



Self-sustained non-periodic activity in networks of spiking neurons: The contribution of local and long-range connections and dynamic synapses

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ABSTRACT

Cortical dynamics show self-sustained activity which is complex and non-periodic. Assemblies of neurons show transient coupling exhibiting both integration and segregation without entering a seizure state. Models to date have demonstrated these properties but have required external input to maintain activity. Here we propose a spiking network model that incorporates a novel combination of both local and long-range connectivity and dynamic synapses (which we call the *LLDS* network) and we present explorations of the network's micro and macro behaviour. At the micro level, the *LLDS* network exhibits self-sustained activity which is complex and non-periodic and shows transient coupling between assemblies in different network regions. At the macro level, the power spectrum of the derived EEG, calculated from the summed membrane potentials, shows a power-law-like distribution similar to that recorded from human EEG. We systematically explored parameter combinations to map the variety of behavioural regimes and found that network connectivity and synaptic mechanisms significantly impact the dynamics. The complex sustained behaviour occupies a transition region in parameter space between two types of non-complex activity state, a synchronised high firing rate regime, resembling seizure, for low connectivity, and repetitive activation of a single network assembly for high connectivity. Networks without synaptic dynamics show only transient complex behaviour. We conclude that local and long-range connectivity and short-term synaptic dynamics are together sufficient to support complex persistent activity. The ability to craft such persistent dynamics in a spiking network model creates new opportunities to study neural processing, learning, injury and disease in nervous systems.

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Introduction

Brain activity exhibits persistent ongoing complex behaviour (Breakspear et al., 2003; Honey et al., 2007) and oscillatory dynamics which approximates a power law (Linkenkaer-Hansen et al., 2001; Kitzbichler et al., 2009). Complex behaviour is hypothesised to support the brain's dynamical flexibility and sophisticated processing capabilities (Breakspear et al., 2003; Buzsaki and Draguhn 2004; Fries, 2005; Lakatos et al., 2008). When the brain enters a mode where it is essentially 'disconnected' from external stimuli, and as such is in a dynamic regime where neural activity is self-sustaining, the activity is focussed in certain cortical regions known as the default, or resting state, network (Greicius et al., 2003). Such activity is associated with memory retrieval, planning and problem solving (Binder et al., 1999; Mazoyer et al., 2001) as well as mind-wandering, or daydreaming (Mason et al., 2007). Brain oscillations in the resting state (relaxed, with eyes closed) are dominated by 8–12 Hz alpha waves, with bursts

of 13–30 Hz beta (in the 17–23 Hz beta-2 band) hypothesised to be associated with spontaneous mental operations during relaxation (Laufs et al., 2003).

Complex dynamical behaviour in brains can be viewed as the ability of different brain regions to both segregate and integrate information (Sporns et al., 2000a; Shanahan, 2008), and to accomplish this dynamically over a broad range of spatial and temporal scales. Segregation of neuronal function in space and time allows for rapid and reliable processing facilitated by specialised and specific brain regions. Integration of brain regions is required to combine and successfully react to information provided by segregated processing, and also to initially *call up* (activate) appropriate segregated regions as processing needs demand. Functional connectivities are therefore continuously established and broken, driven dynamically by combinations of current brain state, perceptual input, gross static (anatomical) connections and fine learned connections within and between brain regions.

A signature characteristic of complex behaviour in brains is transient dynamic phase coupling of brain regions at multiple spatial and temporal scales (Varela et al., 2001). When several brain regions are driven to excitability via mutual connections from other active

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areas and when these regions themselves are interconnected, dynamic coupling can be established between them. Regions need not have direct connections to display correlated activity; such *functional* connectivity is defined solely in terms of correlated activity and is observed over very large structural distances in some network models (Fraiman et al., 2009). In contrast, *effective* connectivity is defined in terms of causal interaction. The properties of effective connectivity, for instance, strength and direction of influence of the ensuing interaction, are dependent on the specific power and timing of the driving influences, which in turn depend on prior states of brain regions from which they themselves are connected, and their own internal states. A slightly different initial state or slight perturbation to the existing state may give rise to very different interactions between regions because of the complex nonlinear mutual interactions within and between several regions at multiple scales. Effective connectivity is also contingent upon properties of the individual network elements—the neurons. Causation between two reciprocally coupled regions can be reversed by simply reversing the phase order of the regions' activities, due to synaptic input being ineffective during a neuron's refractory period (Buzsáki, 2006).

Recently, interest has grown in networks connected using small world principles, due to the ability of these networks to exhibit complex dynamics (Sporns et al., 2000b; Roxin et al., 2004; Riecke et al., 2007; Shanahan, 2008). With carefully chosen parameter values, such networks can exhibit prolonged periods of sustained activity (Roxin et al., 2004; Riecke et al., 2007). When long-range connection densities are low, sustained activity takes the form of periodic activity that can continue indefinitely. When long-range connection densities are high, disordered, complex, non-periodic (chaotic) activity can result, but this activity will eventually cease and return the network to quiescence. For common network topologies, the region of parameter space that exhibits this behaviour is very small (Breakspear et al., 2003; Shanahan, 2008).

It has been recognised that systems exhibiting complex dynamics often tend to be in a transitory, finely balanced state between two distinct states of operation (such as independent vs. synchronised activity), and the complexity can arise from rapid, irregular, cascading, partial transitions between states (Solé et al., 1996). This transitory mode of operation is called a *phase transition* and the state of a system undergoing a phase transition is known as *critical*. Criticality has been investigated extensively in statistical mechanics and thermodynamics, and is readily applicable to a wide variety of natural phenomena (Bak, 1996). Networks of interacting neurons operating in a critical state are hypothesised to balance the conflicting demands of maintaining network stability whilst simultaneously permitting input sensitivity and propagation of information through the network (Beggs and Plenz, 2003). Critical regions in many systems are typically small transition regions between states of low complexity.

The size of a critical region can be influenced by the properties of the network components. Short-term synaptic dynamics is the temporary modification of synaptic weights, usually based on synaptic activity. When a synapse is used, the connection strength between the pre- and post-synaptic neurons can be temporarily increased (facilitated) or decreased (depressed). It has been shown for neural network models that the introduction of short-term synaptic dynamics (specifically activity dependent synaptic depression) can massively enlarge the critical region (Levina et al., 2007). However in Levina et al.'s study, activity was not persistent, relying on external input to maintain it. The random nature of the external input altered the activity that would otherwise be supported by the intrinsic network dynamics. In addition, the network model was greatly simplified and network elements were fully connected.

Measurable properties of a system in a critical state are often distributed according to a power law (Levina et al., 2007). Neural network models that exhibit activity power spectra showing power

laws have been demonstrated (for example see the early work of Freeman, 1991, and Wright and Liley, 1996) and recent work by beim Graben and Kurths, 2008), but none of these models exhibited persistent activity—all relied on some form of external input to continuously drive the dynamics. The question therefore remains—how to generate indefinitely persistent, complex (non-periodic) dynamics in a spiking neural network? Here we report investigations that combined local and long-range connectivity with activity dependent synaptic depression in a spiking network model. The overall goal of these studies was to gain an understanding of persistent complex dynamics within neural networks across a wide range of intrinsic network parameter values. We analysed networks of neurons with varying synaptic efficacies, local and long-range connectivity and a range of short-term synaptic dynamics (which we call Local and Long-range Dynamic Synapse (LLDS) networks), to determine the effects of each component on the ability of the network to exhibit complex persistent activity.

Methods

Neuron and synapse models

All simulations were conducted using the Parallel Circuit Simulator (PCSIM) (Pecevski et al., 2009), a comprehensive, well-supported software package for the simulation of large neural networks.

