

Using sighting records to declare eradication of an invasive species

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Summary

1. A major challenge for eradication managers is deciding when a programme can be deemed successful. Regan *et al.* (2006) were the first to pose this problem within a decision theory framework, minimizing the net expected cost of the decision. The optimal time to declare eradication was based on the number of consecutive surveys in which the species was not found ('absent surveys'). Their formulation used estimates of detectability and persistence – parameters that are often difficult to estimate – to calculate the probability that the invasive species is still present.
2. Here we use a similar decision-making framework but instead predict presence based on the pattern of sightings, using a method developed by Solow (1993a) that assumes a constant sighting rate. This method does not require estimates of detectability and persistence. We find the number of absent surveys after which eradication should be declared, using three approaches: a stochastic dynamic program, which finds the exact optimal solution, a rule of thumb, and an approximation. We then compare these results with a method assuming a declining sighting rate.
3. Both the rule of thumb and approximation give results that are close to the exact optimal solution. The rule of thumb with the declining sighting rate method generally gives a larger optimal number of absent surveys.
4. *Synthesis and applications.* Analysing this problem within a decision theory framework enables us to minimize the expected cost of declaring eradication. By using the more readily available sighting data, we make this framework applicable to a wider range of invasive species. Our approximation is a simple calculation, making it an accessible tool that could be applied by managers of eradication programmes for invasive species.

Key-words: Bayesian statistics, decision theory, detectability, economic costs, presence–absence data, rule of thumb, weed

Introduction

Eradication of invasive species requires the removal of every individual of a species from a target area – for plants this entails the removal of both adults and seeds. There have been many successful eradications of both animals and plants (Mack & Lonsdale 2002; Simberloff 2002, 2003). A major challenge facing eradication managers is deciding when a programme can be declared successful (Morrison *et al.* 2007). Survey techniques are imperfect, and thus, the failure to detect a species does not necessarily mean it is absent. An invasive species can re-emerge if eradication is declared prematurely, and monitoring stopped, with resulting ecological impacts and costs of further management. Despite this risk, eradication is still declared on an *ad hoc* basis (Regan *et al.* 2006), for example, after 3 years without detection (Rejmanek & Pitcairn 2002).

Regan *et al.* (2006) took an economic approach to the question of when to declare eradication, using decision theory. They found the stopping time (based on the number of previous consecutive surveys in which the species is not found, hereafter referred to as absent surveys) that minimizes the net expected cost. This is essentially a trade-off between the cost of continued surveying and the cost if eradication is declared when the species is still present.

Although this work represents a new way of thinking about how we approach setting guidelines for invasive species eradication, its practicality is reduced by the data requirements of the model. To calculate the probability that an invasive species is still present after a number of absent surveys, Regan *et al.* (2006) used probabilities of persistence and detection. These parameters are difficult to estimate for many invasive species. For example, in the field of population viability analysis, uncertainty around the estimates of probabilities of persistence can span zero and one (Ludwig 1999;

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McCarthy, Burgman & Ferson 1996). Similarly, methods for estimating detection probabilities usually require labour-intensive data (MacKenzie *et al.* 2002; Tyre *et al.* 2003; Wintle *et al.* 2004), and detection probabilities for newly invading species are likely to be very uncertain. Instead of estimating these parameters, we can use the presence–absence sighting record of the species.

There are several methods documented in conservation literature that use a species' sighting record to infer persistence. Solow (1993a) used a presence–absence sighting record, and assumed a constant pre-extinction sighting rate. This essentially assumes that the species' population level remains constant prior to extinction. Solow (1993b) described a variation of the equation for use in declining populations where the pre-extinction sighting rate declines. Solow & Roberts (2003) described a nonparametric test, based on the two most recent sightings of the species. These methods and variations are summarized by Solow (2005).

In addition to these, several papers have focused on using sighting records and collection data to identify declining or threatened species. Burgman, Grimson & Ferson (1995) extended the equation from Solow (1993a) to account for multiple sightings within one time period, and explored methods that are sensitive to patterns in sighting data. McCarthy (1998) used five different methods of identifying declining species from museum records, including methods that account for variable collection effort. McInerney *et al.* (2006) modified Solow's equation to remove the influence of the length of the initial sighting period. Of these methods that use sighting data to infer extinction and decline, Solow (1993a) is the only one to include a Bayesian formulation of the probability of presence.

