Continuous or catastrophic solid–liquid transition in jammed systems

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Pasty materials encountered in industry and in earth science are intermediate between solids and liquids either in terms of their internal structure (disordered but jammed) or from a mechanical point of view. Our results indicate that the apparent behavior of a particulate system (soils, suspensions, clays, etc.) can range from liquid-like to soil or solid-like depending on the relative importance of the energy supplied to it and its "state of jamming" which evolves in time, and the transition from one state to another may appear either continuous or catastrophic. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1823531]

In nature and industry there exist a lot of materials such as muds, lavas, snow, cement, mayonnaise, foams, emulsions, melt chocolate, gels, varnishes, sewage sludges, etc., which can appear as solid or liquid depending on circumstances. The transition from one state to the other is of critical importance in various cases, in particular when it occurs abruptly. For example, self-compacting concrete, paints, or drilling fluids flow like low viscosity liquid, rapidly "gelify" at rest thus keeping coarse particles in suspension, but can liquefy again if submitted to sufficient stress and particles sedimentate. Also subaerial or submarine landslides can turn to devastating mud or debris flows as they move downwards[1] or sensitive clayey soils can suddenly liquefy and flow over large distances.[2] In mechanics these materials are generally considered as yield stress fluids, i.e., able to remain indefinitely at rest when submitted to an insufficient shear stress but capable to flow under a larger stress.[3] Along with granular flows, these materials arouse the interest of physicists because they could constitute a fourth state of matter (besides gas, liquids, and solids), i.e., the jammed systems.[4] This jammed state results from the fact that the elements (particles, bubbles, droplets, etc.) confined in a given volume form a continuous network of interactions throughout the sample, which must be broken for flow to occur. The usual, mechanical description of their behavior involves a simple continuous transition from a solid to a liquid behavior beyond the yield stress[3] but it was shown recently that this transition occurs in the form of a "viscosity bifurcation:"[5] under a shear stress (τ) smaller than a critical stress (τ_c) the fluid evolves more or less rapidly towards complete stoppage (infinite viscosity), while under a slightly larger stress the fluid evolves towards rapid flow with a low viscosity. In this context τ_c is an "apparent yield stress" which in particular increases with the time of rest before flow. Here we show experimentally that these different states and evolutions can be obtained with the same material depending on boundary conditions, restructuration time, or solid content. These trends are reproduced by numerical simulations using a simple model taking into account time changes of material viscosity. This provides a general frame for describing the continuous or catastrophic solid–liquid transition of industrial pastes or natural soils, and suggests a continuity between fluid mechanics and soil mechanics based on the evolution of the ratio of the energy supplied to the material and its "jamming energy."

Here it is valuable to focus on a typical yield stress fluid with marked thixotropic character, for the physical effects to be enhanced and more clearly identified. In this aim we used suspensions of bentonite (see Ref. 6), a natural clay, at different solid mass concentrations (ϕ). To reproduce the generic practical situation in which a force is suddenly applied to a mass of material, we poured a given volume of suspension in a dam reservoir at the top of an inclined channel (width: 34 cm, slope: 15°, rough surface to avoid slip). This volume of fluid was left at rest for some time (T) and then we abruptly lifted up the (vertical) gate. This induced a slight (vertical) shearing of the material which likely did not affect its downstream flow. Sedimentation did not occur within the duration of our tests with such materials. We carried out a series of systematic experiments under different times of rest (T), during which the fluid restructured, and for various solid fractions (ϕ). Mainly four flow types were observed (Fig. 1).

- Regime I: For small ϕ or short T the fluid was submitted to a rapid acceleration at the gate opening and flowed rapidly downstream; the flow aspect was that of a gravity current of a simple liquid[7] [Fig. 2(a)].
- Regime II: For intermediate ϕ or T the fluid initially

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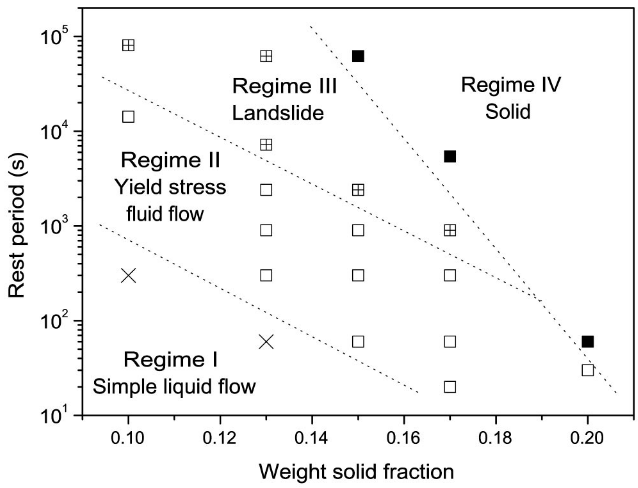

FIG. 1. Distribution of the different flow regimes observed in our experiments as a function of the restructuration time (rest period) and the solid fraction of the material [for a constant mass of fluid (3.7 kg)].

flowed rapidly then its velocity abruptly decreased after some distance; the layer of material slowly flowed during some time then completely stopped; the flow aspect was typical of a layer of a yield stress fluid released over an inclined plane and which stopped flowing when the wall shear stress balanced the yield stress[8,9] [Fig. 2(b)].

