Regret Theory with General Choice Sets

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Abstract

The regret theory of choice under uncertainty proposed by Loomes and Sugden has performed well in explaining and predicting violations of Expected Utility theory. the original version of the model was confined to pairwise choices, which limited its usefulness as an economic theory of choice. Axioms for a more general form of regret theory have been proposed by Loomes and Sugden. In this paper, it is shown that a simple non-manipulability requirement is sufficient to characterize the functional form for regret theory with general choice sets. The stochastic dominance and comparative static properties of the model are outlined. A number of special cases are derived in which regret theory is equivalent to other well–known theories of choice under uncertainty.

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Regret Theory with General Choice Sets

1. Introduction

The Expected Utility (EU) model of choice under uncertainty has dominated its field almost since its formulation by von Neumann and Morgenstern (1944). In recent years a number of alternatives to, and generalizations of, the model have been proposed in response to long–standing criticisms of its axiomatic foundations (Allais 1953) and a number of observed violations of its predictions.

The majority of alternative formulations follow EU in ranking prospects according to some evaluation functional. However, an alternative approach focusing on the notion of pairwise choice between alternatives has been developed by Bell (1982), Fishburn (1983) and Loomes and Sugden (1982, 1983, 1987a). An intuitively attractive presentation of this framework has been the regret theory model of Loomes and Sugden.

Regret theory has been employed with some success to explain the results of experiments which yield violations of EU theory, and has also shown some predictive power in experiments designed to test for the existence of regret effects (Loomes and Sugden 1987b). More recently, Loomes and Sugden have presented a version of regret theory which applies to any finite set of acts. However, there has been little consideration of the properties of functional forms for the general case.

A critical issue in the analysis of regret theory is that the pairwise choices yielded by regret theory are intransitive. On the face of it, this means that an individual with regret theoretic preferences is open to manipulation by a 'money pump' using successive pairwise choices over three or more prospects. Such a money pump would permit the individual's wealth to be driven to zero, given sufficiently many iterations. Loomes and Sugden, in their original paper and subsequently, have argued that in this situation, people with regret theoretic preferences will take the entire choice set into account and will, therefore, not be vulnerable to a money pump. This problem can best be assessed in the context of a version of regret theory which covers multiple, rather than pairwise choices.

In this paper, it is shown that, under plausible assumptions, there is a unique extension of regret theoretic preferences from the pairwise to the general case. The only assumption additional to those imposed in previous presentations of regret theory is that choices should not be influenced by the availability of alternatives which are statewise dominated. It is shown that anyone whose choices are influenced by such clearly inferior alternatives is open to manipulation in the form of a 'money pump'. If irrelevance of statewise dominated alternatives holds, it is shown that regret must be determined solely by the best attainable outcome in each state of the world.

The general form proposed here does permit choices to be influenced by the availability of actions which are regret–dominated, in the sense that there exists another action in the choice set which would be preferred by all people with regret–theoretic preferences. It is shown that regret theory is consistent with the irrelevance of regret–dominated alternatives only when it reduces to the special case of EU theory. This requirement is not as normatively compelling as the irrelevance of statewise dominated alternatives, since it does not obviously give rise to the possibility of a money pump.

The absence of functional forms for a regret–theoretic model of choice with multiple alternatives has also precluded the development of economic models employing regret theory as an alternative to EU theory. Typical economic applications involve comparative static problems in which a control variable varies continuously over some set. Examples which have been analyzed using the EU approach include portfolio theory (Diamond and Stiglitz 1974) and theory of the firm (Sandmo 1971). The comparative static properties of the general form are discussed in this paper. It is shown that, in a number of plausible problem settings, regret theory becomes equivalent to EU or one of its generalizations for which comparative static properties have already been analyzed.

2. Regret Theory – An Outline

The outline of regret theory presented here follows that given by Loomes and Sugden (1987a), with some changes in notation. Regret theory deals with consequences which are assumed to take the form of monetary payments. Preferences over outcomes reflect the usual preference for more money over less and are denoted by the natural order >.