Two other studies tested these statistical methods. Burgman *et al.* (2000) calculated the power of Solow's original equation and a runs test (Grimson, Aldrich & Wanzer Drane 1992) by applying them to data generated from a scenario where the 'true' rate of population decline was known. They found both equations had a type I error rate (probability of detecting a decline when there is none) of less than or equal to the conventional limit of 0.05. Robbirt, Roberts & Hawkins (2006) used herbarium data for endemic Ecuadorian species of *Guzmania* (Bromeliaceae) to compare the results from five different statistical methods with the IUCN (International Union for the Conservation of Nature) listing of each species. The correlation results were not significant in the traditional sense (P value < 0.05), but were close to significant (P value < 0.1).

We use the Bayesian formulation of the equation from Solow (1993a) to calculate the probability that a weed is still extant, given a presence–absence sighting record. This equation assumes a constant pre-extinction sighting rate, meaning the invasive species population level is constant until eradication. We incorporate this into the decision-making framework of Regan *et al.* (2006) to determine the optimal number of absent surveys after which eradication should be declared. First, we examine the analytical solution of the new equation, and find a simple approximation for when to declare eradication. Then, we use stochastic dynamic pro-

gramming to find an exact optimal solution that incorporates the possibility that the weed may be seen in future surveys. In addition, we include a solution with a declining pre-extinction sighting rate, meaning the invasive species population declines prior to eradication. We apply these methods to the example of bitterweed *Helenium amarum*, the same case study used by Regan *et al.* (2006).

Methods

We apply the method from Solow (1993a) to cases in which active surveys for an invasive species are conducted. For an eradication project, we would expect individuals sighted during surveys to be removed. The sighting record is assumed to follow a Poisson process with a constant but unknown pre-extinction sighting rate. We use the number of surveys as a unit for the sighting record. Assuming survey effort is constant, a constant sighting rate implies that the invasive species population level is constant prior to eradication, with reproduction balancing out removed individuals. This describes a very small, stable population subject to rapid extinction.

A total of S surveys have been performed, and the species has been sighted in n of these (s_1, s_2, \dots, s_n). The species has not been seen for the last $S - s_n$ surveys. These surveys do not occur with reference to time, but they must occur far enough apart as to be independent observations. The probability the species is extant given its sighting record \mathbf{s} is:

$$p(\text{extant} | \mathbf{s}) = (1 + \{(1 - \pi)/[\pi B(\mathbf{s})]\})^{-1}, \quad \text{eqn 1}$$

where π is the prior probability the species is extant, independent of the sighting record (Solow 1993a). $B(\mathbf{s})$ is the Bayes factor, the ratio of likelihoods:

$$B(\mathbf{s}) = l(\mathbf{s} | \text{extant})/l(\mathbf{s} | \text{extinct})$$

which, is (Solow 1993a):

$$B(\mathbf{s}) = (n - 1)/[(S/s_n)^{n-1} - 1]. \quad \text{eqn 2}$$

RULE OF THUMB

Regan *et al.* (2006) outlined a 'rule of thumb' for finding when an invasive species should be declared eradicated. Their framework is based on finding the number of consecutive surveys in which the species is not found after which eradication should be declared. The optimal number of consecutive absent surveys is that which gives the lowest net expected cost (NEC). The NEC of stopping after an absent survey is the cost of surveying, plus the expected cost of escape and damage if the species was present but went undetected (Regan *et al.* 2006). The NEC of stopping after d absent surveys (where $d = S - s_n$) is:

$$\text{NEC}(d) = (d - 1)C_s + p(d)C_e,$$

where C_s is the cost of one survey, C_e is the expected cost of escape and damage, and $p(d)$ is the probability that the species is present after d absent surveys.

In Regan *et al.* (2006), $p(d)$ is given by $[h(1 - q)]^d$, where h is the annual probability the species persists and q is the annual probability of detection. Instead, we substituted $p(d)$ with equation 1 to give:

$$\text{NEC}(d) = (d - 1)C_s + C_e/[1 + \{(1 - \pi)/[\pi B(\mathbf{s})]\}], \quad \text{eqn 3}$$

where $B(s)$ is given by equation 2. The minimum NEC (and thus, the optimal value of d , denoted d^*) occurs where the derivative of this equation with respect to d is equal to 0, that is:

$$C_s + \frac{C_e(n-1)^2(\pi-1)\pi y^n}{s_n((1-\pi)y^n - (1-n\pi)y)} = 0 \quad \text{eqn 4}$$

where $y = (s_n + d^*)/s_n$. We cannot rearrange this equation to give d^* directly, and thus instead, we find d^* numerically by calculating equation 3 for a range of integer values of d and identifying the value with the lowest NEC.