The LLDS network was constructed using Izhikevich model cortical neurons (Izhikevich, 2003). These neurons provide for realistic neuron membrane dynamics such as spike frequency adaptation, intrinsic bursting, resonance and bistability, whilst being computationally tractable for large network simulations (Izhikevich, 2004). The Izhikevich model is defined by three equations over two variables, the membrane potential v , and the membrane recovery variable q , which are updated as follows:

$$\dot{v} = 0.04v^2 + 5v + 140 - q + I \quad (1)$$

$$\dot{q} = a(bv - q) \quad (2)$$

$$\begin{aligned} \text{if } v \geq 30 \text{ then} \\ v &\leftarrow g \\ q &\leftarrow q + h \end{aligned} \quad (3)$$

where I is the summed synaptic input current, and a , b , g and h are neuron-specific model parameters. Higher values of q make it harder for the neuron to spike. In this study, all excitatory neurons were modelled as regular-spiking (RS) cells with the parameters $(a,b,g,h) = (0.02, 0.2, -65, 8)$ and the inhibitory neuron as a fast spiking (FS) cell with $(a,b,g,h) = (0.1, 0.2, -65, 2)$.

The synapses were modelled as a postsynaptic current initiated by a presynaptic spike that decayed with a characteristic time constant c . For static synapses (which in the current model applies to all synapses to and from the inhibitory neuron), the magnitude of the synaptic current I_{static} induced when a presynaptic spike occurred was proportional to the synaptic weight w . The rate of change, \dot{I}_{static} , was given by

$$\dot{I}_{\text{static}} = \sum_i \left[\delta(t - t_i) w_i - \frac{I_i}{c_i} \right] \quad (4)$$

where δ is the Dirac delta function, t_i is the time of the last spike from presynaptic neuron i , and w_i is the constant synaptic weight.

To implement short-term synaptic dynamics (which in the current model applies to synapses between all excitatory neurons), a combination of depression and facilitation was used (Markram et al., 1998). For synaptic depression, only an amount u of the currently available proportion of neurotransmitter p was released at the

occurrence of each spike; p recovered back to unity with time constant d . The rate of change of p for neuron i , \dot{p}_i , was given by

$$\dot{p}_i = -\delta(t - t_i)p_i u_i + \frac{(1 - p_i)}{d_i} \quad (5)$$

The initial value of u for all dynamic synapses was set to U , then for synaptic facilitation u was increased on the occurrence of each spike, recovering back to U with time constant f . The rate of change of u for neuron i , \dot{u}_i , was given by

$$\dot{u}_i = -\delta(t - t_i)(1 - u_i)U + \frac{(U - u_i)}{f_i} \quad (6)$$

With dynamic synapses, the calculation for synaptic current, $I_{dynamic}$, becomes

$$I_{dynamic} = \sum_i [w_i(1 - p_i)u_i] \quad (7)$$

The synaptic depression recovery time constant d was set to 50 ms for local and 500 ms for long-range connections. Faster recovery for local connections meant that local assemblies of neurons would be stable for longer time periods, resulting in more slowly evolving complex trajectories of network activity. The synaptic usage factor U was set to 0.25 with a recovery time constant of $f = 1000$ ms for all dynamic synapses. Facilitation has the effect of making synapses stronger during initial spikes, but depleting neurotransmitter reserves faster (and hence effectively reducing synaptic weight). Facilitation results in strong initial synaptic coupling but reduced coupling strength over longer periods, encouraging both the transient grouping of neurons into assemblies and the transient coupling and decoupling of multiple assemblies over time.

Network

The network consisted of $n = 1000$ regular-spiking (RS) pyramidal neurons connected linearly with directional synapses. Each neuron was connected to each of its closest j neighbours with local excitatory weight w_n/j where $w_n = 0.05$ (the end neurons connected to only $j/2$ neighbours). In simulations, j ranged from 0 to 25. For $j = 0$, no neighbourhood connections were made. For even-numbered j , exactly $j/2$ neighbour connections were made to each side of the neuron. For odd and fractional $j > 2$ (e.g. $j = 3, j = 5.25$), an extra connection with weight proportional to the fractional component, modulo $(j/2, 1) \cdot w_n/j$, was made to each of the two neurons just beyond the $j/2$ whole neighbours on each side (i.e. to the two neurons at distance ceiling($j/2$)). For $j < 2$, the only connections were made to the two immediate neighbours with weight modulo $(j/2, 1) \cdot w_n$. Random long-range connections of weight w_r/k where $w_r = w_n$ were then made from each neuron to each other neuron with probability k/n , giving an average of k long-range connections per neuron. In simulations, k ranged from 0 to 25. The scaling of w_n and w_r by j and k respectively ensured that total average synaptic efficacies remained constant as network topology varied¹, except for $j < 2$ for which neighbourhood efficacies decreased linearly to zero. For excitatory pyramidal to pyramidal neuron connections, the synaptic time constant c_{ee} was set to 50 ms to emulate the effect of slow-decay NMDA synaptic receptors and their role in persistent network activity (Wang, 1999). A single inhibitory neuron, a fast-spiking (FS) inhibitory cell, received excitatory input from and projected inhibitory output to every

pyramidal cell with weights $w_{ei} = 1/n$ and $w_{ie} = -0.02$ (time constants $c_{ei} = 10$ ms and $c_{ie} = 25$ ms, static synapses), respectively. Network activity was initiated by a single input pulse at time zero into a random selection of m pyramidal neurons; m ranged from 0 to n . Conduction delays for all connections were set uniformly to 1 ms.

In this study, we show that a combination of dense local neighbourhood connectivity and sparse long-range connectivity is sufficient to support complex non-periodic self-sustained activity. Whilst such connectivity is similar to small world, it differs from small world in two ways. First, the local connectivity need not conform to the small world requirement of cliquishness as defined by a high clustering coefficient (Watts and Strogatz, 1998) (i.e. in the current study, neighbouring nodes need not have other neighbours in common, resulting in a clustering coefficient of zero, which occurs for neighbourhood diameters $j \leq 2$). Second, the proportion of long-range to short-range connections can be much higher than typical small world networks, akin to a high rewiring probability (Watts and Strogatz, 1998), again giving a low clustering coefficient. For large neighbourhood j and small long-range connectivity k , the network is small world; however, for small neighbourhood and large long-range connectivity, the clustering coefficient drops significantly, such that connectivity becomes essentially random. We shall see that complex self-sustained activity can occur even for networks with a clustering coefficient of zero.

Analysis methods

Network activity was analysed over broad ranges of the following network parameters—local neighbourhood diameter j , number of long-range connections k , initiating burst size m , and synaptic depression time constants d and f (both modulated by the same factor in the range 0 to 1). In the final set of simulations where dynamic synapses were removed (i.e. d and f were both set to 0), the excitatory synaptic weights were varied (w_n and w_r were both modulated by a factor in the range 0 to 1). For each parameter combination to be tested, a simulation of 1000 neurons was conducted for 10 s of simulated time with 0.1 ms time-steps using Euler integration. Spike times and membrane potentials of all neurons were recorded at each step. The first second of activity was deemed to be the network settling period during which time the network activity would transition from synchronous bursting (initiated by the input pulse) to non-periodic sustained activity (or to seizure or quiescence depending on the particular parameter combination). The neural activity for this first second was omitted from further analysis.