Consequences arise from the interaction between individual choices and the occurrence of state of the world n, where n is one of a set of N possible states of the world occurring with probability π_n. The case where only two choices, A_i and A_j, are available will be examined first. Suppose that an individual chooses action A_i in preference to A_j, and that the n–th state of the world occurs. The actual consequence is x_{in} while, had he chosen differently, x_{jn} would have occurred. It is assumed that the x_n are members of a set X of consequences, ordered by a preference relationship. The level of satisfaction arising from this choice is denoted $v(x_{in}, x_{jn})$. It is now possible to define

$$v^*(x_{in}, x_{jn}) = v(x_{in}, x_{jn}) - v(x_{jn}, x_{in}), \tag{1}$$

the net advantage, in utility terms, and after taking account of regret and rejoicing, of choosing A_i rather than A_j in the event that state n occurs. The function v is non–decreasing in its first argument and non–increasing in its second, and this property carries over to v^*. In regret theory, A_i is chosen if the expectation of v^*, given by

$$E\,[v^*(x_{in}, x_{jn})] = \sum_{n=1}^{N} \pi_n\, v^*(x_{in}, x_{jn}) \qquad (2)$$

is positive.

As Loomes and Sugden (1986) have pointed out, the functional form given by (1) and (2) guarantees that regret theory will yield preservation of statewise stochastic dominance. That is, if one prospect yields a better outcome in every state, it will be preferred. One the other hand, regret theory does not imply preservation of first stochastic dominance in the ordinary sense. Quiggin (1989) gives an exact characterization of the stochastic dominance properties of regret theory for pairwise choices. Stochastic dominance will hold for A_i over A_j if, and only if, for any state n such that $x_{in} < x_{jn}$, there exists another equally probable state m such that $x_{jm} \leq x_{in} < x_{jn} \leq x_{im}$. This notion can be made more precise for N equiprobable states. Here, A_i regret–dominates A_j (that is, A_i is preferred to A_j for all regret–theoretic preference functionals) if and only if there exists a permutation ρ of the set of states such that

(i) $\rho(\rho(n)) = n$; and

(ii) $x_{i\rho(n)} \geq x_{jn} \quad \forall\, n$

The most obvious difficulty with regret theory is the need to extend the analysis from the case of pairwise choices to that of choices from a finite set of alternatives $\mathcal{A} = \{A_i\colon i = 1 \ldots K\}$. Loomes and Sugden (1982) suggested that this might be done by averaging regret across the available choice pairs using some form of 'action weights'. Loomes and Sugden (1987a) did not develop the action weights approach but proposed a more general formulation in which the function v is extended to be of the form $v(x_{in}, \{x_{kn}\colon k \neq i\})$ and the object is to maximize the expectation of v. It will be simpler in what follows to adopt the logically equivalent representation $v(x_{in}, \{x_{kn}\})$, in which regret is computed with respect to the entire set of alternatives.

Sugden (1992) presents an axiomatization of this general functional form. His axiom set is a modification of that of Savage (1954). The key innovations are that preferences depend on the set of alternatives and that subjective probabilities are axiomatized without reliance on transitivity.

The general form $v(x_{in}, \{x_{kn}\})$ has a number of attractive properties. Provided the set of alternatives is held constant, it will display transitivity. This supports the contention of Loomes and Sugden (1982), that intransitivity in pairwise choice, which is a feature of regret theory, does not imply vulnerability to a 'money pump', when choice is made with the entire set of available actions in mind. Also, statewise stochastic dominance is preserved. That is, if in every state n, $x_{in} \geq x_{jn}$ (with strict inequality in at least one state) then A_i will be preferred to A_j whatever the properties of the other available actions.

One difficulty with this formulation is that preference for A_i over A_j is observable from behavior only if A_i is the most preferred element of the set of available actions. However, Sugden (1992) shows that a unique representation can be inferred from observable choices.

