CHANSON, H. (1999). "The Hydraulics of Open Channel Flow : An Introduction." Butterworth-Heinemann, 1st edition, London, UK, 512 pages (ISBN 0 340 74067 1).

10

Sediment transport mechanisms 1. Bed-load transport

10.1 Introduction

When the bed shear stress exceeds a critical value, sediments are transported in the form of bed-load and suspended load. For bed-load transport, the basic modes of particle motion are *rolling* motion, *sliding* motion and *saltation* motion (Fig. 10.1).

In this chapter, formulations to predict the bed load transport rate are presented. Figure 10.2 shows a natural stream subjected to significant bed-load transport.

> ***Notes***
> 1. Saltation refers to the transport of sediment particles in a series of irregular jumps and bounces along the bed (Fig. 10.1(a)).
> 2. In this section, predictions of bed-load transport are developed for plane bed. Bed form motion and bed form effects on bed-load transport are not considered in this section (see Chapter 12).

Definitions

The sediment transport rate may be measured by weight (units: N/s), by mass (units: kg/s) or by volume (units: m^3/s). In practice the sediment transport rate is often expressed per unit width and is measured either by mass or by volume. These are related by:

$$\dot{m}_s = \rho_s q_s \tag{10.1}$$

where $\dot{m}_s$ is the mass sediment flow rate per unit width, q_s is the volumetric sediment discharge per unit width and ρ_s is the specific mass of sediment.

10.2 Empirical correlations of bed-load transport rate

10.2.1 Introduction

Bed load transport occurs when the bed shear stress τ_o exceeds a critical value $(\tau_o)_c$. In

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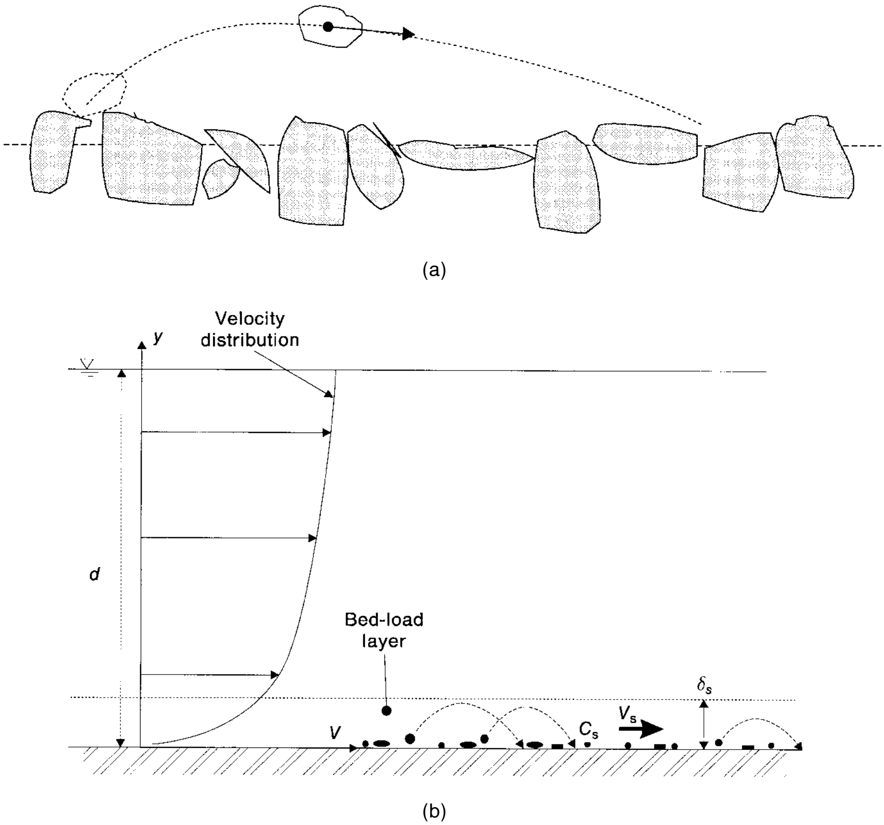

Fig. 10.1 Bed-load motion. (a) Sketch of saltation motion. (b) Definition sketch of the bed-load layer.

dimensionless terms, the condition for bed-load motion is:

$$\tau_* > (\tau_*)_c \qquad \text{Bed-load transport} \tag{10.2}$$

where τ_* is the Shields parameter (i.e. $\tau_* = \tau_o/(\rho(\mathbf{s}-1)gd_s)$) and $(\tau_*)_c$ is the critical Shields parameter for initiation of bed load transport (Fig. 8.4).

10.2.2 Empirical bed load transport predictions

Many researchers attempted to predict the rate of bed load transport. The first successful development was proposed by P.F.D. du Boys in 1879. Although his model of sediment transport was incomplete, the proposed relationship for bed-load transport rate (Table 10.1) proved to be in good agreement with a large amount of experimental measurements.

Subsequently, numerous researchers proposed empirical and semi-empirical correlations. Some are listed in Table 10.1. Graf (1971) and van Rijn (1993) discussed their applicability. The most notorious correlations are the Meyer-Peter and Einstein formulae.

