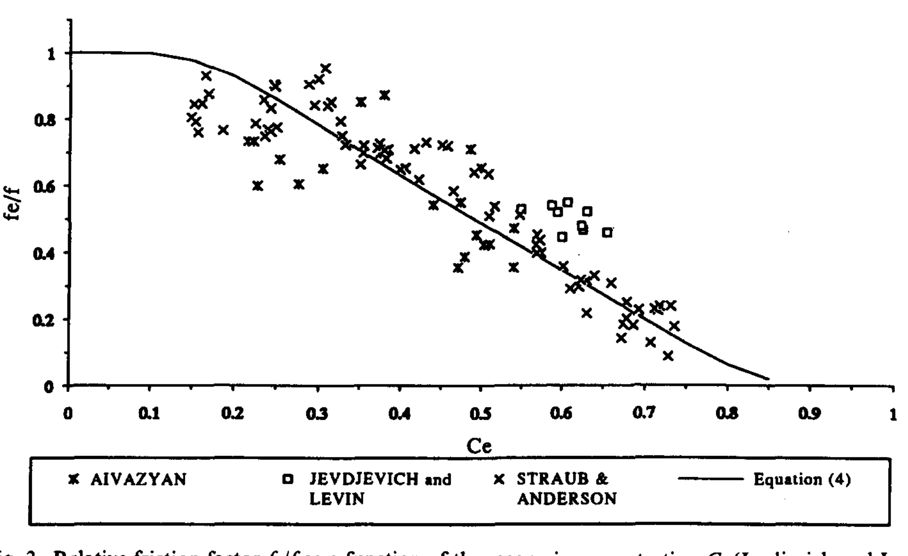


Fig. 3. Relative friction factor f_e/f as a function of the mean air concentration C_e (Jevdjevich and Levin, 1953; Straub and Anderson, 1958; Aivazyan, 1986).

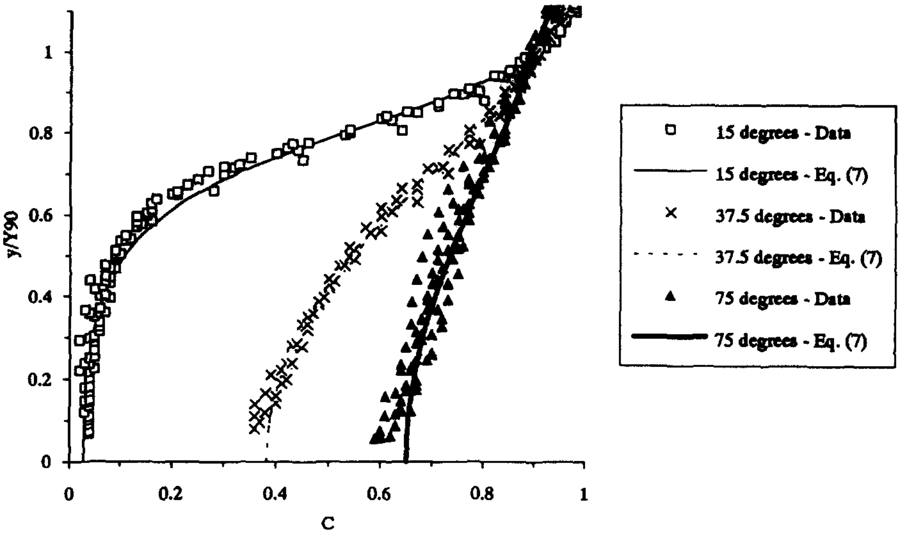


Fig. 4. Air concentration distributions in uniform areated flows for discharges in the range 0.136–0.595 m^2 (data from Straub and Anderson, 1958).

velocity V_{90}, defined as that at Y_{90}, is deduced from the continuity equation for water (Chanson, 1989). Computed values are presented in Table 1, column 4, and a reasonable correlation is:

$$V_{90} = \frac{q_w}{Y_{90} * (0.857 - 0.862 * C_e^{0.888})} \quad (6)$$

From the knowledge of C_e, V_{90} and Y_{90}, the air concentration distribution and the velocity distribution can be computed. Wood (1984) developed a diffusion model of the air bubbles within the air–water mixture that yields to:

$$C = \frac{B'}{B' + e^{-(G' * \cos\alpha * y'^2)}} \quad (7)$$

where $y' = y/Y_{90}$, and B' and G' are functions of the mean air concentration only (Table 1, columns 5 and 6). In Fig. 4 the data of Straub and Anderson (1958) are compared with equation (7). Next to the spillway bottom, however, the data of Cain (1978) obtained on the prototype spillway and Chanson (1988) on the spillway model depart from equation (7) and indicate that the air concentration tends to zero at the bottom (Fig. 5). The re-analysis of the data shows the presence of an air concentration boundary layer, in which the air concentration distribution may be estimated as:

$$C = C_b * \sqrt[3]{\frac{y}{\delta_{ab}}} \quad (8)$$

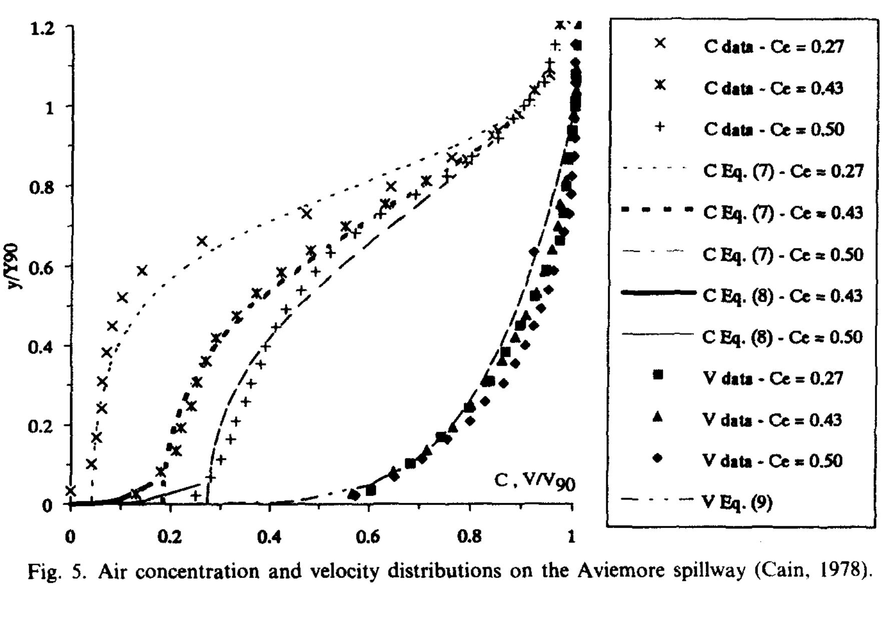
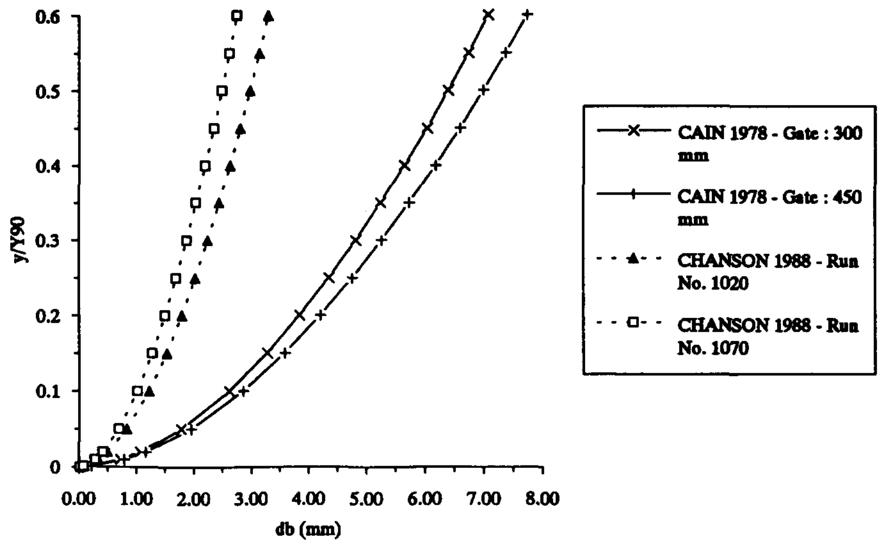


Fig. 5. Air concentration and velocity distributions on the Aviemore spillway (Cain, 1978).