APPROXIMATION

We can also derive an approximate direct expression for d^* . We cannot rearrange equation 4 for d^* , but we can rearrange it to find the value of the cost ratio R (i.e. $R = C_s/C_e$) where the NEC is minimized. We name this value R_{crit} , a critical value of R at which it becomes optimal to stop surveying, that is, for $R < R_{\text{crit}}$ we should keep surveying, while for $R > R_{\text{crit}}$ we should stop. R_{crit} is:

$$R_{\text{crit}} = \frac{(n-1)^2(1-\pi)\pi y^n}{s_n[(1-\pi)y^n - (1-n\pi)y]}$$

and its natural log is:

$$\ln(R_{\text{crit}}) = 2 \ln(n-1) + \ln(1-\pi) + \ln(\pi) + n \ln(y) - 2 \ln[(1-\pi)y^n - (1-n\pi)y]$$

Where y is large, $\ln(R_{\text{crit}})$ is approximately linear with respect to $\ln(y)$. A Taylor series approximation of $\ln(R_{\text{crit}})$ around a large value of y gives:

$$\ln(R_{\text{crit}}) \approx 2 \ln(n-1) + \ln(\pi) - \ln(1-\pi) - n \ln(y)$$

which means

$$R_{\text{crit}} \approx \frac{(n-1)^2 \pi}{s_n(1-\pi)y^n}$$

We can then substitute $y = (s_n + d^*)/s_n$ and rearrange to get the expression:

$$d^* \approx s_n \left(\frac{(n-1)^2 \pi}{s_n(1-\pi)R} \right)^{\frac{1}{n}} - s_n \quad \text{eqn 5}$$

This approximation to the rule of thumb can be used to find the approximate optimal value of d (not necessarily an integer value), without repeatedly calculating the net expected cost.

STOCHASTIC DYNAMIC PROGRAMMING

The rule of thumb (equation 3) and its approximation (equation 5) do not include the possibility that the species may be seen in a future survey, incurring further costs of surveying and possible escape and damage. To incorporate these future expected costs, we can use stochastic dynamic programming. Stochastic dynamic programming is an optimization algorithm that can be applied to any system with a finite number of states, where the dynamics are described by a Markov chain and sequential decisions must be made (Bellman 1957; Lubow 1996; Mangel & Clark 1988). It works backwards over time, finding optimal decisions for each possible management scenario

that take into account future expected costs (Bellman 1957; Lubow 1996; Mangel & Clark 1988).

The formulation of our stochastic dynamic program (SDP) is similar to that in Regan *et al.* (2006). In each time step m (1 to M), there are two possible management decisions: to survey or to stop. The optimal decision is the one with the lowest expected cost. As outlined previously, the species has a sighting record in which it is seen n times in s_n surveys, and then not seen for d surveys. The optimal stopping time for particular values of n and s_n is the smallest d where the expected cost of stopping is less than the expected cost of surveying. The expected cost of stopping is the probability that the species is extant given its sighting record, multiplied by the expected cost of escape and damage:

$$E_{\text{stop}}(m, d, n, s_n) = p(\text{species extant} | n, s_n, d)C_e$$

which substituting equation 1 becomes:

$$E_{\text{stop}}(m, d, n, s_n) = C_e / (1 + \{(1-\pi)/[\pi B(s)]\}),$$

where $B(s)$ is given by equation 2. The expected cost of surveying must encompass two possibilities: the species is detected or not detected. The sighting record can be updated for each case. If the species is detected, the number of sightings n becomes $n+1$, while the most recent sighting s_n becomes s_n+d . The number of absent surveys d becomes 0. If the species is not detected, d becomes $d+1$, while n and s_n remain constant. The expected cost of surveying is thus:

$$E_{\text{survey}}(m, d, n, s_n) = C_s + p(\text{extant})p(\text{detected})E_{\text{opt}}(m+1, 0, n+1, s_n+d) + (1-p(\text{extant})p(\text{detected}))E_{\text{opt}}(m+1, d+1, n, s_n) \quad \begin{matrix} (m < M) \\ (m = M) \end{matrix}$$