The remaining 9 s of the recordings was used to calculate the mean firing rate, r , of the network ($r = T/n/9$, where T is the total number of spikes generated by all neurons). Additionally, the interspike intervals (ISI) for each neuron were individually determined, then all interspike intervals for all neurons were pooled into a single distribution before calculating the standard deviation (ISI-SD). The ISI-SD is a measure of the non-triviality of the network-wide activity train. Networks exhibiting either random uncorrelated activity or seizure-like dynamics both have low ISI-SD, since the lack of any segregation in network activity reduces the spread of interspike intervals, even when the network mean firing rate r is comparable. Seizure states tend to have high firing rates, which further decreases the interspike interval and consequently the ISI-SD. Whilst the value of the ISI-SD is influenced by network mean firing rate, and may be elevated under certain conditions (such as in a network that contains two groups of neurons, both of which fire periodically but at very different frequencies), we demonstrate that for the dynamics evident in the LLDS network, high ISI-SD is an indicator of complex dynamics (for example, see Fig. 1 below). Because the ISI-SD measure of network activity in a seizure or random state is low, whereas for networks in the regime of complex activity is higher, the measure reliably differentiates between these states. The measure for networks with

¹ A single synapse with a spike at time zero will generate total synaptic current $i_t = \int w \cdot \exp(-t/\tau) dt = w \int \exp(-t/\tau) dt$ for $t \geq 0$, where w is the synaptic weight and τ is the synaptic time constant. With n synapses and synaptic normalisation, each synapse will generate total synaptic current $i_n = \int (w/n) \exp(-t/\tau) dt = (w/n) \int \exp(-t/\tau) dt$. The ratio $i_t / i_n = n$, showing that n normalised synapses will generate synaptic current equal to one original synapse.

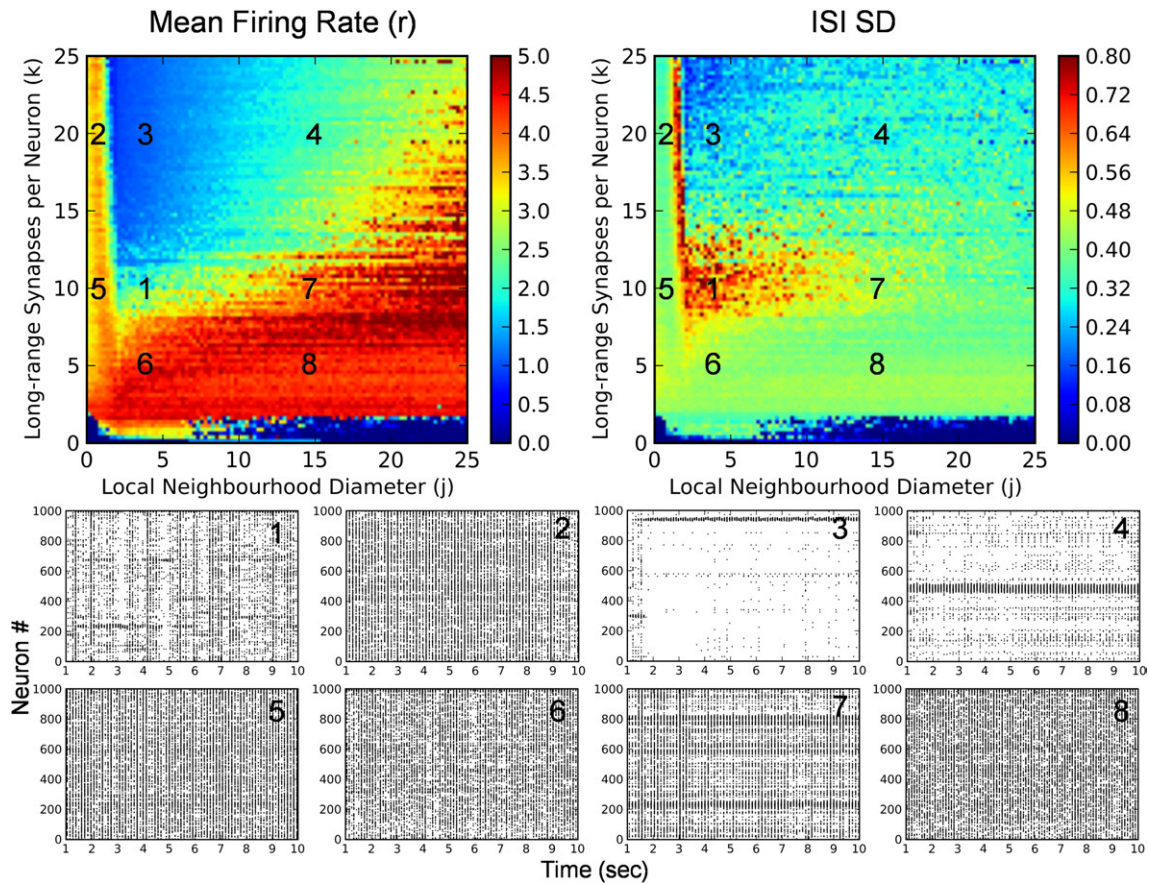


Fig. 1. Network dynamics as a function of local and long-range connectivity: a region of complex persistent activity is bounded by regions of low complexity dynamics as the local and long-range connectivity is varied. Note that each pixel in the heat maps represents one trial (totalling 10,000 parameter combinations with 1 trial per combination). Corresponding pixels in the same position in both heat maps represent the same trial. Top left: Mean firing rate for a network of 1000 neurons, varying the average number of long-range connections per neuron (Y axis) and the local neighbourhood diameter (X axis). Connections weights were scaled to keep mean synaptic efficacies constant in all networks. Colour scale 0–5 Hz. Darkest blue are regions of no activity. Red regions are high firing rate, orange/yellow are moderate, and green/cyan are low. In general, firing rate decreases as long-range connectivity increases; however firing rate remains high for networks of small neighbourhood diameter. Top right: Standard deviation (SD) of the interspike intervals (ISI) for all neurons. ISI-SD is low in regions of random or periodic activity and high in regions of complex activity. Regions of complex activity at moderate firing rates are bounded by regions of low complexity activity at high firing rates and regions of even lower complexity at low firing rates. Both local and long-range connectivity are required to support complex activity, since in regions of either small neighbourhood diameter j or low long-range connectivity k , activity is not complex. Panels: Spike raster plots for the regions labelled 1 to 8 in the heat maps illustrate the broad range of network dynamics (network size $n = 1000$ neurons, initial input burst size $m = 500$ neurons). High firing rate regions (panels 2, 5, 6 and 8) exhibit low-complexity seizure-like states. Low firing rate regions (panels 3 and 4) exhibit low-complexity states with activity tightly focussed in small network neighbourhoods. Complex activity which is neither seizure-like nor focussed (panel 1) occupies the boundary between these states.

no activity, and therefore no activity distribution, was set to zero. The mean firing rates and ISI-SD measures were plotted on two dimensional graphs called *heat maps*, where each point represents a parameter combination and is coloured according to the rate or measure at that point. Heat maps provide a succinct visualisation of large numbers of simulations showing the range of network dynamics in different regions of parameter space. The heat maps in these studies use one pixel per network, and 100×100 networks per map.

In the analysis of network behaviours, some parameter choices result in seizure states with firing frequencies that are outside physiological range. These states occur because the Izhikevich neuron model has no *absolute* refractory period, relying on the modelled membrane dynamics to render a neuron unexcitable based on its recent past spiking activity, and hence implementing a *relative* refractory period only. During real neurons' absolute refractory period, it is impossible for the neurons to spike regardless of the magnitude of the input current, whereas during the relative refractory period it is possible for a spike to occur if the input current is large enough. The reliance by the Izhikevich model on a relative refractory period reliably maintains firing rates for synaptic input currents that are within physiological range. However, in the simulations presented in this study, synaptic currents can exceed physiologically realistic bounds,

particularly for networks which lack short-term synaptic dynamics, which can result in seizure states with extremely high firing rates. In reality, single neurons presented with such high input currents would quickly expire, whilst connected networks of neurons would suffer a simple form of *oscillation death* (Ermentrout and Kopell, 1990), which comprises the super-synchronous firing of all neurons together, such that they all then enter the refractory state simultaneously, resulting in the termination of any further persistent activity. In the simulations presented here, parameter choices that resulted in network seizures at extremely high individual neuron firing rates (>1000 Hz) were deemed to indicate that the network would suffer from oscillation death, and hence enter a sustained period of quiescence.