Reference	Run	Y_{90} (m)	V_{90} (m/s)
CAIN (1978)	Gate 300 mm	0.250	20.0
CAIN (1978)	Gate : 450 mm	0.310	21.7
CHANSON (1988)	No. 1020	0.0394	9.9
CHANSON (1988)	No. 1070	0.0544	17.9

Fig. 6. Bubble size distribution in turbulent shear flows [equation (10)].

where C_b is the air concentration at the outer edge of the air concentration boundary layer and δ_{ab} is the air concentration boundary layer thickness estimated as $\delta_{ab} = 10$–15 mm (Chanson, 1989).

Cain (1978) performed velocity measurements in self-aerated flows on Aviemore spillways. The data were obtained with mean air concentrations in the range 0–50% (Fig. 5). To a first approximation the velocity distribution is independent of the mean air concentration and can be estimated as:

$$\frac{V}{V_{90}} = \left(\frac{y}{Y_{90}}\right)^{1/n} \quad (9)$$

where the exponent n for the roughness of the Aviemore spillway is: $n = 6.0$.

AIR–WATER INTERFACE AREA

In a turbulent shear flow the bubble size is determined by the balance between the surface tension force and the turbulent shear force. The author (Chanson, 1992a) developed a simple model to represent the air bubble size distribution across the flow that relates the maximum bubble size to the velocity gradient:

$$\frac{d_b}{Y_{90}} \sim \sqrt[3]{\frac{2 * n^2}{(\mathrm{We})_e}} * (y')^{2*(n-1)/n} \quad (10)$$

where d_b is the maximum bubble size, $(\mathrm{We})_e$ is the self-aerated flow Weber number defined as: $(\mathrm{We})_e = \rho_w * V_{90}^2 * Y_{90}/\sigma$, ρ_w is the water density and σ is the surface tension. On the prototype spillway, Cain (1978) reported bubble sizes in the range 3–20 mm for an air concentration less than 50%. For $C < 50\%$ the author (Chanson, 1988) observed bubble sizes in the range 0.3–4 mm on the spillway model. In Fig. 6 equation (10) is presented for model and prototype flow conditions. The results show good agreement with Cain's (1978) and Chanson's (1988) observations.

The specific air–water interface area a is defined as the total air–water interface area per unit volume. There is little information available on the bubble size distribution at a certain depth. To a first approximation, equation (10) may provide a first estimate of bubble sizes. High speed photographs (Halbronn *et al.*, 1953, Straub and Lamb, 1953) showed that the shape of air bubbles in self-aerated flows is approximately spherical and the specific interface area can be estimated as:

$$a = 6 * \frac{C}{d_b} \quad \text{air bubbles in water} \quad (11a)$$

$$a = 6 * \frac{1 - C}{d_b} \quad \text{water droplets in air} \quad (11b)$$

It must be noted that equation (11) assumes uniform bubble size at a given depth. When the effects of size distribution at a certain depth are not negligible, equation (11) is questionable.

In self-aerated flows the mean specific interface area a_{mean} is defined as:

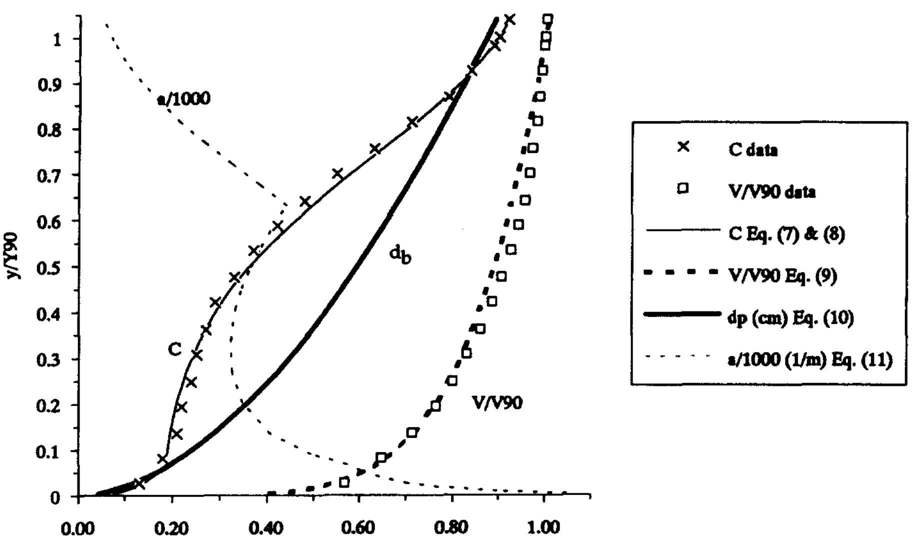


Fig. 7. Air concentration, velocity, bubble size and specific air–water interface area distributions on the Aviemore spillway (Cain, 1978), gate 300 mm, station 503.