The probability that the species is extant is given by equation 1. E_{opt} is the expected cost of future optimal decisions, where the optimal decision gives the lowest expected cost:

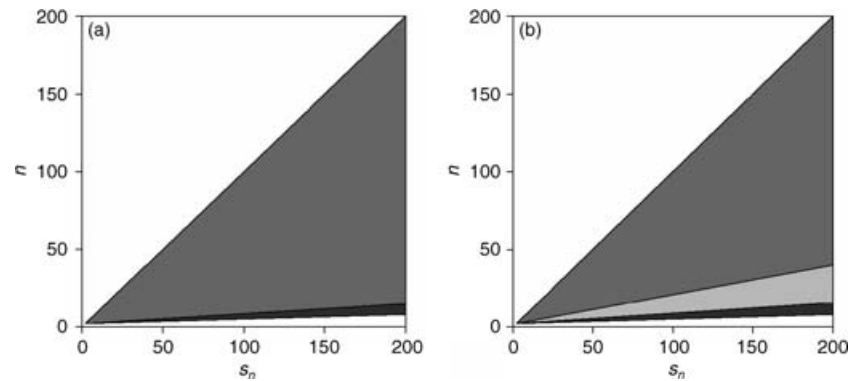
$$E_{\text{opt}}(m, d, n, s_n) = \min[E_{\text{stop}}(m, d, n, s_n), E_{\text{survey}}(m, d, n, s_n)].$$

If the species went extinct immediately after it was last detected in survey s_n , its probability of detection would be n/s_n . However, if the species was extant but undetected for k of the d absent surveys, the detectability estimate would be $n/(s_n+k)$. The value of k could be anywhere between 0 and d . We can determine a general probability of detection by weighting the estimates for each possible k by the probability that the species was present at survey s_n+k , but then went extinct before s_n+k+1 :

$$p(\text{detected} | n, s_n, d) = \sum_{k=0}^d \left[\left(\frac{n}{s_n+k} \right) (p(\text{extant} | n, s_n, k) - p(\text{extant} | n, s_n, k+1)) \right]$$

The SDP is calculated backwards, and thus, it begins by calculating the expected cost of each decision in the final time step M . The optimal decision is the one with the lowest expected cost. It then steps back to the previous time step $M-1$ to calculate the expected cost of each decision, assuming the optimal action is taken in time step M . It continues to step backwards, repeating this process and finding the optimal decision for each time step. The optimal decision for the first time step thus accounts for future possibilities and costs, assuming that all future decisions are optimal. For this reason, we have focused our analysis of SDP results on those from the first time step.

Fig. 1. Performance of the (a) the rule of thumb and (b) the approximation as compared to the exact optimal results from the stochastic dynamic program (SDP). Mid-grey shading indicates combinations of n and s_n where the result was within one absent survey of the SDP result. Dark grey shading indicates where the optimal result was underestimated by more than one absent survey (up to 134 surveys for both methods), and light grey shading indicates where the optimal result was overestimated (by up to 2.23 surveys).



ALTERNATIVE SIGHTING RATE MODEL

The methods described above assume the sighting record of the species represents a Poisson process, with a constant pre-extinction sighting rate. A plausible alternative model is that the pre-extinction sighting rate λ is not constant, but declines with the number of surveys s . We define $\lambda(s)$ as:

$$\lambda(s) = ms^{-a}, \quad \text{eqn 6}$$

where m is a constant, and a is a constant between 0 and 1 (where $a = 0$ corresponds to the constant rate case above). We chose a simpler sighting rate function than that investigated by Solow (1993b), to enable us to calculate the Bayes factor. The Bayes factor for this model (integrated over all possible values of m) is:

$$B(s) = (n(1-a) - 1) / [(S/s_n)^{n(1-a)-1} - 1]$$

Details of this calculation are provided in Supporting Material, Appendix S1. To investigate the optimal decisions under this model, we substituted this Bayes factor into the rule of thumb (equation 3) to calculate the net expected cost of stopping after d absent surveys.

