unlikely that they will change to the Darcy-Weisbach model any time soon. As a result, practitioners and students must be familiar with the Hazen-Williams equation and its use.

Balance between Complexity and Accuracy

The comment by the discusser suggesting the necessity for balance between the computational effort required for accuracy and the complexity is indeed important. In many contexts, the effort required is in the implementation of program code and will arguably be worthwhile because it is made just once. Nevertheless, a detailed complexity analysis of the various methods available is long overdue.

Laminar Flow Friction Factor Formula Implementation

It is correctly stated that the Darcy-Weisbach friction factor for the laminar flow region can be written $f = 64/\mathsf{R}$. And it does indeed follow that the networks in which the flows in all pipes are laminar lead to systems of equations that are linear. However, the fact that the resistance factor for laminar flow is independent of flow is very important for the correct implementation of program coding to solve the systems of equations for which zero flows may occur (Elhay and Simpson 2011). Consider the case of a pipe in which the head loss is modeled by the Darcy-Weisbach formula. The head loss for this pipe is given for friction factor f by

$$h_f = f\frac{LV^2}{2gD} = f\frac{LQ^2}{2gDA^2} = f\frac{8LQ^2}{\pi^2 gD^5} \tag{1}$$

Now, for laminar flow

$$f = \frac{64}{\mathsf{R}} = \frac{64\nu}{VD} = \frac{16\pi\nu D}{Q} \tag{2}$$

and so, substituting Eq. (2) into Eq. (1) gives the head loss for laminar flow as

$$h_L = \left(\frac{128\nu}{\pi g}\right)\frac{L}{D^4}Q = r_L Q \tag{3}$$

with the obvious definition for the laminar flow resistance factor r_L. Because r_L is independent of flow, it is important, when writing program code dealing with laminar flows that are very small or zero, to use Eq. (3) rather than Eq. (1) because the term for the pipe on the diagonal of the top left block (which itself is a diagonal matrix) of the Jacobian is the differential with respect to Q of the term $r_L Q$. A zero flow does not affect this term, although using Eq. (1) with f replaced by $64/\mathsf{R}$ leads to division by zero when $Q = 0$.

Replacing the Smooth Cubic Spline Transition Region Approximation of Friction Factor by a Jump Discontinuity

It was also suggested by the discusser that for the transition from laminar to turbulent flow friction factors should be modeled by a jump discontinuity rather than by the smooth Dunlop cubic spline approximation used in the original paper. This suggestion brings with it some hazards. An assumption underpinning the Newton method is that the function being zeroed is differentiable and therefore continuous. Violating this assumption, as the discusser's suggestion does, may lead to unpredictable results and/or a divergence that may not otherwise occur. However, the shape of the Dunlop cubic spline could itself misdirect iterates. The Dunlop spline used in the original paper is precisely the one used in EPANET so that a fair comparison could be made between the formulas proposed in the paper and EPANET.

Alternative Approximations to the Colebrook-White Formula

Many different approximations for the Colebrook-White formula, which is used in the computation of the Darcy-Weisbach friction factor, exist in the literature. The discusser presents an interesting discussion of some of these, including the Swamee and Jain approximation used in the original paper. Perhaps one could choose, from among the more accurate approximations, one for which the the formulas in the Jacobian are more comfortable. Once again, the authors used the Swamee and Jain approximation to allow a fair comparison with EPANET.

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Discussion of "Inception Point Relationship for Flat-Sloped Stepped Spillways" by Sherry L. Hunt and Kem C. Kadavy

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The authors presented an interesting contribution to the topic of stepped spillways on moderate slopes. Their work serves as a reminder that the flow properties on moderate-slope stepped spillways, including embankment stepped spillways, may differ from those observed on steep stepped spillways. The authors compared new physical data in terms of the location L_i of the inception point of free-surface aeration with a correlation originally developed for steep stepped spillways [Eq. (1), Chanson (1994)] and a newer correlation Eq. (2):

$$\frac{L_i}{k_s} = 9.8(\sin\,\theta)^{0.08}\mathsf{F}_*^{0.71} \tag{1}$$

$$\frac{L_i}{k_s} = 6.1(\sin\,\theta)^{0.08}\mathsf{F}_*^{0.86} \tag{2}$$

where θ = channel slope; k_s = step roughness height [$k_s = h\,\cos(\theta)$]; and F_* = Froude number defined in terms of the step roughness height and reduced gravity, $\mathsf{F}_* = q/\sqrt{g(\sin\,\theta)k_s^3}$, with g the gravity acceleration. Eqs. (1) and (2) are compared in Fig. 1 with a broader range of physical data, including data from the original paper (Baker 1994; Haddad 1998; Chanson and Toombes 2002; Gonzalez and Chanson 2004; Carosi and Chanson 2008). Design engineers must be aware of the limitations of the correlations, as shown in Fig. 1, in which a prototype data set is