Results

The LLDS network has distinct modes of operation depending on the local and long-range connectivity (see Fig. 1). If the long-range connection density is low, the network remains in a quiescent state indefinitely—mean firing rate r is zero for most cases of fewer than approximately $k < 2$ long-range connections per neuron (Fig. 1, top left). In the small subregion in this region of low long-range connectivity where activity is sustained (for neighbourhood

diameters of between approximately $1 < j < 6$ neurons), the activity is not complex, as indicated by the low ISI-SD (standard deviation of the interspike interval) of approximately 0.3 (Fig. 1, top right). As the long-range connection density is increased (moving up along the Y axis), network activity transitions rapidly into a large region of high firing rate (left, $r > 4$ Hz), low complexity (right, $\text{ISI-SD} \approx 0.4$ to 0.5) dynamics, which dominates for long-range connectivity $2 < k < 8$ (see panels 6 and 8). In this region, the network is in a synchronous semi-seizure state. This region of low complexity dynamics extends, with a slight decrease in both ISI-SD and mean firing rate, for all networks with a small neighbourhood diameter $j < 1.5$ (approximately) regardless of the long-range connectivity (see panels 2 and 5). Networks with larger neighbourhood diameter $2 < j < 12$ and intermediate long-range connectivity $8 < k < 13$, occupy a regime where mean firing rate drops ($2 < r < 3$ Hz) and ISI-SD increases substantially, indicating the emergence of complex activity patterns (see panel 1). For higher long-range connectivity $k > 13$, network activity enters a regime of very low complexity activity at firing rates that increase as neighbourhood diameter increases (see panels 3 and 4). In this region, the network is in a state where sustained activation of a single assembly occurs. In summary, too few long-range connections result in activity failure. Slightly more long-range connectivity or a small neighbourhood diameter results in low complexity, high firing rate, seizure-like dynamics, whilst too many long-range connections also result in low complexity dynamics, with activity concentrated in a small region of the network and network firing rate dependent on neighbourhood diameter. In between the low and high firing rate extremes for respectively high and low long-range connection density, there is a region where activity at moderate firing rates is complex. Both local and long-range connectivity are simultaneously required to support this complex activity (Fig. 1, top right and panel 1). With either small

neighbourhood diameter j or low long-range connectivity k , self-sustained activity is not complex.

Network activity within the self-sustaining activity region exhibits regular oscillations in the alpha band at approximately 9.5 Hz (see Figs. 2 and 3). Due to the neighbourhood connections, neurons tend to fire in local groups, or assemblies. Assemblies that fire once tend to fire again over successive alpha cycles, often increasing in size over several cycles. The entire assembly may then remain silent for several cycles, before sometimes beginning to fire again and repeating the increase in size over successive cycles (see Fig. 2). Assemblies and the individual neurons of which they are comprised do not fire in repeating patterns. Even though the network as a whole oscillates in the alpha band, neurons and assemblies fire sporadically and (apparently) unpredictably, at times bursting strongly, and other times remaining quiescent for tens of seconds, or even minutes in longer simulations. These unpredictable burst lengths and intervals between bursts are reminiscent of the 'neuronal avalanches' described in cortical circuits in vitro (Beggs and Plenz, 2003), and recently in awake monkeys (Petermann et al., 2009). In the current network model, the potentially long delays between firings of single neurons or assemblies can give rise to very low network average firing rates whilst still enabling network activity to be self-sustained. Neurons can also fire individually rather than being part of an assembly (see the scattering of single raster points in Fig. 2). This firing is not regular and resembles noise, but is generated by the deterministic rules governing the network dynamics.

The electro-encephalogram (EEG) that would be recorded from this network at any moment in time, $\text{eeg}(t)$, can be estimated by summing the membrane potentials of all neurons:

$$\text{eeg}(t) = \sum_i v_i(t) \quad (8)$$

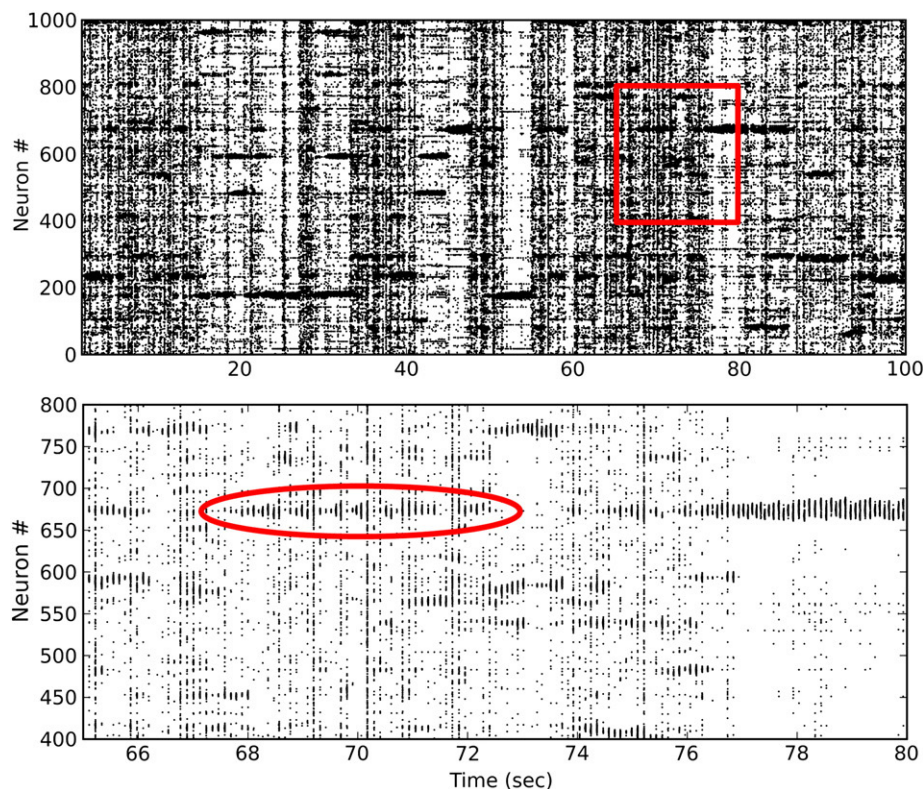


Fig. 2. Micro dynamics of network activity shows non-periodic complex behaviour (network size $n = 1000$ neurons, initial input burst size $m = 500$ neurons, long-range connectivity $k = 10$ neurons, local neighbourhood diameter $j = 4$ neurons, i.e. $X = 4$, $Y = 10$ in Fig. 1; identical parameters used as for Panel 1). Top: A raster plot of activity of all 1000 neurons in the network shows how neurons can be continuously active for short durations, several seconds, or several tens of seconds, and can be quiescent for similar periods of time (see Fig. 3 for the power spectrum of the derived EEG of this network). Bottom: Expanded view of the sub-region highlighted in the top. Neurons organise into assemblies, which may burst over successive alpha cycles, often increasing in size as they do so, then may remain silent for a short time, then burst again (see region inside red oval showing approximately 50 alpha cycles).