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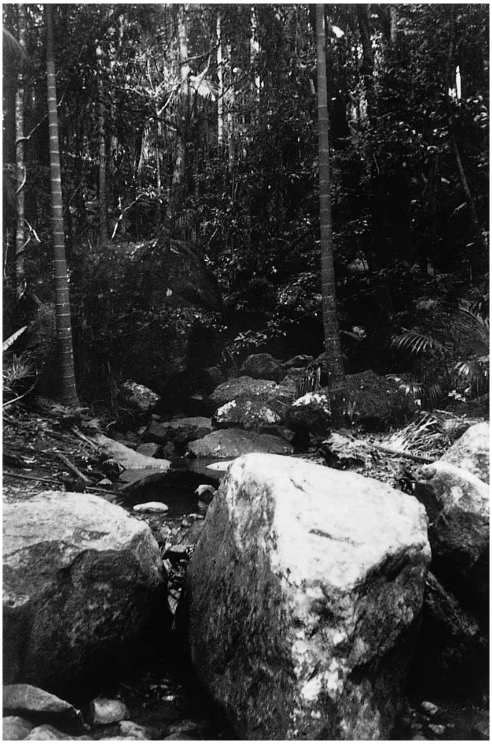
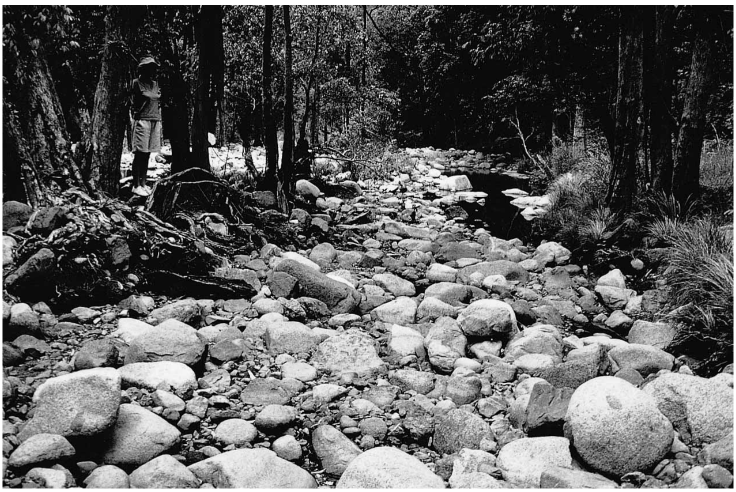


(a)



(b)

Fig. 10.2 Bed-load transport in natural streams. (a) South Korrumbyn Creek NSW, Australia looking upstream (25 April 1997). Stream bed 2 km upstream of the Korrumbyn Creek weir, fully-silted today (Appendix A2.1). (b) Cedar Creek QLD, Australia looking downstream (9 December 1992) (courtesy of Mrs J. Hacker). Note the coarse material left after the flood.

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Table 10.1 Empirical and semi-empirical correlations of bed load transport

Reference (1)	Formulation (2)	Range (3)	Remarks (4)
Boys (1879)	$q_s = \lambda\tau_o(\tau_o - (\tau_o)_c)$		λ was called the characteristic sediment coefficient.
	$\lambda = \frac{0.54}{(\rho_s - \rho)g}$ Schoklitsch (1914)		Laboratory experiments with uniform grains of various kinds of sand and porcelain.
	$\lambda \propto d_s^{-3/4}$ Straub (1935)	$0.125 < d_s < 4\,\text{mm}$	Based upon laboratory data.
Schoklitsch (1930)	$q_s = \lambda'(\sin\theta)^k(q - q_c)$ $q_c = 1.944 \times 10^{-2} d_s(\sin\theta)^{-4/3}$	$0.305 < d_s < 7.02\,\text{mm}$	Based upon laboratory experiments.
Shields (1936)	$\frac{q_s}{q} = 10\frac{\sin\theta}{s}\frac{\tau_o - (\tau_o)_c}{\rho g(\mathbf{s}-1)d_s}$	$1.06 < \mathbf{s} < 4.25$ $1.56 < d_s < 2.47\,\text{mm}$	
Einstein (1942)	$\frac{q_s}{\sqrt{(\mathbf{s}-1)gd_s^3}} = 2.15\exp\left(-0.391\frac{\rho(\mathbf{s}-1)gd_s}{\tau_o}\right)$	$\frac{q_s}{\sqrt{(\mathbf{s}-1)gd_s^3}} < 0.4$ $1.25 < \mathbf{s} < 4.25$ $0.315 < d_s < 28.6\,\text{mm}$	Laboratory experiments. Weak sediment transport formula for sand mixtures. Note: $d_s \approx d_{35}$ to d_{45}.
Meyer-Peter (1949,1951)	$\frac{\dot{m}^{2/3}\sin\theta}{d_s} - 9.57(\rho g(\mathbf{s}-1))^{10/9} = 0.462(\mathbf{s}-1)\frac{(\rho g(\dot{m}_s)^2)^{2/3}}{d_s}$	$1.25 < \mathbf{s} < 4.2$	Laboratory experiments. Uniform grain size distribution.
	$\frac{q_s}{\sqrt{(\mathbf{s}-1)gd_s^3}} = \left(\frac{4\tau_o}{\rho(\mathbf{s}-1)gd_s} - 0.188\right)^{3/2}$		Laboratory experiments. Particle mixtures. Note: $d_s \approx d_{50}$.
Einstein (1950)	Design chart $\frac{q_s}{\sqrt{(\mathbf{s}-1)gd_s^3}} = \not{f}\left(\frac{\rho(\mathbf{s}-1)gd_s}{\tau_o}\right)$	$\frac{q_s}{\sqrt{(\mathbf{s}-1)gd_s^3}} < 10$ $1.25 < \mathbf{s} < 4.25$ $0.315 < d_s < 28.6\,\text{mm}$	Laboratory experiments. For sand mixtures. Note: $d_s \approx d_{35}$ to d_{45}.
Schoklitsch (1950)	$\dot{m}_s = 2500(\sin\theta)^{3/2}(q - q_c)$ $q_c = 0.26(\mathbf{s}-1)^{5/3}d_{40}^{3/2}(\sin\theta)^{-7/6}$		Based upon laboratory experiments and field measurements (Danube and Aare rivers).
Nielsen (1992)	$\frac{q_s}{\sqrt{(\mathbf{s}-1)gd_s^3}} = \left(\frac{12\tau_o}{\rho(\mathbf{s}-1)gd_s} - 0.05\right)\sqrt{\frac{\tau_o}{\rho(\mathbf{s}-1)gd_s}}$	$1.25 < \mathbf{s} < 4.22$ $0.69 < d_s < 28.7\,\text{mm}$	Re-analysis of laboratory data.