A more fundamental difficulty is the possibility that forms of manipulation other than the simple money pump may permit the exploitation of individuals with regret–theoretic preferences. The object of the present paper is to show that, if such exploitation is not permitted, the class of functional forms is restricted to those in which regret is defined relative to the best attainable outcome in a given state of the world.

3. Irrelevance of Statewise Dominated Alternatives

The most obvious danger of a procedure dependent on the set of alternatives is that manipulation of the set of the alternatives may yield irrational choices. This is the basis of the 'money pump' argument against intransitivity in pairwise choices. If a functional form for v is to be normatively acceptable, it must be proof against manipulation[1].

The simplest form of manipulation arises if the introduction of clearly unattractive alternatives influences decisions over the remaining choices. This permits the construction of a money pump as follows. Suppose that in a pairwise choice *A* is preferred to *B*, but that this choice can be manipulated by the addition of unattractive alternatives Z_1, Z_2, and so on, to the choice set. Each time the addition of a Z_i results in a switch of preferences between A and *B*, the manipulator can extract a payment δ.

This money pump is immune from the argument put forward by Loomes and Sugden (1987a) to show that intransitivity *per se* does not expose a person with regret–theoretic preferences to a money pump. Their point is that once a given option has been considered, the manipulator cannot erase it from the memory of the victim and thereby remove it from the choice set. Hence, it is impossible to repeatedly exploit a preference for *A* from $\{A, B\}$, *B* from $\{B, C\}$ and *C* from $\{C, A\}$. After the first two choices have been offered the effective choice set becomes $\{A, B, C\}$. Because the manipulation considered above relies solely on the addition of new elements to the choice set, it is not affected by Loomes and Sugden's argument.

It is particularly easy to introduce statewise dominated alternatives into the choice set. For example, for any action A_i in the choice set, someone wishing to manipulate choices could propose actions of the form $A_i - \varepsilon$, that is, "choose A_i then pay me ε". While such an action will never be selected, its availability may affect the relative attractiveness of other members of the choice set for some functions *v*. The action weight model has this undesirable property, assuming that all available actions are given positive weights.

[1] The existence of a feasible money pump represents a decisive normative to any normative theory of choice under uncertainty. It is also important in assessing a positive theory, since it is unlikely that a feasible money pump will go unexploited. Hence if a positive theory suggests that money pumps are feasible, it yields a strong and testable prediction. Thus, the research agenda for variants of regret theory that do not display independence of statewise dominated alternatives should be to determine the nature of the money pumps theoretically available and check for the existence (or nonexistence) of institutions that exploit them.

A second advantage for the manipulator is that, regardless of the decision–maker's preferences, the introduction of statewise dominated alternatives into the choice set, in the manner described above, does not involve any risk of loss. Even if the decision–maker should violate monotonicity and choose the dominated alternative, the manipulator receives a strictly non–negative profit.

It is natural, therefore, to consider conditions on the function v^* which ensure that the introduction of statewise dominated alternatives will not affect the ranking of any other pair of actions. We require:

Irrelevance of statewise dominated alternatives (ISDA): Let A_i be the most preferred element of the set of alternatives A, and suppose there are some $A_r, A_s \in \mathsf{A}$ such that A_r statewise dominates A_s. Then A_i isthe most preferred element of the set of alternatives $\mathsf{A}^* = \mathsf{A} - \{A_s\}$. ■

Somewhat surprisingly, this requirement is sufficient to give a fairly complete characterization of the properties of v. We being by defining

$$v^*(x_{in},\ x_{jn},\ \{x_{kn}\}\ ) = v(x_{in}, \{x_{kn}\}\ ) - v(x_{jn}, \{x_{kn}\}\ )$$

E[v^*] is positive if and only if A_i is preferred to A_j given the set of alternatives A.