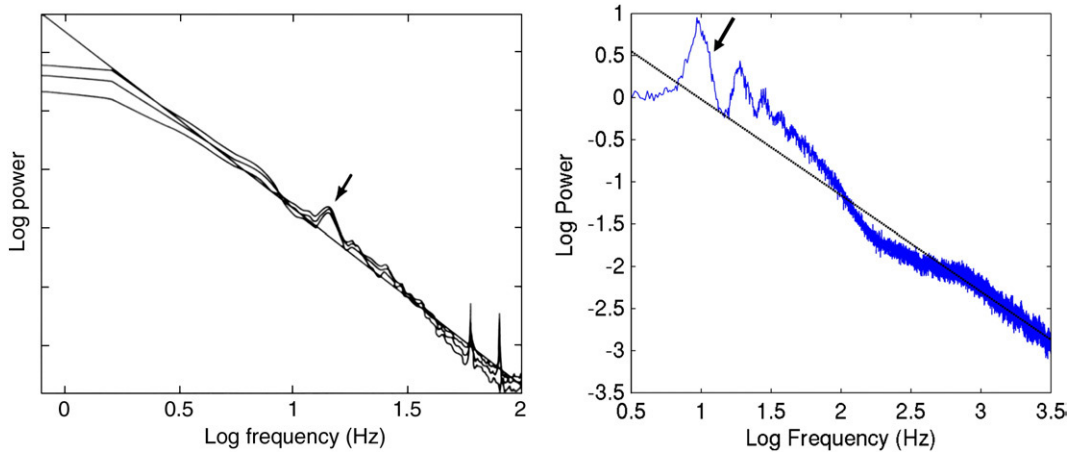


Fig. 3. Power spectrum of the derived EEG of network activity shows close similarities to that recorded from humans. Both show peaks at around 10 Hz and both show approximate power law relations over several orders of magnitude. Left: The power spectrum of the EEG recorded from human temporal lobe during sleep (figure from Buzsáki, 2006, incorporating results from Freeman et al., 2000) shows an approximate power law relation and a peak just above 10 Hz (arrowed). Right: Power spectrum for the LLDS network activity shown in Fig. 2 shows a similar approximate power law relation. The strongest peak is just below 10 Hz (arrowed), in the alpha band (8–12 Hz) which dominates cortical activity in the alert resting state. Additional peaks are also evident in the beta-2 (17–23 Hz), beta-3 (24–30 Hz) and gamma (31–80 Hz) bands. Power in the beta-2 band is characteristic in the cortex of spontaneous mental operations whilst in the relaxed state (Laufs et al., 2003). This power spectrum was generated by averaging the individual power spectra for each 10 s period in the 100 s simulation shown in Fig. 2. The dashed line fits the power relation $p = 1/f^\alpha$ with $\alpha = 1.1$.

where $v_i(t)$ is the membrane potential of neuron i at time t , yielding the simulated EEG time-series. Due to the vertical alignment of dendrites, a (real) dendritic potential, when summed from many cortical neurons, contributes to the local field potential and ultimately to the EEG as recorded from the scalp (Nunez and Srinivasan, 2006). Neurons in this study are simulated as point entities with no dendritic processes, so the neuron (soma) membrane potential is the closest estimate of the dendritic potential. This process of calculating the estimate of the EEG is similar to that used by beim Graben and Kurths (2008). The simulated EEG for the studied network and the EEG power spectrum show similarities to that recorded from humans (Freeman et al., 2000; Buzsáki, 2006), and an approximate power law relation, when the network is operating in the region of controlled persistent activity (see Fig. 3).

Correlations in activity between different regions in the network appear and disappear as activity evolves over time (see Fig. 4). Correlations change as effective connectivities shift between assemblies. In this network, there are no external inputs and all activity is generated from activity elsewhere in the network, hence regions of high correlation indicate likely effective connections.

When the network is simulated for longer periods of time, persistent activity continues and neither periodic activity nor activity failure eventuate (see Fig. 5). Although there is a propensity for some assemblies to fire more often than others due to chance concentrations of long-range connections, the activity remains self-sustaining and unpredictable. It is plausible that regions of relatively higher network activity over time may correspond to structural hubs that exist in the cortex; if that is the case, then the network activity over long time frames reflects the underlying network structure, whilst over shorter periods the effective connectivities may vary considerably, as has already been demonstrated. This variation in dynamic connectivity dependent upon the considered time frame replicates the result of Honey et al. (2007), which showed that, in simulated networks constructed to resemble cortical structure, functional connectivities over short time frames are highly variable whilst at longer time frames the connectivities reflect known anatomical connections. However, the current study replicates this result with no external input to the network. Simulations of networks of 100,000 neurons display similar dynamics.

We have shown how local and long-range connectivity is required in the current study for the support of persistent complex non-periodic activity (recall Fig. 1). In a similar manner, we explored the

contribution of short-term synaptic dynamics. Reducing the synaptic depression time constant with a factor ϵ between 0 and 1 shows that, with no synaptic depression (i.e. $\epsilon = 0$), the network enters a seizure state as soon as sufficient input is applied to sustain activity

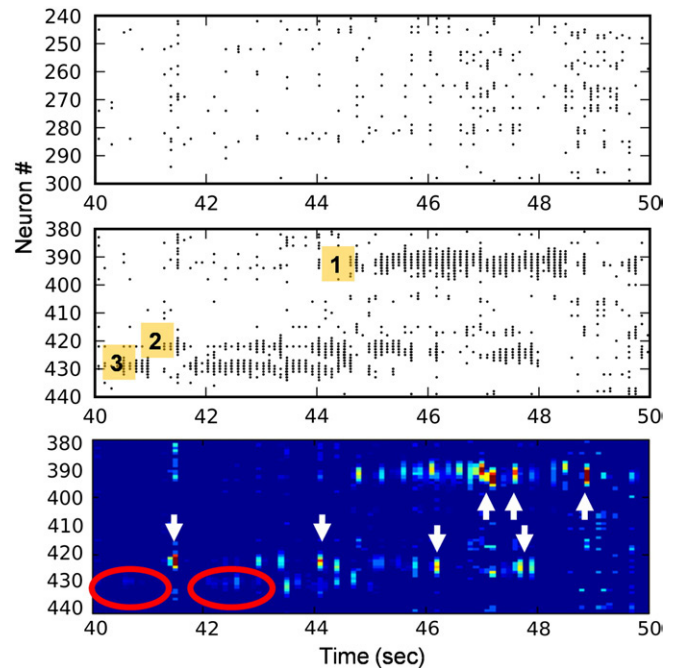


Fig. 4. Transient correlations in network activity show segregation and integration of network activity over time. Top: Network activity in neurons 240 to 300 (Y axis) in the time range from 40 to 50 s (X axis) shows continuous activity but no activation of assemblies. Centre: Network activity in neurons 380 to 440 shows activation of several assemblies. Correlations in activity of these assemblies shift over time as assembly 3 ceases firing, assembly 2 continues and assembly 1 begins. Bottom: Correlation of the activity of each of the neurons shown in the centre with the summed activity of the neurons shown in the top. Regions where the summed activity has high correlation with individual neurons in the range 380 to 440 are arrowed. Regions of conspicuous absence of strong correlation are circled in red; in these regions, there is strong assembly activity (centre panel) but only weak correlations. The regions of high correlation vary over the course of several seconds as effective connectivities are dynamically established and extinguished between neurons 380 to 440 and neurons 240 to 300.

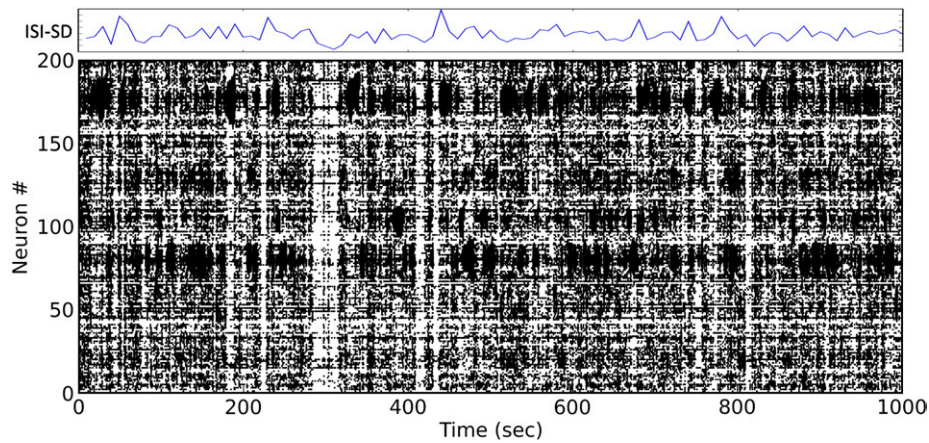


Fig. 5. Persistent complex dynamics over long time frames. Top: ISI-SD for each 10 s period of network activity (ISI-SD on the Y axis ranges from 0.4 to 1.1). Bottom: Network activity is shown for 1000 s of simulated time in a network of $n=1000$ neurons (first 200 neurons shown; all parameters as for Fig. 2). The initial burst consisted of $m=500$ neurons activated at time zero, then the network was allowed to run with no further external input. No periodic activity, seizures, or loss of persistent activity occurred. In simulations of 10,000 s duration (10^8 simulation time-steps), complex persistent activity continued unabated (data not shown). Note that no adjustments to network parameters are required for simulations of from 500 to 100,000 neurons to display qualitatively and quantitatively similar dynamics.