Notes: $\dot{m}$ = mass water flow rate per unit width; $\dot{m}_s$ = mass sediment flow rate per unit width; q = volumetric water discharge; q_s = volumetric sediment discharge per unit width; $(\tau_o)_c$ = critical bed shear stress for initiation of bed load.

Note

P.F.D. du Boys (1847-1924) was a French hydraulic engineer. In 1879, he proposed a bed load transport model, assuming that sediment particles move in sliding layers (Boys 1879).

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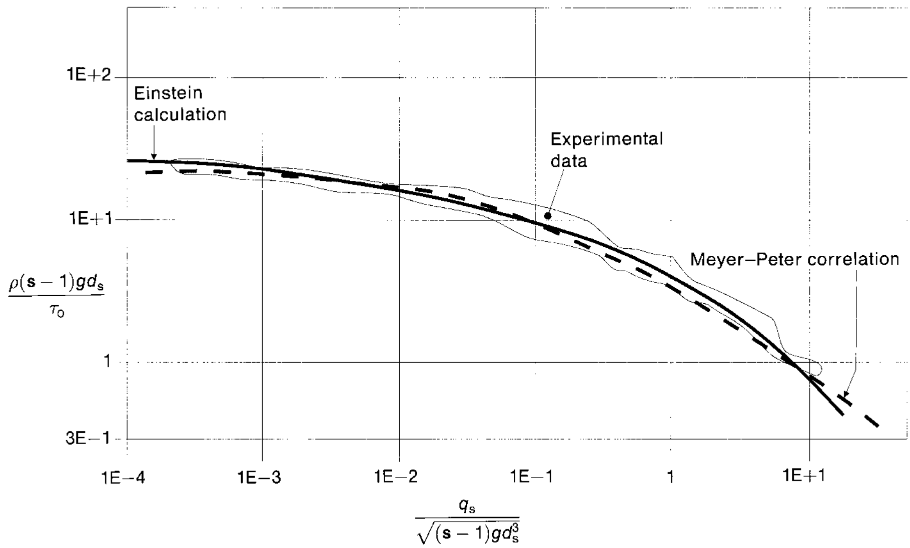

Fig. 10.3 Bed-load transport rate: comparison between Meyer-Peter formula, Einstein calculation and laboratory data (Meyer-Peter *et al.* 1934, Gilbert 1914, Chien 1954).

Discussion

The correlation of Meyer-Peter (1949, 1951) has been popular in Europe. It is considered most appropriate for wide channels (i.e. large width to depth ratios) and coarse material. Einstein's (1942, 1950) formulations derived from physical models of grain saltation, and they have been widely used in America. Both the Meyer-Peter and Einstein correlations give close results (e.g. Graf 1971, p. 150), usually within the accuracy of the data (Fig. 10.3).

It must be noted that empirical correlations should not be used outside of their domain of validity. For example, Engelund and Hansen (1972) indicated explicitly that Einstein's (1950) bed-load transport formula differs significantly from experimental data for large amounts of bed load (i.e. $q_s/\sqrt{(\mathbf{s}-1)gd_s^3} > 10$).

10.3 Bed-load calculations

10.3.1 Presentation

Bed-load transport is closely associated with inter-granular forces. It takes place in a thin region of fluid close to the bed (sometimes called the *bed-load layer* or saltation layer) (Fig. 10.1, 10.4). Visual observations suggest that the bed-load particles move within a region of less than 10 to 20 particle-diameter heights.