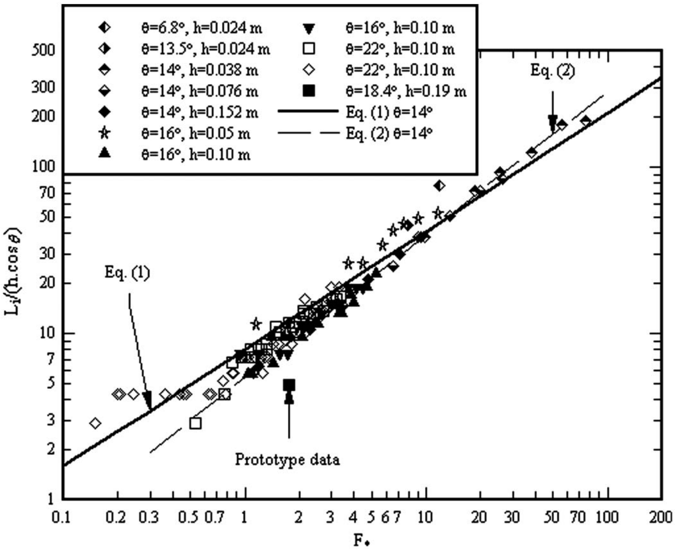

Fig. 1. Comparison of Eqs. (1) and (2) for dimensionless location L_i of the inception point of free-surface aeration for moderate-slope stepped spillway using a prototype data set [$\theta = 18.4°$, $h = 0.19$ m (Baker 1994)] and several laboratory data sets [$\theta = 6.8$ and 13.5° and $h = 0.024$ m (Haddad 1998); $\theta = 14°$ and $h = 0.038$, 0.076, and 0.152 m (the authors'); $\theta = 16°$ and $h = 0.05$ and 0.1 m (Chanson and Toombes 2002; Gonzalez and Chanson 2004); and $\theta = 22°$ and $h = 0.05$ and 0.1 m (Chanson and Toombes 2002; Carosi and Chanson 2008)]

Fig. 2. High-shutter-speed photograph, looking upstream for $d_c/h = 1.86$, highlighting the rapid onset of self-aeration for free-surface aeration down a very flat stepped channel [$\theta = 3.4°$, $q = 0.15\ \mathrm{m^2/s}$, $h = 0.0715$ m (Chanson and Toombes 2002)]

reported. Both correlations overestimated the location of the inception point by a factor of two to three. Fig. 1 is presented with a log-log scale, which tends to smooth the data scatter and inaccuracy of the preceding correlations. Whereas Eqs. (1) and (2) might be used as predesign guidelines, Fig. 1 highlights the limitations of these semiempirical correlations to predict accurately the prototype flow properties and the needs for solid physical modeling during the design process.

The authors' original contribution and the preceding review implicitly assume that the location of the inception point of free-surface aeration corresponds to the intersection of the outer edge of the developing boundary layer with the free surface. Whereas this assumption is widely accepted for moderate to steep smooth and stepped chutes, it has been argued that it is untrue for very flat slopes ($\theta < 5$–10°) (Anwar 1994; Chanson 1997). On smooth chutes, Anwar (1994) stressed that the development of free-surface instabilities enables the onset of free-surface aeration. Physical observations showed that free-surface aeration takes place very rapidly along a flat-slope channel and that air entrainment may occur far upstream of the intersection of the outer edge of the developing turbulent boundary layer with the free surface (Arreguin and Echavez 1986; Chanson 1997). On a flat stepped chute ($\theta = 3.4°$, $h = 0.0715$, and 0.143 m), the observations of Chanson and Toombes (2002) showed that the inception point of free-surface aeration took place upstream of the intersection between the free surface and the boundary layer's outer edge (Fig. 2). For example, Eqs. (1) and (2) overestimate the inception-point position by a factor of two for the experiment shown in Fig. 2. The onset of self-aeration on very flat smooth and stepped chutes is not trivial. It is believed to be linked with turbulent structures acting next to the free surface, including longitudinal vortices (Levi 1965; Anwar 1994).