(see Fig. 6). It is only with $\varepsilon > 0.1$ (approximately) that non-seizure persistent activity appears. For values of ε approaching 1, network firing rate decreases, and activity becomes more complex as evidenced by the standard deviation of the interspike intervals (see Fig. 6).

To further elucidate the transition to seizure in the case of purely static synapses, ε was set to zero and the excitatory synapses in the network were modulated by a factor η from 0 to 1. For $\eta > 0.25$ (approximately), the network entered an out-of-physiological-range seizure state (leading to oscillation death) as soon as sufficient input was applied to sustain activity (see Fig. 7). For most $\eta < 0.18$, activity failed to be self-sustaining except for a small region for $\eta < 0.08$ and input burst size $m > 500$ where seizure at moderate network firing rates occurred. For $0.18 < \eta < 0.25$ (approximately), there is a small region where complex activity occurred transiently but was not persistent. This is the phase transition region, or critical zone, between activity failure below and seizure at very high rates above. This zone is increased in size when synaptic dynamics are applied to the synapses (Levina et al., 2007), by making it harder for the network to enter a

seizure state (see Fig. 6) due to synaptic weights depressing further when network firing rate increases, and thus having a homeostatic effect on the firing rate. The other consequence of synaptic dynamics for network activity is the extension of the time before persistent activity fails, perhaps indefinitely.

Discussion

The goal of this research was to gain an understanding of persistent complex dynamics in spiking neural networks: specifically, to investigate the effects of varying local and long-range connectivity and short-term synaptic dynamics on the ability of the network to exhibit activity that is both complex and indefinitely self-sustained. This study shows that local and long-range connectivity (refer to Fig. 1) and short-term synaptic dynamics (refer to Fig. 6) are sufficient to generate persistent complex dynamics in a spiking neural network simulation. Removal of either component from the network reliably resulted in seizure or activity failure, whilst introducing graded levels

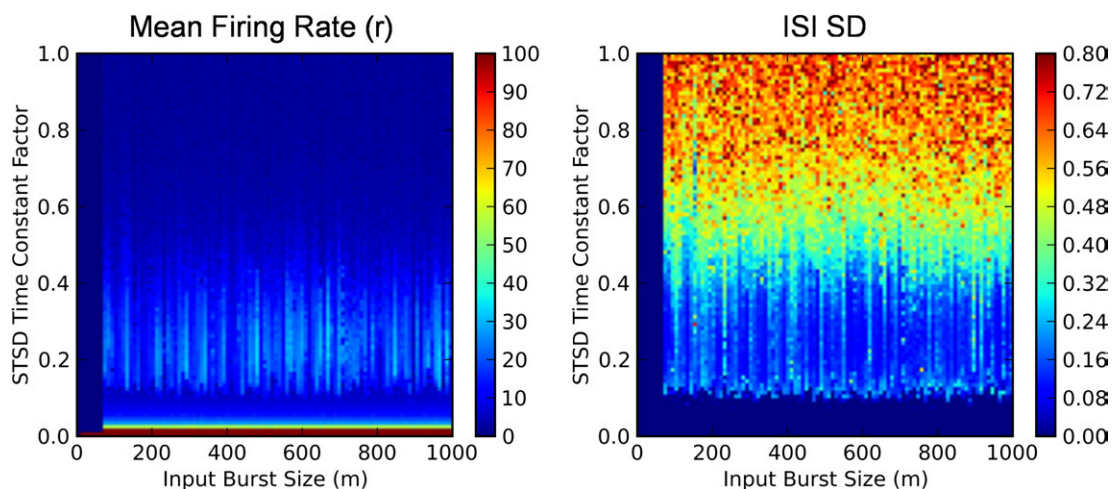


Fig. 6. Network behaviour with varying degrees of synaptic dynamics (network size $n=1000$ neurons, long-range connectivity $k=10$ neurons, local neighbourhood diameter $j=4$ neurons). As the short-term synaptic dynamics time constant is modulated by a factor ε between 0 and 1, the network enters seizure when ε is close to zero (see the horizontal red zone at the bottom of the left graph), and enters a region of complex activity as ε approaches one (see top of right graph). Left: Mean firing rate for all neurons in the network. The colour scale range (0–100 Hz) is increased compared to Fig. 1 in order to show the higher firing rates in the seizure region. Dark blue represents regions of no activity. Red regions are out-of-range activity (approximately 5000 Hz firing; in networks of real cortical neurons these regions would result in *oscillation death* and hence the silencing of network activity—see Methods). Green and cyan are seizures of different firing rates. All lighter blues are regions of persistent non-periodic activity. Without synaptic dynamics ($\varepsilon=0$), the network invariably enters seizure (see the horizontal red zone at the bottom of the graph). As ε increases, the network firing rate decreases in general. Right: Standard deviation of the interspike intervals (ISI-SD) of all neurons in the network. As the time constant factor ε approaches one, the ISI-SD increases, indicating increasingly complex network activity.

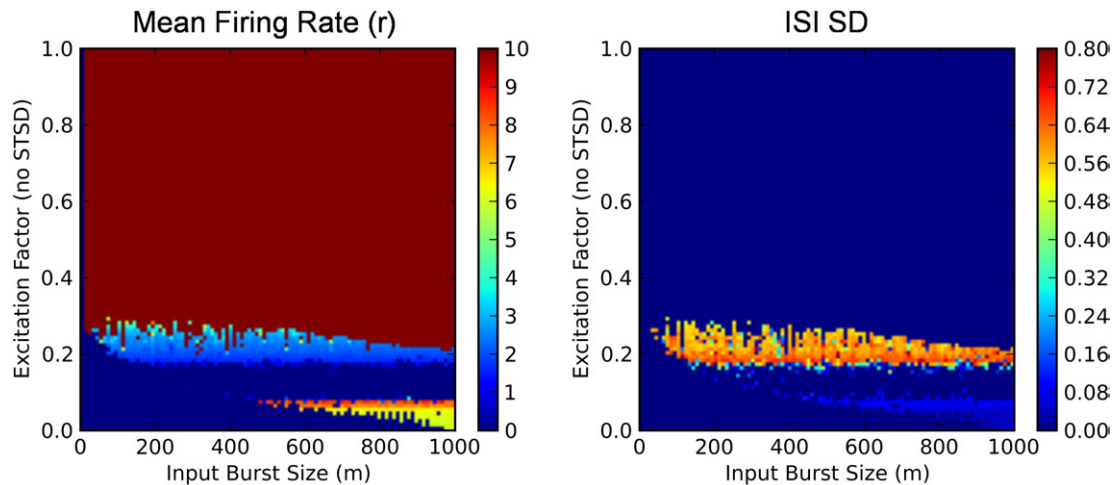


Fig. 7. Small and unstable phase transitions occur without synaptic dynamics (network size $n = 1000$ neurons, long-range connectivity $k = 10$ neurons, local neighbourhood diameter $j = 4$ neurons). Left: Mean firing rate for a network with no short-term synaptic dynamics, varying the static excitatory synapse weight by a factor of between 0 and 1 (Y axis) and the initial input burst size (X axis), shows that the critical region of complex behaviour is small. The mid-blue region on and just above an excitation factor of 0.2 is the critical region from activity failure (dark blue) to out of range activity (red). Network activity in this zone exhibits complex dynamics but it is not sustained indefinitely; the activity will enter seizure or quiescence if simulated for long enough. This zone is analogous to Roxin et al.'s (2004) study which discovered a critical zone of complex behaviour in small world spiking networks that would eventually fail (Roxin et al., 2004; Riecke et al., 2007). However, in the Roxin et al. study, the network activity was in a semi-synchronous regime that we would classify as seizure; the network was in an oscillation state where each neuron fired regularly, with some chaotic jitter, in most or all oscillation periods. Right: Note that activity in the critical zone is not as complex as networks with synaptic dynamics, as indicated by the generally lower ISI-SD (compare with Fig. 6). For the relationship of the colour scale to network behaviour see Fig. 6.