CASE STUDY

We apply the method to the eradication of *Helenium amarum* (bitterweed) in Queensland, Australia – the example that was used in Regan *et al.* (2006). *H. amarum* is toxic to stock, and if ingested causes vomiting, diarrhoea, and production of bitter undrinkable milk. It was first found in Queensland in 1953, and an eradication programme began in the same year. After 3 years of herbicide and manual removal, only isolated patches of plants remained. Between 1988 and 1992, no plants were detected, and the weed was declared eradicated (Tomley & Panetta 2002). This eradication seemed to have been successful (Regan *et al.* 2006), but in March 2007 a small infestation of *H. amarum* was discovered at the site of original occupancy, and control activities are currently underway (D. Panetta, personal communication).

We used the best estimate parameters in Regan *et al.* (2006) and the raw sighting data of *H. amarum* to parameterize the models. The best estimate for the cost ratio $R(C_f/C_c)$ is 0.00282, meaning the cost of escape is 354 times the cost of surveying (Regan *et al.* 2006). The sighting record for *H. amarum* comprises 178 surveys ($S = 178$) between 1953 and 1992. Until the last sighting in 1987, 169 surveys were conducted ($s_n = 169$), and *H. amarum* was seen in 142 of those surveys ($n = 142$). *H. amarum* was declared eradicated after 9

consecutive absent surveys in the period 1988–1992 ($d = 9$). We used a non-informative prior probability of presence π of 0.5. In the SDP, we found optimal decisions for 20 consecutive choices ($M = 20$), and we discuss the optimal decisions for the first of these as they incorporate the possible future outcomes of the next 19 choices. For the alternative model of sighting rate, we used WinBUGS software (Spiegelhalter *et al.* 2003) to fit the declining sighting rate model (equation 6) to the raw sighting data for *H. amarum*. The estimate was $\alpha \approx 0.1$ when using a uniform prior.

Results

PERFORMANCE OF RULE OF THUMB AND APPROXIMATION

The exact optimal number of absent surveys before stopping (calculated with the SDP) decreases as the sighting frequency n/s_n increases (Supporting Material, Fig. S2). For most combinations of n and s_n , the rule of thumb gives results that are within one absent survey of the exact optimal results from the SDP (Fig. 1a). Larger differences occur when the sighting frequency is very low, with the rule of thumb underestimating the exact optimal result. Similarly, the approximation gives results that are within one absent survey of the exact optimal for most combinations of n and s_n , and tends to underestimate the optimal result when the sighting frequency is very low (Fig. 1b). The approximation also tends to slightly overestimate the optimal result (by up to three surveys) when the sighting frequency is moderately low.

Using the constant sighting rate model, the rule of thumb, SDP and approximation all give an optimal stopping time of 13 absent surveys ($d^* = 13$) for *H. amarum* (Fig. 2: rule of thumb; Supporting Material, Fig. S3: SDP, approximation $d^* = 13.15$). The SDP finds the optimal decision, to keep surveying or stop, for every possible sighting record (every combination of n , s_n and d). If there have been 12 consecutive absent surveys, it is optimal to stop surveying for many combinations of n and s_n , but not for the initial sighting record of *H. amarum* (Supporting Material, Fig. S3a). After 13 consecutive absent surveys, it is optimal to stop surveying for *H. amarum* (Supporting Material, Fig. S3b). As the number of absent surveys increases, the number of combinations for which it is optimal to stop surveying increases.

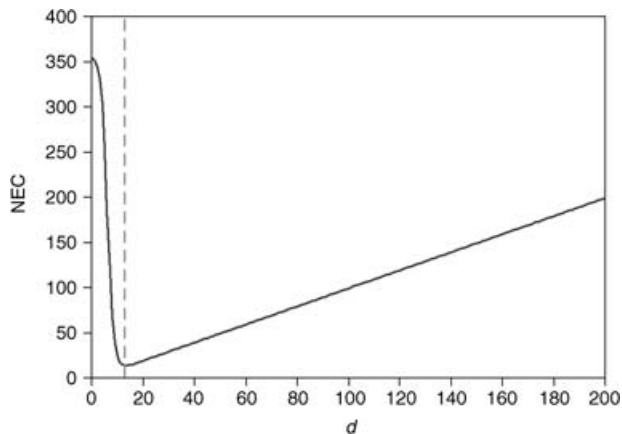


Fig. 2. The net expected cost (NEC) as a function of the number of absent surveys for *H. amarum*, calculated using the rule of thumb (equation 3). The lowest NEC occurs after 13 absent surveys (marked with dotted line), making this the optimal number after which to declare eradication.