$$a_{\text{mean}} = \frac{1}{Y_{90}} * \int_0^{Y_{90}} a * \mathrm{d}y \tag{12}$$

where Y_{90}, C, d_b and a are computed from equations (2), (7), (8), (10) and (11). An example of air concentration, velocity, bubble size and mean specific interface area distributions is presented in Fig. 7, and compared with the velocity and air concentration data measured by Cain (1978) on the Aviemore spillway.

On the prototype spillway, the flow depth is large compared to the air concentration boundary layer thickness. To a first approximation it is reasonable to neglect the effects of the air concentration boundary layer on the mean specific interface area. The author investigated the effects of the mean air concentration and Weber number on the specific interface area using equations (7), (8) and (11). The results indicate that equation (12) can be correlated as:

$$a_{\text{mean}} * Y_{90} = 4.193 * (C_e)^{1.538} * \sqrt[3]{(\mathrm{We})_e} \tag{13}$$

neglecting the effects of δ_{ab}/Y_{90}. As an example, values of $V_{90} = 20$ m/s and $Y_{90} = 1$ m imply $(\mathrm{We})_e = 5.4 \times 10^6$, and for a mean air concentration $C_e = 0.30$, equation (13) yields: $a_{\text{mean}} = 101\ \mathrm{m}^{-1}$.

Gulliver *et al.* (1990) re-analysed high-speed photographs taken during Straub and Anderson's (1958) experiments of a sectional view of self-aerated flows through a glass side wall. Their analysis of the photographs suggested that the specific area is independent of the distance from the channel surface, the velocity and the air concentration (i.e. $a = a_{\text{mean}}$), and that the mean specific area may be estimated as:

$$a_{\text{mean}} = 6.49 * \frac{C_e}{(d_b)_{\text{obs}}} \tag{14}$$

where $(d_b)_{\text{obs}}$ is the maximum bubble diameter observed: $(d_b)_{\text{obs}} = 2.7$ mm. For $C_e = 0.30$, equation (14) gives: $a_{\text{mean}} = 721\ \mathrm{m}^{-1}$. Comparison between equations (13) and (14) shows that equation (14) overestimates the air–water interface area. It is suggested that the photographic technique used by Gulliver *et al.* (1990) gives the bubble size distribution in the side wall boundary layer, that is characterized by higher shear stress and smaller bubble sizes than on the centre-line, hence larger interface areas. It must, however, be emphasized that equation (13) is based upon equation (11) and hence upon the assumption of uniform bubble size distribution at a certain depth.

APPLICATION

Equation (1) shows that the rate of aeration (e.g. oxygenation, nitrogenation) is directly proportional to the air–water interface area. In uniform self-aerated flows, all the flow properties, including the mean specific interface area, can be deduced from the channel slope, the discharge and the roughness height only. Equation (13) provides a simple correlation to estimate the air–water surface area.

Considering a 20° concrete spillway discharge $q_w = 10\ \mathrm{m}^2/\mathrm{s}$, the uniform average air concentration is: $C_e = 0.31$ [equation (3)]. For a non-aerated friction factor $f = 0.018$, the aerated friction factor is: $f_e = 0.0138$ [equation (4)]. The mean flow depth is: $d = 0.371$ m [equation (5)] and the characteristic velocity V_{90} equals 33.6 m/s [equation (6)]. The mean specific interface area becomes: $a_{\text{mean}} = 258\ \mathrm{m}^{-1}$ [equation (13)].

CONCLUSION

This paper summarizes the hydraulic properties of self-aerated flows. In the uniform flow region, the flow characteristics are functions of the slope, discharge and roughness only. A method is developed to estimate the air–water interface area distribution and the mean interface area. For prototype spillways the mean specific interface area is a function of the mean air concentration and aerated flow Weber number only.

For the estimation of the rate of air–water gas transfer, this method enables a rapid calculation of the interface area. However, it must emphasized that the calculation of the coefficient of transfer K_L is empirical and that further measurements of the air bubble size distribution in self-aerated flows are required.

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