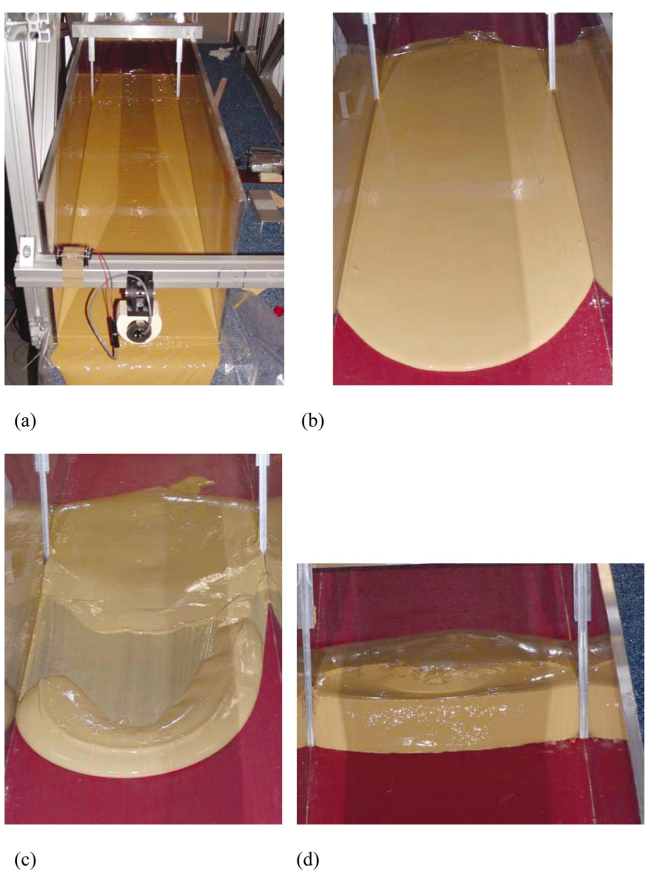

FIG. 2. (Color). Typical aspects of the flow in the different regimes: (a) Type I, simple liquid wave ($\phi=10\%;T=5$ min.); (b) Type II, yield stress fluid layer ($\phi=15\%;T=1$ min.); (c) Type III, landslide ($\phi=15\%;T=40$ min.); (d) Type IV, solid at rest ($\phi=15\%;T=1035$ min.).

- Regime III: For large ϕ or T the whole material started to flow, then separated into a tail which remained attached to the initial dam and stopped flowing, and a front which went on flowing downstream over some distance then stopped [Fig. 2(c)]. In the front part the shear was localized along the wall, the rest of material above more or less moving in mass (this was not a wall slip since the surge front part left behind it a thin layer of material); these characteristics were reminiscent of the aspect of some landslides; in some cases the moving part advanced over a larger distance than in the Regime II, for example reaching the downstream extremity of the channel with some velocity.
- Regime IV: In that case the material slightly deformed but never flowed downstream even after several days [Fig. 2(d)].

These flow regimes surprisingly correspond to the overall motion characteristics usually observed with material types *a priori* considered as distinct: (i) sudden release of a simple liquid, (ii) flow and stoppage of a simple pasty (yield stress) fluid, (iii) landslide, (iv) deformable solid. Our results show that, in contrast with the usual separation of materials in distinct categories, the apparent behavior of a jammed material under continuously varying conditions can cover all possible liquid or solid aspects.