During the bed-load motion, the moving grains are subjected to hydrodynamic forces, gravity force and inter-granular forces. Conversely the (submerged) weight of the bed load is transferred as a normal stress to the (immobile) bed grains. The

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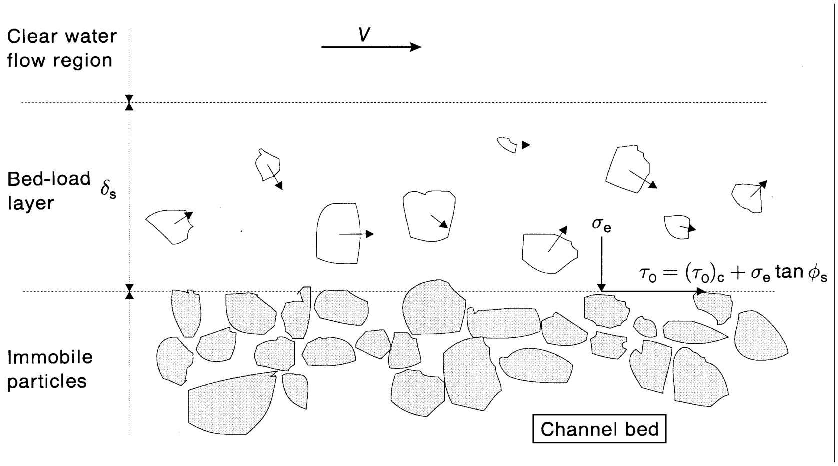

Fig. 10.4 Sketch of bed-load motion at equilibrium.

normal stress σ_e exerted by the bed load on the immobile bed particles is called the *effective stress* and it is proportional to:

$$\sigma_e \propto \rho(\mathbf{s} - 1)g \cos\theta C_s \delta_s \tag{10.3}$$

where δ_s is the bed-load layer thickness, C_s is the volumetric concentration of sediment in the bed-load layer and θ is the longitudinal bed slope.

The normal stress increases the frictional strength of the sediment bed and the boundary shear stress applied to the top layer of the immobile grains becomes:

$$\tau_o = (\tau_o)_c + \sigma_e \tan\phi_s \tag{10.4}$$

where $(\tau_o)_c$ is the critical bed shear stress for initiation of bed load and ϕ_s is the angle of repose.

Notes

1. The concept of effective stress and associated bed shear stress (as presented above) derives from the work of Bagnold (1956, 1966).
2. For sediment particles, the angle of repose ranges usually from 26° to 42° and hence $0.5 < \tan\phi_s < 0.9$. For sands, it is common to choose: $\tan\phi_s \approx 0.6$.

10.3.2 Bed load transport rate

The bed load transport rate per unit width may be defined as:

$$q_s = C_s \delta_s V_s \tag{10.5}$$

where V_s is the average sediment velocity in the bed-load layer (Fig. 10.1(b)).

Physically the transport rate is related to the characteristics of the bed-load layer: its mean sediment concentration C_s, its thickness δ_s which is equivalent to the average

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saltation height measured normal to the bed (Fig. 10.1 and 10.4) and the average speed V_s of sediment moving along the plane bed.

> ### *Notes*
> 1. A steady sediment transport in the bed-load layer is sometimes called a (no-suspension) *sheet-flow*.
> 2. Note that the volumetric sediment concentration has a maximum value. For rounded grains, the maximum sediment concentration is 0.65.

10.3.3 Discussion

Several researchers have proposed formulae to estimate the characteristics of the bed-load layer (Table 10.2). Figure 10.5 presents a comparison between two formulae. Overall the results are not very consistent. In practice there is still great uncertainty on the prediction of bed load transport.

Note that the correlations of van Rijn (1984a) are probably more accurate to estimate the saltation properties (i.e. C_s, δ_s/d_s and V_s/V_*) (within their range of validity).