of each back into the network resulted in differing interspike interval standard deviations, reflecting differing levels of dynamical complexity (see Figs. 1 and 6).

The LLDS network exhibits several broad categories of activity. (1) Cessation of activity results when the initial input burst is not sufficient to cause the firing of a large enough number of postsynaptic neurons to sustain further activity. (2) Low complexity seizure-like dynamics at relatively high firing rates (4–5 Hz) occurs for low long-range connectivity because the absence of long-range connections allows the local interactions to fully drive the dynamics, settling into a state where each neuron fires at the same time as its neighbours. Neighbours, with no or few other driving influences, tend to synchronise. (3) Conversely and somewhat counter-intuitively, networks with large numbers of long-range connections tend to settle into states of sustained firing of a single local assembly. This can be understood as follows: high long-range connectivity effectively connects all network elements together with few intervening synapses. When network integration is high, segregation of activity becomes unlikely or impossible, and as such the network operates as a unit entity. In this case, however, seizure does not result, because the large numbers of weaker long-range connections have an averaging effect on network activity and do not strongly propagate synchronous events to highly specific targets; global inhibition is therefore able to exert a stabilising effect, and the relatively stronger but less-numerous local connections cause activity to collapse to a single attractor state. (4) Between these seizure and attractor regimes lies the region of complex persistent dynamics, where the number of long-range connections is sufficient to disrupt the neighbourhood-induced synchrony, but not so much that network-wide integration ensues.

Networks in the region of parameter space that resulted in self-sustained complex dynamics also resulted in interesting macrodynamics in their derived EEGs. In the current study, we have shown that even with the removal of external input, network activity can be persistent in a complex, non-periodic dynamical regime where the power spectrum of the simulated EEG approximates a power law over several orders of magnitude, similar to that observed in recordings from human cortex (see Fig. 3). Power law dynamics are not new in simulation: In several neural network simulation studies,

network activity that approximates power law dynamics has been demonstrated (Wright and Liley, 1996; beim Graben and Kurths, 2008). However, the models in these studies required the injection of input to the networks from external sources to sustain network activity.

Systems exhibiting complex, power-law dynamics often tend to be in a critical (phase transition) state (Bak, 1996; Solé et al., 1996). We have shown that adding short-term synaptic dynamics to a spiking network with local and long-range connectivity creates network dynamics that reliably support persistent, complex non-periodic activity. A recent study has already shown that incorporating synaptic dynamics in simulations of spiking networks can dramatically increase the size of the critical region that supports complex behaviour (Levina et al., 2007); that is, the critical region or phase transition zone between two modes of non-complex network behaviour, such as quiescence and seizure, is expanded over a much larger range of network parameters. However, Levina et al. used fully connected networks and the network required external input to maintain activity. The introduction of short-term synaptic dynamics to the network in this study has two important effects on network dynamics. First, increasing the size of the critical region means that the network does not need to be maintained in a precisely tuned state to achieve desirable dynamics, suggesting that it will be less sensitive to perturbation, noise and damage. Second, the complex behaviour can continue for long periods of time: we have shown that combining local and long-range connectivity and synaptic dynamics allows small networks of just 1000 neurons to remain in a complex dynamic regime for at least 10,000 s. In terms of constraints imposed on biological organisms, 10,000 s is more than sufficient time to enable adaptive processes at synapses to alter the network connectivity and hence the dynamics (e.g. protein expression in a synapse can take place in as little as 1 h (3600 s); Mayford et al., 1995). Furthermore, there are homeostatic processes for the regulation of synaptic efficacy and neuronal firing rates that operate over much shorter timescales (Schulz, 2006), helping to preclude seizure states or loss of sustained activity. Finally, external environmental inputs are also likely to regularly impinge on the network, perturbing its state and altering the complex trajectory of activity. Together, these points suggest that the demonstrated ability of local and long-range connectivity and

synaptic dynamics to maintain complex network behaviour for long periods exceeds the requirements of organisms operating at biologically relevant timescales.

Model networks connected using small world principles have shown both complex behaviour and activity that can be self-sustained for some time before failure (Sporns et al., 2000b; Roxin et al., 2004; Riecke et al., 2007; Shanahan, 2008). The current study affords two extensions to these results. First, dynamical complexity of the LLDS network activity, as measured by the standard deviation of the interspike interval, is higher, since the complex activity regimes in the previous studies involved semi-periodic population or cluster oscillations. Despite the individual neuron spike times in each of these oscillations being unpredictable or chaotic, the distribution of interspike intervals was narrowed by the oscillatory network dynamics. In the current study, no such population-wide oscillations occur, individual neurons can remain quiescent for long and unpredictable durations, and when neurons fire they often repeatedly burst as components of an activated assembly. Such complex dynamics dramatically enlarge the spread of the interspike interval distribution. An additional effect of long interspike intervals is the maintenance of sustained activity at very low mean network firing rates. Second, eventual activity failure in the previous studies was a common if not inevitable outcome. In the current study, activity failure occurs only for networks with very low long-range connection density; for all other network instances, even those that do not demonstrate complex dynamics, activity is sustained for exceedingly long durations.

Brains operating in a complex dynamical regime show both functional segregation and functional integration over a range of spatial and temporal scales (Sporns et al., 2000a; Shanahan, 2008). Brain regions continuously couple and decouple based on the current dynamical state, anatomical connectivity, and the influence of external input. We have shown that, even in a state where the network is disconnected from external influences, the combination of local and long-range connectivity and synaptic dynamics in a spiking network model causes correlations between assemblies to appear and disappear transiently over extended periods of time. This transient dynamic phase coupling is characteristic of complex non-periodic behaviour (Sporns et al., 2000a).

The importance of local and long-range connectivity and short-term synaptic dynamics for maintaining complex persistent network dynamics suggests that some neuropathology may be related to disruptions in these network properties. A common complaint in aging is failure of a well-known memory to surface when required at a given moment (reported as “nothing comes to mind”). It is not clear what underlies this phenomenon, or why it is more frequently reported by older people, but it is conceivable that it relates to a failure to sustain or propagate activity. Such an idea has received support from studies that have demonstrated reduced activity in specific regions with aging (Rypma and D'Esposito, 2000). The converse is for people with depression or post traumatic stress disorder, when an unwanted pattern repeats pathologically. The problem of repetitive thoughts may be related to a failure of the variability of sustained activity—effectively, the cortical dynamics fall into a single trajectory. Past models, relying on external input to sustain complex network activity, have lacked the capacity to isolate and investigate these dynamics, as the application of external input perturbs the network trajectories and obscures the characteristics under investigation. The network architecture presented here allows for the clear investigation of dynamics related to sustained cortical activity in both healthy and damaged or diseased brains.