In order to have a clearer view of the physical origin of these effects a possibility consists in attempting to reproduce them with a physically meaningful model. In this aim we can use a simple model[5] proposed for explaining viscosity bifurcation effects, and which relies on classical thixotropy concepts: the instantaneous viscosity ($\mu=\tau/\dot{\gamma}$, in which $\dot{\gamma}$ is the rate of shear) of the fluid is a function of the actual *state of structure* λ, which expresses as $\mu=\mu_0(1+\lambda^n)$, in which μ_0 and $n(>1)$ are two material parameters. The thixotropic character of the fluid has its origin in the time variations of the state of structure, which result from the competition between restructuration process at a rate mainly depending on material properties and destructuration process at a rate proportional to the flow rate ($\dot{\gamma}$): $d\lambda/dt=1/\theta-\alpha\lambda\dot{\gamma}$, in which θ is the characteristic time of restructuration. It may be shown[10] that under controlled stress such a material exhibits an apparent yield stress (τ_c) which increases with the structure state at the initial instant denoted λ_0: $\tau_c(\lambda_0)\approx\mu_0\lambda_0^{n-1}/\alpha\theta$ (for $\lambda_0\gg 1$). Moreover, under controlled shear stress, stable can only be obtained for a shear rate larger than the critical value $\dot{\gamma}_c=(n-1)^{1/n}/\alpha\theta$. We deduce that $\alpha\theta$ is an important characteristic time of the model, which in particular decreases as the solid fraction increases (since $\dot{\gamma}_c$ increases[11]). Previous thixotropy models had been mainly compared with few macroscopic data but it was shown with the help of coupled magnetic resonance imaging (MRI) rheometry that this model not only predicts the peculiar qualitative trends of paste flows (viscosity bifurcation, shear localization) but is also capable to well reproduce local flow characteristics under steady and transient conditions.[10] The above yield stress expression gives a first, though incomplete, explanation to the transition from Regime III to Regime IV: when the time of rest (or equivalently λ_0) or the

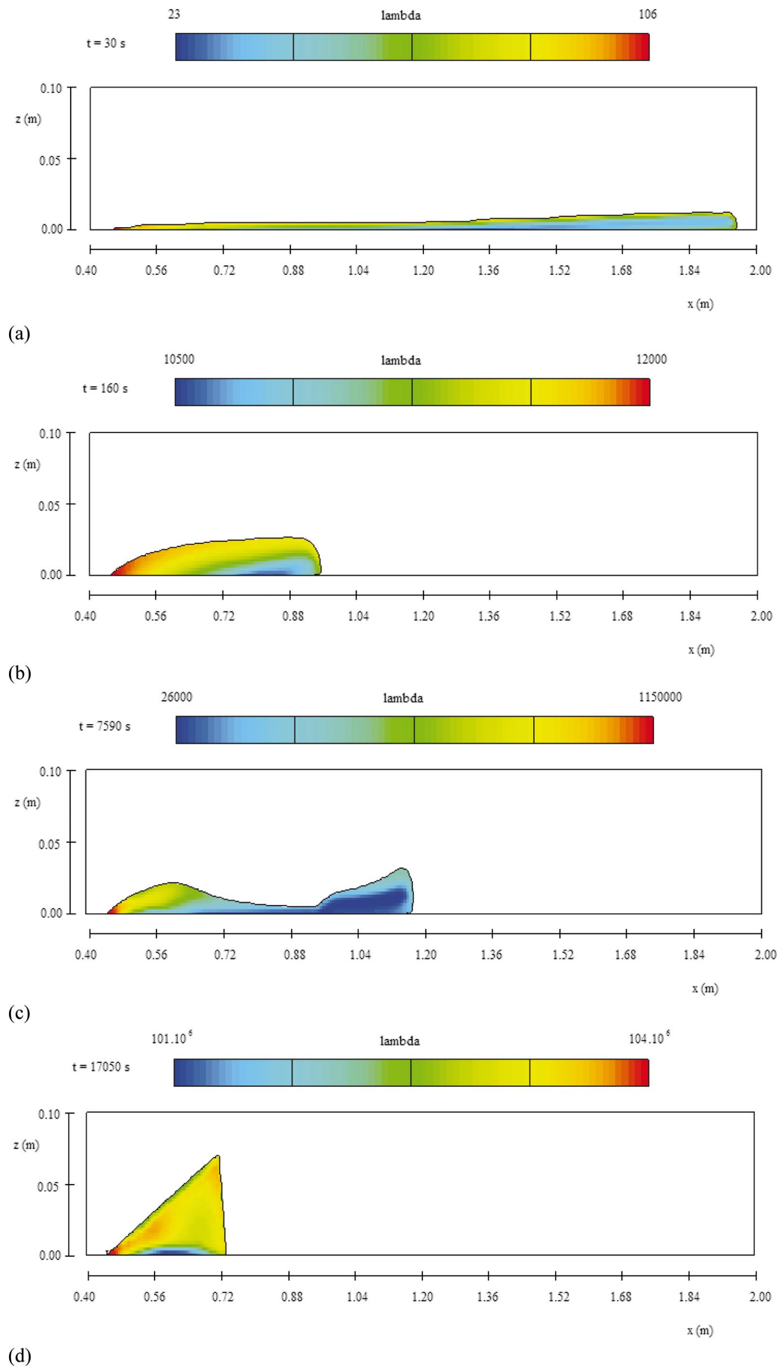

FIG. 3. (Color). Aspect of the flow from 2D simulations with FLOW 3D of the channelized flow after a certain time, as described in the text, under rheological conditions: $n=1.067$ (the value deduced from complete measurements in Ref. 10), $\mu_0=0.1$ Pa.s and (a) $\lambda_0=100, \alpha\theta=0.1$ (Regime I); (b) $\lambda_0=10\,000, \alpha\theta=0.05$ (Regime II); (c) $\lambda_0=1\,000\,000, \alpha\theta=0.025$ (Regime III); (d) $\lambda_0=100\,000\,000, \alpha\theta=0.0125$ (Regime IV). The colors correspond to the different values of the structure parameter.

concentration are too large (or equivalently when $\alpha\theta$ is too small) the apparent yield stress is so large that the mud does not start to flow.