Table 10.2 Bed-load transport rate calculations

Reference (1)	Bed-load layer characteristics (2)	Remarks (3)
Fernandez-Luque and van Beek (1976)	$\frac{V_s}{V_*} = 9.2\left(1 - 0.7\sqrt{\frac{(\tau_*)_c}{\tau_*}}\right)$	Laboratory data $1.34 \le \mathbf{s} \le 4.58$ $0.9 \le d_s \le 3.3$ mm $0.08 \le d \le 0.12$ m
Nielsen (1992)	$C_s = 0.65$ $\frac{\delta_s}{d_s} = 2.5(\tau_* - (\tau_*)_c)$ $\frac{V_s}{V_*} = 4.8$	Simplified model.
Van Rijn (1984a,1993)	$C_s = \frac{0.117}{d_s}\left(\frac{\nu^2}{(\mathbf{s}-1)g}\right)^{1/3}\left(\frac{\tau_*}{(\tau_*)_c} - 1\right)$ $\frac{\delta_s}{d_s} = 0.3\left(d_s\left(\frac{(\mathbf{s}-1)g}{\nu^2}\right)^{1/3}\right)^{0.7}\sqrt{\frac{\tau_*}{(\tau_*)_c} - 1}$ $\frac{V_s}{V_*} = 9 + 2.6\log_{10}\left(d_s\left(\frac{(\mathbf{s}-1)g}{\nu^2}\right)^{1/3}\right) - 8\sqrt{\frac{(\tau_*)_c}{\tau_*}}$	For $\frac{\tau_*}{(\tau_*)_c} < 2$ and $d_s = d_{50}$. Based on laboratory data $0.2 \le d_s \le 2$ mm $d > 0.1$ m $Fr < 0.9$
	$C_s = \frac{0.117}{d_s}\left(\frac{\nu^2}{(\mathbf{s}-1)g}\right)^{1/3}\left(\frac{\tau_*}{(\tau_*)_c} - 1\right)$ $\frac{\delta_s}{d_s} = 0.3\left(d_s\left(\frac{(s-1)g}{\nu^2}\right)^{1/3}\right)^{0.7}\sqrt{\frac{\tau_*}{(\tau_*)_c} - 1}$ $\frac{V_s}{V_*} = 7$	$d_s = d_{50}$. Based on laboratory data $0.2 \le d_s \le 2$ mm $d > 0.1$ m $Fr < 0.9$

Notes: V_* = shear velocity; $(\tau_*)_c$ = critical Shields parameter for initiation of bed load.

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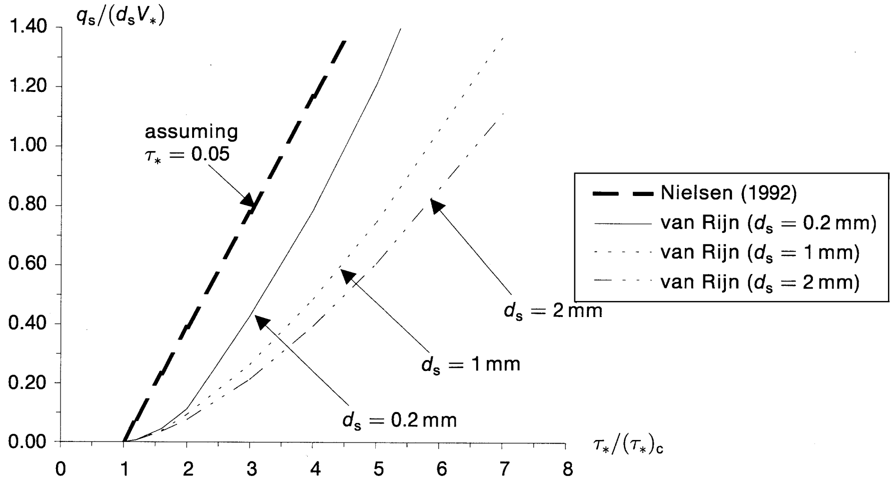

Fig. 10.5 Dimensionless bed-load transport rate $q_s/(d_s V_*)$ as a function of the dimensionless Shields parameter $\tau_*/(\tau_*)_c$ (Table 10.2).

Notes

1. The calculations detailed in Table 10.2 apply to flat channels (i.e. $\sin\theta < 0.001$ to 0.01) and in absence of bed forms (i.e. plane bed only).
2. For steep channels several authors showed a strong increase of bed-load transport rate. It is believed that the longitudinal bed slope affects the transport rate because the threshold conditions (i.e. initiation of bed load) are affected by the bed slope, the sediment motion is changed with steep bed slope and the velocity distribution near the bed is modified.

Discussion

The prediction of bed-load transport rate is *not* an accurate prediction. One researcher (van Rijn 1984a) stated explicitly that:

> the overall inaccuracy [...] may not be less than a factor 2

10.4 Applications

10.4.1 Application No. 1

The bed-load transport rate must be estimated for the Danube river (Central Europe) at a particular cross-section. The known hydraulic data are: flow rate of about 530 m^3/s, flow depth of 4.27 m, bed slope being about 0.0011. The channel bed is a sediment mixture with a median grain size of 0.012 m and the channel width is about 34 m.

Predict the sediment-load rate using the Meyer-Peter correlation, the Einstein formula, and equation (10.5) using both Nielsen and van Rijn coefficients.

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First calculations

Assuming a wide channel, the mean shear stress and shear velocity equals respectively:

$$\tau_o = \rho g d \sin\theta = 998.2 \times 9.81 \times 4.27 \times 0.0011 = 46.0\,\text{Pa}$$

$$V_* = \sqrt{g d \sin\theta} = 0.215\,\text{m/s}$$

The Shields parameter equals:

$$\tau_* = \frac{\tau_o}{\rho(\mathbf{s}-1) g d_s} = \frac{46.0}{998.2 \times 1.65 \times 9.81 \times 0.012} = 0.237$$

assuming $s = 2.65$ (quartz particles). And the particle Reynolds number is:

$$Re_* = \frac{V_* d_s}{\nu} = \frac{0.215 \times 0.012}{1.007 \times 10^{-6}} = 2558$$

For these flow conditions (τ_*, Re_*), the Shields diagram predicts sediment motion:

$$\tau_* = 0.237 > (\tau_*)_c \approx 0.05$$

Note that V_*/w_o is small and the flow conditions are near the initiation of suspension. In the first approximation, the suspended sediment transport will be neglected.