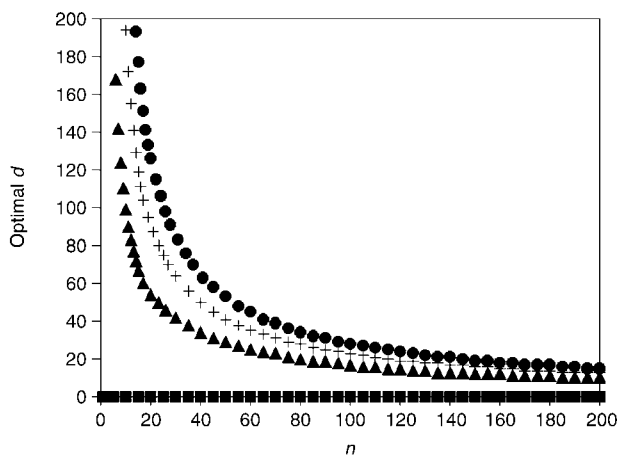


Fig. 3. SDP results for different values of the cost ratio $R(C_s/C_e)$, for $s_n = 200$. Squares, $R = 1$ or 0.1 ; triangles, $R = 0.01$; crosses, $R = 0.001$; circles, $R = 0.0001$. The prior probability of presence is kept constant at $\pi = 0.5$.

SENSITIVITY OF OPTIMAL SOLUTION

The results from the SDP are sensitive to the ratio of the cost of surveying to the cost of escape ($R = C_s/C_e$). For a particular combination of n and s_n , decreasing this ratio R increases the optimal number of absent surveys. For $s_n = 200$, the results for $R = 1$ and $R = 0.1$ are the same – stop surveying even if the weed has been seen in the previous survey ($d^* = 0$) (Fig. 3). For smaller cost ratios ($R = 0.01, 0.001, 0.0001$), the optimal number of absent surveys decreases as n increases (Fig. 3). The difference between results for each cost ratio is greater for smaller values of n (Fig. 3).

The SDP results are also sensitive to the prior probability of species presence (π). Higher values of the prior give a larger optimal number of absent surveys (Fig. 4). As with the cost ratio, the difference is larger for small values of n . The difference is also greater around the edges of the range for π , that is,

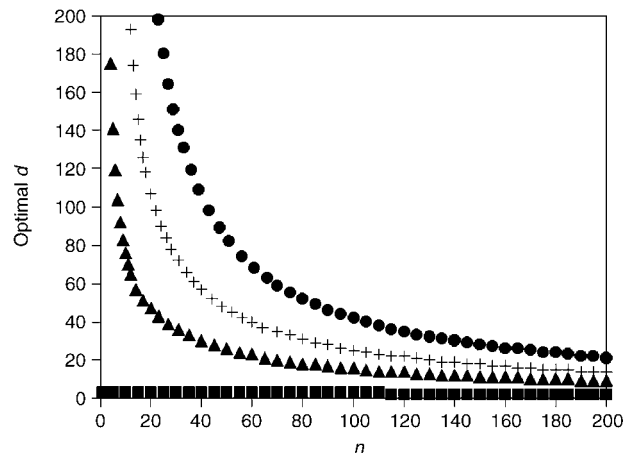


Fig. 4. SDP results for different values of the prior π , for $s_n = 200$. Squares, $\pi = 0.0001$; triangles, $\pi = 0.1$; crosses, $\pi = 0.9$; circles, $\pi = 0.9999$. The cost ratio is kept constant at $R = 0.00282$.