We implemented this constitutive equation in the software FLOW 3D and simulated these flows (in 2D, i.e., without lateral walls) under different values of $\alpha\theta$ and initial values of the restructuration (λ_0). The computational results are qualitatively similar to the experimental ones. In particular, exactly the same flow regimes can be identified (Fig. 3) with decreasing $\alpha\theta$ (and thus increasing minimum apparent yield stress, which also increases with the solid fraction) and increasing λ_0. Thus a phase diagram similar to that of Fig. 1 could be drawn in a plane λ_0 vs $\alpha\theta$. Using now the same software applied to a 3D flow motion, hence taking into account the lateral walls, the results appeared to be even more similar to our experimental results. The qualitative agreement between our simulations and reality means that the basic physical ingredients of this model are likely to be at the origin of the observed phenomena. This suggests that, in contrast with usual models devoted to a specific field, a

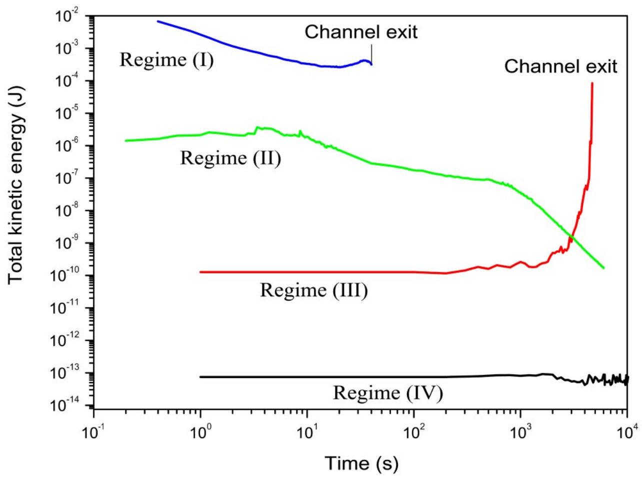

FIG. 4. (Color online). Total kinetic energy of the system as a function of time in the 2D simulations described in Fig. 3 and corresponding to the different flow regimes described in the text.

model of this type is capable to reproduce the different stages of flow of natural mass movements which turn from solid to liquid or from liquid to solid.

We computed the total kinetic energy of the system in time for the different flow regimes with the help of the software (cf. Fig. 4). The different regimes observed also correspond to strictly different time evolutions of the kinetic energy of the system, which provides a practical means for distinguishing them. A striking point is the peculiar evolution of the kinetic energy in Regime III: after a long period during which it does not vary, the kinetic energy dramatically increases and becomes much larger than the corresponding kinetic energy in Regime II under the same boundary conditions but with lower initial apparent viscosity. These trends are in perfect qualitative agreement with our experimental observations, which further confirms the ability of our model to describe various complex phenomena and in particular the above unexpected effect (Regime III) which precisely corresponds to catastrophic events observed in practice. Note that the flow characteristics in each regime completely differ: in Regime II the fluid is more or less homogeneously sheared; in Regime III, when the kinetic energy is large, the fluid is mainly sheared in a thin layer close to the solid plane which ensures the rapid motion of a rigid mass of material above it.

More generally, our results suggest that there is a continuity of a liquid suspension behavior towards the soil behavior as the solid fraction or the restructuration time (i.e., the *degree of jamming*) increases. The particle rearrangements during restructuration at rest leads to the formation of a stronger network of interactions somewhat analogous to the strengthening effect resulting from the increase of interaction number by unit volume when increasing the solid fraction. Existing MRI data concerning the internal flow characteristics of colloidal suspensions in steady state, simple shear lead to an analogous conclusion: the shear localizes in a region of thickness decreasing with the solid fraction or with the restructuration time while the shear rate remains almost uniform in the sheared region.[11] Thus, under a given rotation velocity (Ω) of the inner cylinder, the material appears as a liquid for a low degree of jamming, as a yield stress fluid for an intermediate value, and as a plastic material (with a shear localized along the larger stress region) for a large degree of jamming. However, a material with a large degree of jamming may now appear as a yield stress fluid or as a liquid for a sufficiently large value of Ω, or a material with a low degree of jamming may appear as plastic solid under a sufficiently low value of Ω. This shows that the flow regime mainly depends on the relative importance of the energy supplied to the system and the initial degree of jamming of the material.

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