Approach No. 1: Meyer-Peter correlation

For the hydraulic flow conditions, the dimensionless parameter used for the Meyer-Peter formula is:

$$\frac{\rho(\mathbf{s}-1) g d_s}{\tau_o} = \frac{998.2 \times 1.65 \times 9.81 \times 0.012}{46} = 4.215$$

Application of the Meyer-Peter formula (for a sediment mixture) leads to:

$$\frac{q_s}{\sqrt{(\mathbf{s}-1) g d_s^3}} = 0.66$$

Hence: $q_s = 0.0035\,\text{m}^2/\text{s}$.

Approach No. 2: Einstein function

For the hydraulic flow conditions, the dimensionless parameter used for the Einstein formula is:

$$\frac{\rho(\mathbf{s}-1) g d_{35}}{\tau_o}$$

In the absence of information on the grain size distribution, it will be assumed: $d_{35} \approx d_{50}$. It yields:

$$\frac{\rho(\mathbf{s}-1) g d_{35}}{\tau_o} \approx 4.215$$

For a sediment mixture, the Einstein formula gives:

$$\frac{q_s}{\sqrt{(\mathbf{s}-1) g d_s^3}} = 0.85$$

Hence: $q_s = 0.0045\,\text{m}^2/\text{s}$.

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Approach No. 3: bed-load calculation (equation (10.5))

The bed load transport rate per unit width equals:

$$q_s = C_s \delta_s V_s \tag{10.5}$$

Using Nielsen's (1992) simplified model, it yields:

$$q_s = 0.65 \times 2.5(\tau_* - (\tau_*)_c) d_s 4.8 V_*$$
$$= 0.65 \times 2.5 \times (0.237 - 0.05) \times 0.012 \times 4.8 \times 0.215 = 0.0038\,\text{m}^2/\text{s}$$

With the correlation of van Rijn (1984a), the saltation properties are:

$$C_s = 0.00145$$
$$\frac{\delta_s}{d_s} = 31.6$$
$$\frac{V_s}{V_*} = 7$$

and the sediment transport rate is: $q_s = 0.00083\,\text{m}^2/\text{s}$.

Summary

	Meyer-Peter	Einstein	Eq. (10.5) (Nielsen)	Eq. (10.5) (van Rijn)
Q_s (m^3/s)	0.119	0.153	0.128	0.0281 (?)
q_s (m^2/s)	0.0035	0.0045	0.0038	0.00083 (?)
Mass sediment rate (kg/s)	314	405	339	74.6 (?)

Four formulae were applied to predict the sediment transport rate by bed-load. Three formulae give reasonably close results. Let us review the various formulae.

Graf (1971) commented that the Meyer-Peter formula 'should be used carefully at [...] high mass flow rate', emphasizing that most experiments with large flow rates used by Meyer-Peter *et al.* (1934) were performed with light sediment particles (i.e. lignite breeze, $\mathbf{s} = 1.25$). Graf stated that one advantage of the Meyer-Peter formula is that 'it has been tested with large grains'.

The Einstein formula has been established with more varied experimental data than the Meyer-Peter formula. The present application is within the range of validity of the data (i.e. $q_s/\sqrt{(\mathbf{s}-1)gd_s^3} = 0.85 \ll 10$).

Equation (10.5) gives reasonably good overall results using Nielsen's (1992) simplified model. In the present application, the grain size (0.012 m) is large compared with the largest grain size used by van Rijn (1984a) to validate his formulae (i.e. $d_s \leq 0.002$ m). Hence it is understandable that equation (10.5) with van Rijn's formulae is inaccurate (and not applicable).

For the present application, it might be recommended to consider the Meyer-Peter formula, which was developed and tested in Europe.

Note

All bed-load formulae would predict the maximum bed-load transport rate for a stream in equilibrium. This transport capacity may or may not be equal to the

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actual bed-load if the channel is subjected to degradation or aggradation (see Chapter 12).

10.4.2 Application No. 2

A wide stream has a depth of 0.6 m and the bed slope is 0.0008. The bed consists of a mixture of heavy particles ($\rho_s = 2980\,\text{kg/m}^3$) with a median particle size $d_{50} = 950\,\mu\text{m}$.

Compute the bed-load transport rate using the formulae of Meyer-Peter and Einstein, and equation (10.5) for uniform equilibrium flow conditions.

First calculations

Assuming a wide channel, the mean shear stress and shear velocity equals respectively:

$$\tau_o = \rho g d \sin\theta = 998.2 \times 9.80 \times 0.6 \times 0.0008 = 4.70\,\text{Pa}$$

$$V_* = \sqrt{g d \sin\theta} = 0.069\,\text{m/s}$$

The Shields parameter equals:

$$\tau_* = \frac{\tau_o}{\rho(\mathbf{s}-1) g d_s} = \frac{4.70}{998.2 \times 1.98 \times 9.80 \times 0.00095} = 0.255$$

The particle Reynolds number is:

$$Re_* = \frac{V_* d_s}{\nu} = \frac{0.069 \times 0.00095}{1.007 \times 10^{-6}} = 65.1$$

For these flow conditions (τ_*, Re_*), the Shields diagram predicts sediment motion:

$$\tau_* = 0.255 > (\tau_*)_c \approx 0.05$$

Note that V_*/w_o is less than 0.7. In the first approximation, the suspended sediment transport is negligible.

