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Short Communication

Explicit equations for critical depth in open channels with complex compound cross sections. A discussion

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ABSTRACT

In open channel hydraulics, the notion of critical flow conditions and critical depth are not restricted to open channel flows with hydrostatic pressure distributions. This contribution shows an extension of the concept of critical flow conditions linked with the minimum specific energy, as introduced by Bakhmeteff [1] and extended by Liggett [9] and Chanson [5]. It demonstrated that the critical depth may be defined more broadly including when the pressure field is not hydrostatic.

The authors developed a series of expression for the critical depth in open channels with irregular channel cross-sections. It is believed that the article thrust and its conclusion missed a key point. The work is restricted to an open channel flow motion with hydrostatic pressure distributions although it was not stated explicitly. In turn the readers could be misled to assume that the results may apply to a wide range of open channel situations including weirs, spillway crests, and gates. Fig. 1 illustrates some flow situations in which the flow is critical but the pressure distributions are not hydrostatic. It is shown herein that the critical depth may be derived more broadly for flow situations with non-hydrostatic pressure distributions.

At critical flow conditions, the specific energy is minimum [1,2,9]. The cross-sectional averaged specific energy H is commonly expressed following Chanson [5]

$$H=\frac{1}{A}\times\iint\left(\frac{v_x^2}{2\times g}+z+\frac{P}{\rho\times g}\right)\times dA=\beta\times\frac{V^2}{2\times g}+\Lambda\times y \tag{1}$$

where A is the wetted cross-section area, y the flow depth, P the pressure, V the depth-averaged velocity, v_x the longitudinal velocity component, z the vertical elevation above the crest, g the gravity constant, ρ the water density, β the Boussinesq momentum correction coefficient, and Λ a pressure correction coefficient

$$\Lambda=\frac{1}{2}+\frac{1}{A}\times\iint\frac{P}{\rho\times g\times y}\times dA \tag{2}$$

For an uniform flow above a flat rectangular invert with streamlines parallel to the crest, the velocity distribution is uniform ($\beta=1$), the pressure is hydrostatic ($\Lambda=1$), and Eq. (1) equals the classical result: $H=1.5\times y_c$ where y_c is the critical depth. For an irregular channel cross-section with uniform velocity distribution ($\beta=1$) and hydrostatic pressure ($\Lambda=1$), the differentiation of Eq. (1) with respect of the flow depth gives

$$\frac{Q^2}{g\times\left(A^3/B\right)}=1 \quad \text{Hydrostatic pressure distribution} \tag{3}$$

at critical flow conditions [8,3]. In many practical applications, the velocity distributions are not uniform, the streamlines were not parallel to the invert everywhere (Fig. 1) and the pressure gradient is not hydrostatic. In turn Eq. (3) becomes inapplicable.

In the general case, the specific energy is minimum at critical flow conditions [8,9]. For a wide channel, the flow depth y must satisfy one of four physical solutions [5]

$$\frac{y}{H}\times\Lambda=\sqrt[3]{\frac{1-2\times\beta\times{C_D}^2\times\Lambda^2}{27}+\Lambda^3\times\sqrt{\Delta}}+\sqrt[3]{\frac{1-2\times\beta\times{C_D}^2\times\Lambda^2}{27}-\Lambda^3\times\sqrt{\Delta}}+\frac{1}{3}\quad \Delta>0 \tag{4a}$$

$$\frac{y}{H}\times\Lambda=\frac{2}{3}\quad \Delta=0 \tag{4b}$$

$$\frac{y}{H}\times\Lambda=\frac{2}{3}\times\left(\frac{1}{2}+\cos\frac{\varepsilon}{3}\right)\quad \Delta<0,\ \text{Solution S1} \tag{4c}$$