In summary, the empirical correlations fit well with a broader range of physical data and may provide suitable predesign guidelines for stepped spillways with slopes $\theta > 10°$. However, their accuracy is limited, and the design engineers must fully comprehend their limitations, as illustrated in Fig. 1, in which a prototype data set differs significantly from the predictive correlations. Physical modeling is strongly recommended during the final design stages of a stepped spillway. Furthermore, the authors' correlations should not be used for very flat slope stepped chutes ($\theta < 5$–10°) because the onset of free-surface aeration takes place upstream of the intersection of the outer edge of the turbulent boundary layer with the free surface. Self-aeration on very flat slope chutes is believed to be linked with the interactions of vortical structures with the free surface, yielding to its deformation and air entrapment.

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Closure to "Inception Point Relationship for Flat-Sloped Stepped Spillways" by Sherry L. Hunt and Kem C. Kadavy

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The writers thank the discusser for the interest shown and for the opportunity to further discuss the inception-point relationship for flat-sloped stepped spillways. It is a pleasure to have an open dialogue with other researchers who have similar research.

The writers agree with the discusser in that design engineers should be aware of the limitations provided by the development of inception-point relationships, especially since most of the data collected by researchers, including data in the original paper, were visually based observations (Chanson and Toombes 2002; Gonzalez 2005; Gonzalez and Chanson 2007; Chanson and Carosi 2007; Meireles and Matos 2009). The subjectivity of these observations can lead to ±5% or more error in these measurements, as indicated in the original paper. The error becomes more magnified the closer the inception point is to the spillway crest. Additionally, these inaccuracies are less noticeable in log-log scale, as the discusser indicated.

The discusser stated that an overestimation of the inception-point location could be as much as a factor of two or three with prototype data, and supported this conclusion with a data point from Baker (1994). The writers have conducted recent research within a similar range of parameters [i.e., $\theta = 18.4°$; $h = 0.152$ and 0.305 m; and unit discharges (q) from 0.146 to 1.88 $\mathrm{m^3/s \cdot m}$] and found relatively good agreement with the current relationship (Hunt and Kadavy 2011), as illustrated in Fig. 1. One possibility for the difference in results is that Baker (1994) and Baker and Gardiner (1994) conducted research in a trapezoidal channel with wedge block sloped steps and a turbulent high-velocity inlet, whereas the writers conducted research in a rectangular channel with uniform steps and an uncontrolled broad-crested inlet, providing for a smooth flow entrance condition with flow passing through critical depth. Chanson (2006) and Pfister and Hager (2011) indicated that the inception point can be affected by turbulence generated by inflow conditions. Additional data recorded by the writers from this new prototype testing facility are expected to be presented in future publications.

The discusser is correct, in that the writers assumed that the inception-point location of the free-surface aeration corresponded to the intersection of the outer edge of the developing boundary layer with the free surface. This assumption is widely accepted in the research arena for moderate to steep smooth and stepped chutes, as the discusser indicates, but it is also supported by velocity profile data illustrating the change in the velocity profile shape as the turbulent boundary layer develops. For example, Fig. 2 shows velocity profiles near the broad-crested entrance, $L/L_i = 0.10$, to slightly downstream of the inception point, $L/L_i = 1.05$, for a prototype stepped spillway with $\theta = 18.6°$, $q = 1.86\ \mathrm{m^3/s \cdot m}$, and $h = 0.152$ m. Velocities were collected by using a back-flushing pitot tube coupled with a differential pressure transducer. Moving from $L/L_i = 0.10$ to $L/L_i = 1.05$, the velocity profiles became increasingly less uniform, indicating the growing turbulent boundary layer. The turbulent boundary layer reached the free surface when the velocity profiles became similarly shaped near the inception point, $L/L_i = 1.0$. The velocity profiles $L/L_i = 0.67$ to $L/L_i = 1.0$ also showed the irregularity of the free surface attributable to the approaching boundary layer. The velocity data near the surface was affected by the pitot tube being in and out of the water because of the irregular free surface. These velocity data points are not valid measurements but were included in Fig. 2 for illustration purposes. The onset of free-surface instabilities, as illustrated in Fig. 3, were also noted by Hunt and Kadavy (2008) for $\theta = 14°$. Design engineers should be aware that these instabilities upstream of the intersection between the free surface and the boundary layer outer edge may occur and should account for these instabilities in the design of stepped spillways, especially for extremely flat slopes ($\theta < 5$ to 10°), as the discusser points out.