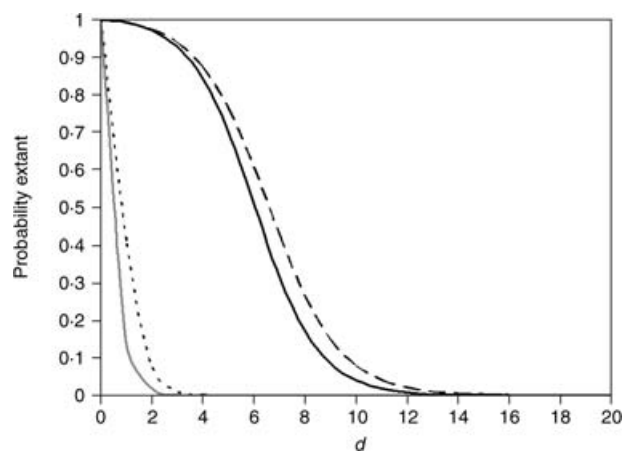


Fig. 5. The probability that *H. amarum* is extant as a function of the number of consecutive absent surveys (shown up to $d = 20$), calculated with four different methods. The solid grey line is calculated as in the rule of thumb in Regan *et al.* (2006), and the dotted line is calculated as in the SDP in Regan *et al.* (2006). Both of these used the best estimate parameters for *H. amarum* described in Regan *et al.* (2006), and assumed surveys are conducted annually. The solid black line is calculated using Solow's equation, and the dashed line is calculated using the declining sighting rate equation. Both of these used the sighting record for *H. amarum* ($s_n = 169, n = 142$), and $\pi = 0.5$ as the prior probability it is extant. The declining sighting rate equation used $a = 0.1$.

when π is close to 0 or 1. For example, the difference in results between $\pi = 0.9$ and $\pi = 0.9999$ is similar to that between $\pi = 0.1$ and $\pi = 0.9$.

DECLINING SIGHTING RATE MODEL

When using the parameters for *H. amarum* (including $a = 0.1$), the rule of thumb with the declining sighting rate model gave an optimal d of 14, as compared with 13 from the original model. For any value of d , the declining sighting rate equation gives a higher probability that the species is extant than Solow's equation, although both give much higher probabilities than the methods used in Regan *et al.* (2006) (Fig. 5).

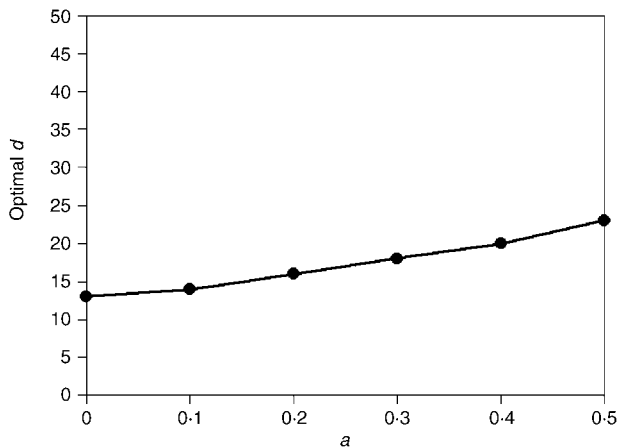


Fig. 6. The optimal number of consecutive absent surveys found using the declining sighting rate equation, for different values of parameter a . All other parameters values are set for *H. amarum*. When $a = 0$, the declining sighting rate equation is equivalent to Solow's equation.

The parameter a in this model represents the magnitude of decline in sighting rate over the original sighting period. When $a = 0$, this model is equivalent to the constant sighting rate model, and it gives the same optimal number of absent surveys (Fig. 6). The optimal number of absent surveys then increases as a increases, but the increase up to $a = 0.5$ is gradual: when $a = 0.5$, $d^* = 23$.

Discussion

Regan *et al.* (2006) provided the first decision theoretic method for determining when to declare eradication of an invasive species. We examine alternative methods for calculating the probability an invasive species is extant, for use in their decision-making framework. Although our results are not directly comparable with those in Regan *et al.* (2006) as they are in different units (surveys vs. annual surveys), it is clear that the method used to calculate this probability influences the optimal decision of when to declare eradication. The question is then: which method is more suitable? A benefit of methods that use sighting data is that they are amenable to power analysis through simulation, and thus, their accuracy in different circumstances can be determined (see Burgman *et al.* 2000). Otherwise, the method used may simply depend on the type of data available. Regan *et al.*'s (2006) method requires estimates of annual detectability and annual probability of persistence. When these estimates are not available, methods that make use of the raw sighting data may be preferable. However, they instead require an estimate of the prior probability of presence.

The prior probability of presence could be obtained by eliciting probabilities from one or more experts (McCarthy 2007), or by summarizing the success and failure of similar eradication attempts. For example, in a review of goat *Capra hircus* eradications from islands, Campbell & Donlan (2005) found 120 documented successes and 10 documented failures. In this case, the prior probability of an unsuccessful eradication

(and thus, the prior probability that goats are still present on an island following an eradication attempt) is 0.08. In some cases, probability of eradication can be predicted, a priori, as a function of characteristics of the site or species. For example, Campbell & Donlan (2005) show that eradication of goats from larger islands was less successful than from smaller islands. Similarly, eradication success will be a function of the level of effort. McCarthy & Masters (2005) illustrated this approach for predicting annual survival rates of birds by constructing regression models based on previously published data. A meta-analysis of the eradication literature would provide a means for generating prior probabilities of eradication failure, based on the characteristics of the eradication programme.