PROPOSITION 1: Assume that **ISDA** holds. Then v^* may be represented by a function ζ of the form

$$v^*(x_{in}, x_{jn},\ \{x_{kn}\}\ ) = \zeta\,(x_{in}, x_{jn}, \max\,(\{x_{kn}\})) \tag{3}$$

Proof: Throughout the proof attention will be confined to the case of three equiprobable states of the world. (If **ISDA** holds in general, it must hold for arbitrary probability distributions, including, in particular, those with three equiprobable states). The result is proved first for the case where A has three elements. Choose a, b, c, d, e such that

(i) $a \geq b, c$

(ii) In a pairwise choice $(a, a, c) \sim (b, b, a)$

(iii) $d < c$

(iv) $e < a$

From (ii)

$$2v^*(a, b, \{a, b\}) + v^*(c, a, \{a, c\}) = 0$$

For any d satisfying (iii), the prospect (d, d, c) is statewise dominated by (a, a, c). Hence

$$2v^*(a, b, \{a, b, d\}) + v^*(c, a, \{a, c\}) = 0$$

This can only be true if $v^*(a, b, \{a, b, d\})$ is independent of $d < a$.

Similarly, the prospect (b, b, e) is statewise dominated by (b, b, a). Hence for arbitrary e satisfying (iii)

$$2v^*(a, b, \{a, b\}) + v^*(c, a, \{a, c, e\}) = 0$$

This can only be true if $v^*(c, a, \{a, e, c\})$ is independent of $e < a$.

Combining the results for these two cases, we find that for any α, β, γ, $v^*(\alpha, \beta, \{\alpha, \beta, \gamma\})$ can depend on γ only if $\gamma > \alpha$, $\gamma > \beta$, that is, if $\gamma = \max\{\alpha, \beta, \gamma\}$. Hence v^* may be represented by a function of the form ζ given in (3). This completes the proof for the case where **A** has three elements.

The remainder of the proof is a straightforward induction. Assuming that the result holds for any m prospects, it is always possible to construct an additional dominated prospect to extend the result to the case of $m + 1$ prospects. Consider in particular the case when (i)–(iv) are satisfied, when $m - 1$ of the prospects yield (a, a, c) and one yields (b, b, a). The inductive hypothesis ensures that indifference is maintained. The addition of dominated prospects (d, d, c) and (b, b, e) completes the inductive step. ■

Some minor complications arise in extending this result from v^* to v. We have:

Corollary 1: Assume that **ISDA** holds. Then, if the number of alternatives is N, v may be represented by functions $\phi:\Re^2 \to \Re$, $\eta:\Re^{N} \to \Re$ where ϕ is independent of N and

$$v(x_{in}, \{x_{kn}\}) = \phi(x_{in}, \max\{x_{kn}\}) + \eta(\{x_{kn}\}). \tag{4}$$

Proof: Suppose that for all real numbers x_{in}, x_{jn} and sets $\{x_{kn}\}$ and $\{x'_{kn}\}$ with $\max\{x_{kn}\} = \max\{x'_{kn}\}$and $x_{in}, x_{jn} \in \{x_{kn}\}, \{x'_{kn}\}$, v satisfies

$$v(x_{in}, \{x_{kn}\}) - v(x_{in}, \{x'_{kn}\}) = v(x_{jn}, \{x_{kn}\}) - v(x_{jn}, \{x'_{kn}\}) \tag{5}$$

Then for each real M choose an arbitrary set $\{x_{kn}\}_M$ with $\max\{x_{kn}\}_M = M$ and define

$$\phi(x_{in}, \{x_{kn}\}) = v(x_{in}, \{x_{kn}\}_{\max\{x_{kn}\}}).$$

and

$$\eta(\{x_{kn}\}) = v(M, \{x_{kn}) - v(M, \{x_{kn}\}_{max\{x_{kn}\}})$$

By (5), the functions ϕ and η are independent of the arbitrary choice of $\{x_{kn}\}_M$, and (4) is satisfied.