Conclusion

Many studies have investigated the link between the brain's complex dynamics and its processing capabilities (Sporns et al.,

2000a; Breakspear et al., 2003; Buzsáki and Draguhn, 2004; Fries, 2005; Lakatos et al., 2008; Shanahan, 2008), and even between brain dynamics and mind wandering (Mason et al., 2007). These studies emphasise the diverse dynamics, and hence flexible processing capabilities, conferred by complex activity in the brain. The contribution we have made is to show that local and long-range connectivity and short-term synaptic dynamics (the LLDS network) are sufficient to support persistent, complex, non-periodic activity in networks of spiking neurons. Additionally, the dynamical regime evident in the network activity, on both a micro-scale (transient coupling between regions) and macro-scale (derived EEG), is similar to that observed in brains. The ability to craft such persistent dynamics in a spiking network model creates new opportunities to study neural processing, learning, injury and disease in nervous systems.

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References

- Bak, P., 1996. *How Nature Works*. Copernicus New York, NY, USA.
- Beggs, J.M., Plenz, D., 2003. Neuronal avalanches in neocortical circuits. *J. Neurosci.* 23 (35), 11167–11177.
- beim Graben, P., Kurths, J., 2008. Simulating global properties of electroencephalograms with minimal random neural networks. *Neurocomputing* 71 (4–6), 999–1007.
- Binder, J.R., Frost, J.A., et al., 1999. Conceptual processing during the conscious resting state: A functional MRI study. *J. Cogn. Neurosci.* 11 (1), 80–93.
- Breakspear, M., Terry, J.R., et al., 2003. Modulation of excitatory synaptic coupling facilitates synchronization and complex dynamics in a nonlinear model of neuronal dynamics. *Network* 52, 151–158.
- Buzsáki, G., 2006. *Rhythms of the Brain*. Oxford University Press, USA.
- Buzsáki, G., Draguhn, A., 2004. Neuronal oscillations in cortical networks. *Science* 304 (5679), 1926–1929.
- Ermentrout, G.B., Kopell, N., 1990. Oscillator death in systems of coupled neural oscillators. *SIAM J. Appl. Math.* 125–146.
- Fraiman, D., Balenzuela, P., et al., 2009. Ising-like dynamics in large-scale functional brain networks. *Phys. Rev. E* 79 (6), 61922.
- Freeman, W.J., 1991. Predictions on neocortical dynamics derived from studies in paleocortex. In: Basar, E., Bullock, T.H. (Eds.), *Induced Rhythms of the Brain*. Birkhaeuser Boston Inc, Cambridge MA.
- Freeman, W.J., Rogers, L.J., et al., 2000. Spatial spectral analysis of human electrocorticograms including the alpha and gamma bands. *J. Neurosci. Methods* 95 (2), 111–121.
- Fries, P., 2005. A mechanism for cognitive dynamics: neuronal communication through neuronal coherence. *Trends Cogn. Sci.* 9 (10), 474–480.
- Greicius, M.D., Krasnow, B., et al., 2003. Functional connectivity in the resting brain: a network analysis of the default mode hypothesis. *Proc. Natl. Acad. Sci.* 100 (1), 253–258.
- Honey, C.J., Kötter, R., et al., 2007. Network structure of cerebral cortex shapes functional connectivity on multiple time scales. *Proc. Natl. Acad. Sci.* 104 (24), 10240–10245.
- Izhikevich, E.M., 2003. Simple model of spiking neurons. *IEEE Trans Neural Netw.* 14 (6), 1569–1572.
- Izhikevich, E.M., 2004. Which model to use for cortical spiking neurons? *IEEE Trans Neural Netw.* 15 (5), 1063–1070.
- Kitzbichler, M.G., Smith, M.L., et al., 2009. Broadband criticality of human brain network synchronization. *PLoS Computat. Biol.* 5 (3).
- Lakatos, P., Karmos, G., et al., 2008. Entrainment of neuronal oscillations as a mechanism of attentional selection. *Science* 320 (5872), 110–113.
- Laufs, H., Krakow, K., et al., 2003. Electroencephalographic signatures of attentional and cognitive default modes in spontaneous brain activity fluctuations at rest. *Proc. Natl. Acad. Sci.* 100 (19), 11053–11058.
- Levina, A., Herrmann, J.M., et al., 2007. Dynamical synapses causing self-organized criticality in neural networks. *Nat. Phys.* 3 (12), 857–860.
- Linkenkaer-Hansen, K., Nikouline, V.V., et al., 2001. Long-range temporal correlations and scaling behavior in human brain oscillations. *J. Neurosci.* 21 (4), 1370–1377.
- Markram, H., Wang, Y., et al., 1998. Differential signaling via the same axon of neocortical pyramidal neurons. *Proc. Natl. Acad. Sci.* 95 (9), 5323–5328.
- Mason, M.F., Norton, M.I., et al., 2007. Wandering minds: the default network and stimulus-independent thought. *Science* 315 (5810), 393–395.
- Mayford, M., Abel, T., et al., 1995. Transgenic approaches to cognition. *Curr. Opin. Neurobiol.* 5 (2), 141–148.
- Mazoyer, B., Zago, L., et al., 2001. Cortical networks for working memory and

- executive functions sustain the conscious resting state in man. *Brain Res. Bull.* 54 (3), 287–298.
- Nunez, P.L., Srinivasan, R., 2006. *Electric Fields of the Brain: The Neurophysics of EEG*. Oxford University Press, USA.
- Pecevski, D., Natschläger, T., et al., 2009. PCSIM: a parallel simulation environment for neural circuits fully integrated with Python. *Front. Neuroinformatics* 3 (11).
- Petermann, T., Thiagarajan, T.C., et al., 2009. Spontaneous cortical activity in awake monkeys composed of neuronal avalanches. *Proc. Natl. Acad. Sci.* 106 (37), 15921–15926.
- Riecke, H., Roxin, A., et al., 2007. Multiple attractors, long chaotic transients, and failure in small-world networks of excitable neurons. *Chaos: An Interdisciplinary J. Nonlinear Sci.* 17 (026110).
- Roxin, A., Riecke, H., et al., 2004. Self-sustained activity in a small-world network of excitable neurons. *Phys Rev Lett* 92 (198101).
- Rypma, B., D'Esposito, M., 2000. Isolating the neural mechanisms of age-related changes in human working memory. *Nat. Neurosci.* 3 (5), 509–515.
- Schulz, D.J., 2006. Plasticity and stability in neuronal output via changes in intrinsic excitability: it's what's inside that counts. *J. Exp. Biol.* 209 (24), 4821–4827.
- Shanahan, M., 2008. Dynamical complexity in small-world networks of spiking neurons. *Phys. Rev. E* 78 (4), 041924.
- Solé, R.V., Manrubia, S.C., et al., 1996. Phase transitions and complex systems. *Complexity* 1 (4), 13–26.
- Sporns, O., Tononi, G., et al., 2000a. Connectivity and complexity: the relationship between neuroanatomy and brain dynamics. *Neural Netw.* 13 (8–9), 909–922.
- Sporns, O., Tononi, G., et al., 2000b. Theoretical neuroanatomy: relating anatomical and functional connectivity in graphs and cortical connection matrices. *Cereb. Cortex* 10 (2), 127–141.
- Varela, F., Lachaux, J.P., et al., 2001. The brainweb: phase synchronization and large-scale integration. *Nat. Rev. Neurosci.* 2 (4), 229–239.
- Wang, X.J., 1999. Synaptic basis of cortical persistent activity: the importance of NMDA receptors to working memory. *J. Neurosci.* 19 (21), 9587–9603.
- Watts, D.J., Strogatz, S.H., 1998. Collective dynamics of 'small-world' networks. *Nature* 393, 440–442.
- Wright, J.J., Liley, D.T.J., 1996. Dynamics of the brain at global and microscopic scales: Neural networks and the EEG. *Behav. Brain Sci.* 19 (2), 285–294.