Approach No. 1: Meyer-Peter correlation

For the hydraulic flow conditions, the dimensionless parameter used for the Meyer-Peter formula is:

$$\frac{\rho(\mathbf{s}-1) g d_s}{\tau_o} = \frac{998.2 \times 1.98 \times 9.80 \times 0.00095}{4.7} = 3.91$$

Application of the Meyer-Peter formula (for a sediment mixture) leads to:

$$\frac{q_s}{\sqrt{(\mathbf{s}-1) g d_s^3}} = 0.76$$

Hence: $q_s = 9.82 \times 10^{-5}\,\text{m}^2/\text{s}$.

Approach No. 2: Einstein function

For the hydraulic flow conditions, the Einstein formula is based on the d_{35} grain size. In the absence of information on the grain size distribution, it will be assumed: $d_{35} \approx d_{50}$.

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For a sediment mixture, the Einstein formula gives:

$$\frac{q_s}{\sqrt{(\mathbf{s}-1)gd_s^3}} \approx 1$$

Hence: $q_s = 1.29 \times 10^{-4}\,\text{m}^2/\text{s}$.

Approach No. 3: bed-load calculation (equation (10.5))

The bed-load transport rate per unit width equals:

$$q_s = C_s \delta_s V_s \tag{10.5}$$

Using Nielsen's (1992) simplified model, it yields:

$$q_s = 0.65 \times 2.5 \times (\tau_* - (\tau_*)_c)d_s 4.8 V_*$$

$$= 0.65 \times 2.5 \times (0.255 - 0.05) \times 0.00095 \times 4.8 \times 0.069 = 1.05 \times 10^{-4}\,\text{m}^2/\text{s}$$

With the correlation of van Rijn (1984a), the saltation properties are:

$$C_s = 0.019$$

$$\frac{\delta_s}{d_s} = 5.848$$

$$\frac{V_s}{V_*} = 7$$

and the sediment transport rate is: $q_s = 0.5 \times 10^{-4}\,\text{m}^2/\text{s}$.

Summary

	Meyer-Peter	Einstein	Eq. (10.5) (Nielsen)	Eq. (10.5) (van Rijn)
q_s (m^2/s)	0.98×10^{-5}	1.2×10^{-4}	1.05×10^{-4}	0.5×10^{-4}
Mass sediment rate (kg/s/m)	0.29	0.38	0.31	0.15

All the formulae give consistent results (within the accuracy of the calculations!).

For small-size particles, (i.e. $d_{50} < 2\,\text{mm}$), the formulae of van Rijn are recommended because they were validated with over 500 laboratory and field data.

Note, however, that 'the overall inaccuracy of the predicted [bed-load] transport rates may not be less than a factor 2' (van Rijn 1984a, p. 1453).

10.4.3 Application No. 3

The North Fork Toutle river flows on the north-west slopes of Mount St. Helens (USA), which was devastated in May 1980 by a volcanic eruption. Since 1980 the river has carried a large volume of sediment.

Measurements were performed on 28 March 1989 at the Hoffstadt Creek bridge. At that location the river is 18 m wide. Hydraulic measurements indicated that the flow depth was 0.83 m, the depth-averaged velocity was 3.06 m/s and the bed slope was $\sin\theta = 0.0077$. The channel bed is a sediment mixture with a median grain size of 15 mm and $d_{84} = 55\,\text{mm}$.

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Predict the sediment-load rate using the Meyer-Peter correlation, the Einstein formula, and equation (10.5) using both Nielsen and van Rijn coefficients.

First calculations

Assuming a wide channel ($d = 0.83\,\text{m} \ll 18\,\text{m}$), the mean shear stress and shear velocity equals respectively:

$$\tau_o = \rho g d \sin\theta = 998.2 \times 9.81 \times 0.83 \times 0.0077 = 62.6\,\text{Pa}$$

$$V_* = \sqrt{g d \sin\theta} = 0.25\,\text{m/s}$$

The Shields parameter equals:

$$\tau_* = \frac{\tau_o}{\rho(\mathbf{s} - 1) g d_s} = 0.258$$

assuming $\mathbf{s} = 2.65$ (quartz particles) and using $d_s = d_{50}$. And the particle Reynolds number is:

$$Re_* = \frac{V_* d_s}{\nu} = 3725$$

For these flow conditions (τ_*, Re_*), the Shields diagram predicts sediment motion:

$$\tau_* = 0.258 > (\tau_*)_c \approx 0.05$$

Note that V_*/w_o is small ($V_*/w_o \approx 0.5$) and the flow conditions are near the initiation of suspension. In the first approximation, the suspended sediment load will be neglected.