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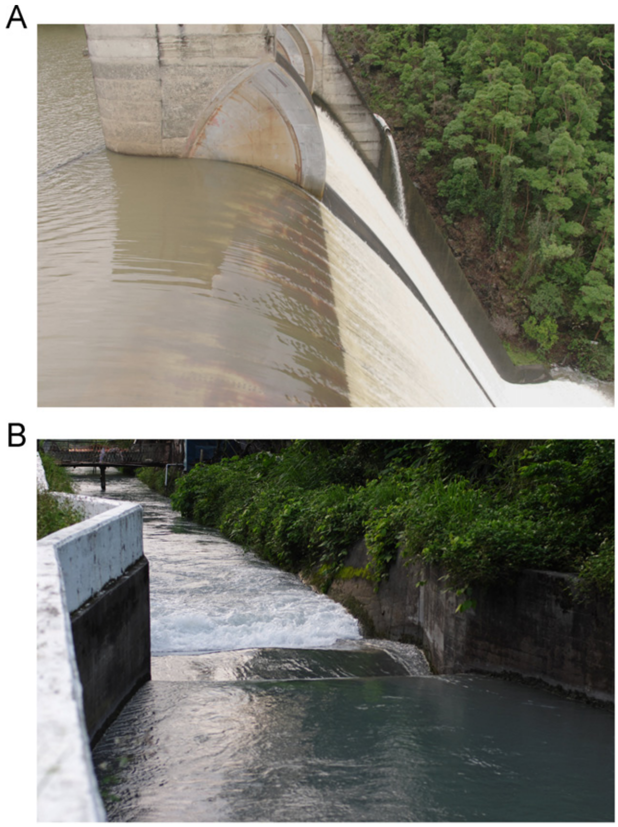

Fig. 1. Critical flow conditions in open channels. (A) Overflow above the Little Nerang dam spillway crest on 28 December 2010—Head above crest: 0.4 m, q=0.43 m^2/s, Q=14 m^3/s. (B) Undular flow in a Venturi flume along an irrigation canal near Hualien on 10 November 2010—Flow from foreground to background.

$$\frac{y}{H}\times\Lambda=\frac{2}{3}\times\frac{1-\cos(\varepsilon/3)+\sqrt{3\times\left(1-(\cos(\varepsilon/3))^2\right)}}{2}\qquad \Delta<0,\ \text{Solution S3} \tag{4d}$$

where C_D is a dimensionless discharge, $\cos\varepsilon=1-2\times\beta\times C_D^2\times\Lambda^2$ and the discriminant Δ equals

$$\Delta=\frac{4\times\beta\times C_D^2\times\Lambda^2}{(3\times\Lambda)^6}\times\left(\beta\times C_D^2\times\Lambda^2-1\right) \tag{5}$$

Eq. (4) expresses the flow depth at critical flow conditions in the general case when $\beta>1$ and $\Lambda\neq1$. Eq. (4) is tested against a series of experimental data in Fig. 2 with the dimensionless water depth $y\times\Lambda/H_1$ at critical flow conditions being a function of the dimensionless discharge $\beta\times C_D^2\times\Lambda^2$, where β and Λ were calculated based upon the pressure and velocity distribution data and C_D was calculated

$$C_D=\frac{\int_0^y v_x\times dz}{\sqrt{g\times((2/3)\times H)^3}} \tag{6}$$

In Fig. 2, the physical data showed a good agreement with the theory, in particular with the solutions $S1$ and $S3$ ($\Delta<0$) (Fig. 2).

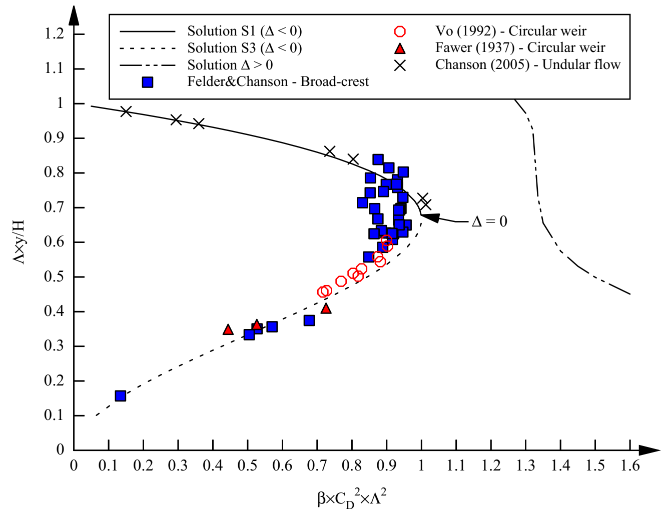

Fig. 2. Dimensionless critical depth $y\times\Lambda/H$ as a function of the dimensional discharge $\beta\times C_D^2\times\Lambda^2$—Comparison between analytical solutions (Eq. (4)), broad-crested weir data [7], circular crested weir data [6,10] and undular flow data [4].

The agreement between Eq (4) and data highlighted that the notion of critical flow conditions may be applied broadly to open channel flows with non-uniform velocity and non-hydrostatic pressure distributions.

In summary, the notion of critical flow conditions and critical depth are not restricted to open channel flows with hydrostatic pressure distributions. This discussion showed an extension of the concept of critical flow conditions linked with the minimum specific energy, as introduced by Bakhmeteff [1]. It demonstrated that the critical depth may be defined more broadly including when the pressure field is not hydrostatic.

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