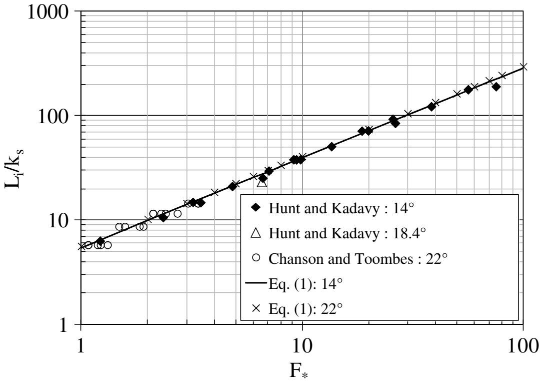

Fig. 1. Prototype data representing $\theta = 18.4°$, $h = 0.305$ m, and unit discharge $(q) = 1.79\ \mathrm{m^3/s \cdot m}$ from a recent study conducted by Hunt and Kadavy (2011) compared to data from Chanson and Toombes (2002) and the original paper

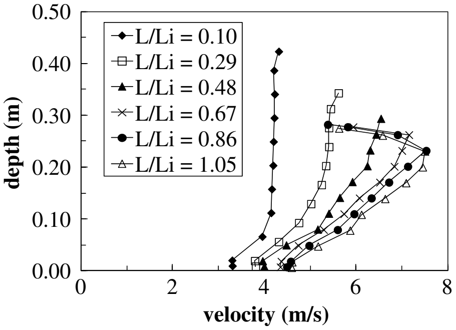

Fig. 2. Velocity profiles for a stepped spillway with $\theta = 18.4°$ and $h = 0.152$ m tested at a unit discharge $(q) = 1.86\ \mathrm{m^3/s \cdot m}$

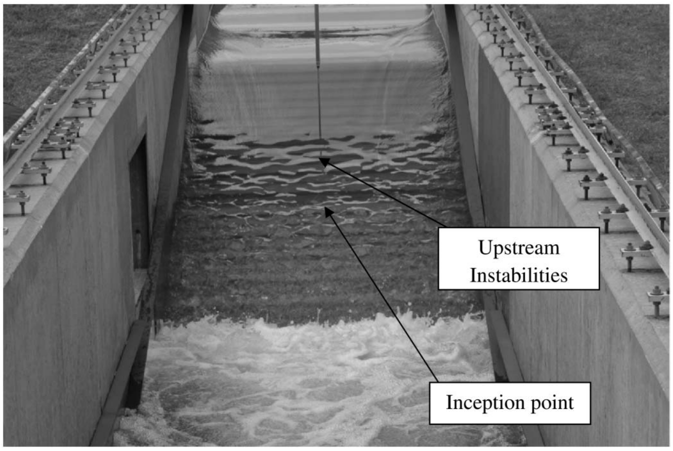

Fig. 3. Inception point and upstream instabilities for $0.28\ \mathrm{m}^3/\mathrm{s} \cdot \mathrm{m}$ flow overtopping 14° stepped spillway with step heights of 0.038 m

In conclusion, a newly collected prototype data point fit the empirical correlation presented in the original paper well, as illustrated in Fig. 1. The writers feel with appropriate engineering judgment that this correlation could be used for the final design stages of a stepped spillway with a broad-crested entrance, given the supporting prototype data; however, the writers agree with the discusser that the correlation may be better suited for stepped spillways with slopes $\theta > 10°$. The potential of air entrapment that can lead to the onset of free-surface aeration upstream of the intersection of the outer edge of the turbulent boundary layer with the free surface is a concern for extremely flat slopes (i.e., $\theta < 10°$), as indicated by the discusser.

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