In the absence of such a meta-analysis, we initially chose a prior of $\pi = 0.5$ for *H. amarum*, and analysed the results over the entire range of π . We found the exact optimal results were sensitive to this prior probability, with sensitivities being more pronounced where the initial sighting frequency was very low. The exact optimal results were also more sensitive to changes in the cost ratio R when the sighting frequency was very low. For cryptic species with fewer sightings in the original sighting period, it is important to have reliable estimates of these parameters.

As mentioned in the Methods section, we have applied Solow's equation to active surveys of species. In Solow (1993a), the pre-extinction sighting rate is constant over time, which is not sensible when surveys are being carried out – it implies the probability of sighting during a survey is the same as the probability of sighting between surveys. We have instead applied Solow's method using the number of surveys as a unit for the sighting record. This means that the sighting rate is constant for each survey, regardless of how much time has passed between surveys. However, enough time must pass for each survey to be considered an independent observation. If a species has been surveyed at regular time intervals throughout its sighting period, then the number of surveys can be used as an index for time.

The interpretation of the sighting record in units of surveys rather than time has little impact when assuming a constant sighting rate. However, care needs to be taken when interpreting results for our declining sighting rate model. The declining sighting rate can be interpreted as modelling a declining population: as the size of the population decreases, the probability of sighting an individual declines. However, the sighting rate does not decline over time, but declines with the number of surveys s . This would be reasonable if eradication efforts occur when surveys occur, for example, if herbicide is applied while surveying.

We have provided two different models for the pre-extinction sighting rate: constant and declining. The constant sighting rate model is suitable for very small populations that are relatively stable but subject to rapid extinction (Solow 1993b). This is a reasonable description of a population where declaring eradication is a possibility. We are focusing on populations that have already been reduced to a very low level, and thus, if they are still declining, they are not likely to be declining at a great rate for any length of time. This was apparent

when fitting the declining sighting rate model to our case study of *H. amarum*. If the decline in sighting rate is only slight, our results suggest that the optimal number of absent surveys is not greatly different under the constant sighting rate model. In general, assuming a declining sighting rate increases the optimal number of absent surveys compared with a constant sighting rate.

There are numerous other methods to calculate the probability of presence from sighting data (see Introduction), each with their own assumptions about the sighting record and species population. Within this decision-making framework, we could substitute a number of different methods, depending on the species in question and the type of data available. If available data are more detailed than a presence-absence record, then methods that make use of this additional information (e.g. Solow *et al.* 2008) would be preferable. We are currently examining the range of results that might be achieved given plausible alternative models for how the probability of presence declines with the number of consecutive surveys in which a species is not found. Another way to account for uncertainty as to the most appropriate method of calculating the probability of presence is to find a decision that will deliver a satisfactory outcome and is most robust to uncertainty (Ben-Haim 2006).

We have explored here three different methods for finding the optimal number of absent surveys before declaring eradication: the SDP, the rule of thumb, and an approximation. These three methods trade off accuracy and accessibility. The SDP calculates the exact optimal result, but the method and the results it produces are both quite difficult to understand and interpret. Computer programming skills are required to conduct the analysis and summarize the results. The rule of thumb is more intuitive, but still requires time and some technical skills to calculate the net expected cost over a range of values of *d* to find the minimum. The approximation is a simple calculation that could be performed quickly with a calculator, making it an accessible tool that could be applied by decision-makers and managers of invasive species eradication programmes. The rule of thumb and approximation perform well when compared with the exact optimal results from the SDP. Accuracy of these methods is only a concern for the most cryptic species where the initial sighting frequency is very low.

By using decision theory methods to declare eradication of invasive species, decisions can be made that are scientific, transparent, and justifiable. We have outlined a method whereby such systematic decisions can be made even with sparse data. By finding a simple approximate solution, we make it possible to make systematic decisions without a complex quantitative analysis. This increase in practicality and accessibility will hopefully facilitate use of decision theory by managers of invasive species eradication programmes.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1. Derivation of Bayes factor for declining sighting rate model.

Fig. S1. Optimal number of absent surveys before declaring eradication for a range of sighting frequencies.

Fig. S2. Optimal decisions for different sighting records, using the costs for *H. amarum*.

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