Approach No. 1: Meyer-Peter correlation

For the hydraulic flow conditions, the dimensionless parameter used for the Meyer-Peter formula is:

$$\frac{\rho(\mathbf{s} - 1) g d_s}{\tau_o} = 3.87$$

using $d_s = d_{50}$. Application of the Meyer-Peter formula (for a sediment mixture) would lead to:

$$\frac{q_s}{\sqrt{(\mathbf{s} - 1) g d_s^3}} = 0.78$$

Hence: $q_s = 0.0057\,\text{m}^2/\text{s}$.

Approach No. 2: Einstein function

For the hydraulic flow conditions, the dimensionless parameter used for the Einstein formula is:

$$\frac{\rho(\mathbf{s} - 1) g d_{35}}{\tau_o}$$

In the absence of information on the grain size distribution, we assume: $d_{35} \approx d_{50}$.

For a sediment mixture, the Einstein formula gives:

$$\frac{q_s}{\sqrt{(\mathbf{s} - 1) g d_s^3}} \sim 40$$

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But note that the flow conditions are outside of the range of validity of the formula. That is, the Einstein formula should not be used.

Approach No. 3: bed-load calculation (equation (10.5))

The bed-load transport rate per unit width equals:

$$q_s = C_s \delta_s V_s \tag{10.5}$$

Using Nielsen's (1992) simplified model, it yields:

$$q_s = 0.65 \times 2.5 \times (\tau_* - (\tau_*)_c) d_s 4.8 V_* = 0.0061\,\mathrm{m^2/s}$$

With the correlation of van Rijn (1984a), the saltation properties are:

$$C_s = 0.00129$$

$$\frac{\delta_s}{d_s} = 38.97$$

$$\frac{V_s}{V_*} = 7$$

And the sediment transport rate is: $q_s = 0.0013\,\mathrm{m^2/s}$.

Summary

	Meyer-Peter	Einstein	Eq. (10.5) (Nielsen)	Eq. (10.5) (van Rijn)	Data
Q_s (m^3/s)	0.10	N/A	0.11	0.024	
q_s (m^2/s)	0.0057	N/A	0.0061	0.0023	
Mass sediment rate (kg/s)	274	N/A	290	63	205.2

Pitlick (1992) described in-depth the field study performed at the Hoffstadt Creek bridge on the North Fork Toutle river. On 28 March 1989, the main observations were:

$$d = 0.83\,\mathrm{m}, \quad V = 3.06\,\mathrm{m/s}, \quad \sin\theta = 0.0077, \quad f = 0.054, \quad \tau_o = 63\,\mathrm{N/m^2},$$
$$C_s = 0.031, \quad \dot{m}_s = 11.4\,\mathrm{kg/m/s}$$

The channel bed was formed in dunes (up to 0.16 m high).

Discussion

First let us note that two methods of calculations are incorrect: the Einstein formula and equation (10.5) using van Rijn's correlation. The flow conditions and sediment characteristics are outside of the range of applicability of Einstein's formula as $q_s/\sqrt{(\mathbf{s}-1)g d_s^3} > 10$. In addition the grain size (0.015 m) is larger than the largest grain size used by van Rijn (1984a) to validate his formulae (i.e. $d_s \leq 0.002$ m). Secondly it is worth noting that the Meyer-Peter formula and equation (10.5) using Nielsen's correlations give reasonable predictions.

This last application is an interesting case: it is well documented. The river flow is characterized by heavy bed-load transport and the bed-forms are a significant feature of the channel bed.

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10.5 Exercises

Numerical solutions to some of these exercises are available from the Web at www.arnoldpublishers.com/support/chanson

Considering a 20 m wide channel, the bed slope is 0.00075 and the observed flow depth is 3.2 m. The channel bed is sandy ($d_s = 0.008$ m). Calculate: (a) mean velocity, (b) mean boundary shear stress, (c) shear velocity, (d) Shields parameter and (e) occurrence of bed-load motion. If bed-load motion occurs, calculate: (f) bed-load layer sediment concentration, (g) bed-load layer thickness, (h) average sediment velocity in bed-load layer and (i) bed-load transport rate. (Assume that the equivalent roughness height of the channel bed equals the median grain size. Use the Nielsen simplified model.)

Considering a wide channel, the discharge is 20 m^2/s, the observed flow depth is 4.47 m and the bed slope is 0.001. The channel bed consists of a sand mixture ($d_{50} = 0.011$ m). Calculate the bed-load transport rate using: (a) Meyer-Peter correlation, (b) Einstein function, (c) Nielsen simplified model and (d) van Rijn correlations. (Assume that the equivalent roughness height of the channel bed equals the median grain size.)

A 25 m wide channel has a bed slope of 0.0009. The bed consists of a mixture of light particles ($\rho_s = 2350$ kg/m^3) with a median particle size $d_{50} = 1.15$ mm. The flow rate is 7.9 m^3/s. Calculate the bed-load transport rate at uniform equilibrium flow conditions using the formulae of Meyer-Peter, Einstein, Nielsen and van Rijn. (Assume that the equivalent roughness height of the channel bed equals the median grain size.)

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