Hence, if (4) does not hold it must be possible to select real numbers x_{in}, x_{jn} and sets $\{x_{kn}\}$ and $\{x'_{kn}\}$ with $\max\{x_{kn}\} = \max\{x'_{kn}\}$and $x_{in}, x_{jn} \in \{x_{kn}\}, \{x'_{kn}\}$, so that

$$v(x_{in}, \{x_{kn}\}) - v(x_{in}, \{x'_{kn}\}) \neq v(x_{jn}, \{x_{kn}\}) - v(x_{jn}, \{x'_{kn}\}) \tag{5}$$

Adding and rearranging yields

$$v^*(x_{in}, x_{jn}, \{x_{kn}\}) \neq v^*(x_{in}, x_{jn}, \{x'_{kn}\})$$

contradicting Proposition 1. ■

It is natural to normalize by disregarding the 'nuisance' term $\eta(\{x_{kn}\})$ which will never affect v^* and hence will never change the ranking of any two alternatives. This yields the representation

$$v(x_{in}, \{x_{kn}\}) = \phi(x_{in}, \max(\{x_{kn}\})) \tag{6}$$

That is, the regret associated with a given action i, assuming state n to occur, depends only on the actual outcome x_{in} and the best possible outcome that could have been attained in state n. The preference functional (6) satisfies **ISDA**, so the converses of Proposition 1 and Corollary 1 are true.

Although it has been derived from non-manipulability criteria, rather than from consideration of the psychology of choice, this result is in accord with the intuition behind regret theory. Regret theory is concerned with the difference between the return obtained from a given choice, conditional on the occurrence of a particular state of the world, and the return which might have been obtained had one chosen differently. It is natural to focus regrets of this kind on the best outcome which was forgone as a result of the choice actually made.

Symmetry might lead us to suppose that the worst outcome which could have occurred in a given state of the world would also be relevant, but the argument above shows why this is not the case. It is always possible to augment the set of alternatives with options which turn out arbitrarily badly in a given state of the world, and hence the minimum outcome is not well–defined.

Thus whereas the original version of regret theory might, with equal justice, have been called rejoicing theory, the general model proposed here is strictly a theory of regret. It is arguable whether this enhances or diminishes the psychological plausibility of the model. On the one hand, symmetry is generally an appealing property. On the other, both the choice of name for the original model and most of the subsequent informal discussion focus attention on regret rather than rejoicing.

The fact that regret depends only on the best available outcome means that preferences in any two states with the same maximum outcome M may be represented by a common function $\phi(\bullet; M)$ which has the usual properties of a von Neumann–Morgenstern utility function. When the maximum is the same in all states, we have

Corollary 2: Let max $\{x_{kn}\} = M, \forall j$. Then preferences may be represented by an expected utility functional with $U(\bullet) = \phi(\bullet; M)$.

The main use of this result is a negative one. If M is constant, then the phenomena analyzed by regret theory cannot be present. Either EU theory will suffice to explain observed behavior, or some alternative explanation must be sought for deviations from EU theory.

An example where Corollary 2 applies arises when the choice is between strategies to achieve a given objective that do not differ in cost. It is common to feel regret *ex post* if a strategy fails, when, for the realized state of the world an alternative strategy would have succeeded. Nonetheless, *ex ante* minimization of expected regret is achieved by selecting the strategy with the highest probability of success, just as is maximization of expected utility. For in all states of the world, either at least one strategy achieves the objective, in which case the return is some constant M, or all strategies fail, in which case the state may be disregarded.

Proposition 1 also permits the derivation of general regret–theoretic preferences from observations on choices over pairs of alternatives. Denote the best outcome available in state n, for a given choice set $\{\boldsymbol{x}\}$, by $M_n\{\boldsymbol{x}\}$.

For a pairwise choice, suppose without loss of generality that $x_{in} \geq x_{jn}$, that is, $M_n\{x_\mathrm{i}, x_\mathrm{j}\} = x_{in}$. In regret theory, we are concerned only with differences of the form

$\phi(z, x) - \phi(y, x)$. Hence, we can normalize $\phi(x, x)$ to any value we choose. Impose the natural normalization

$$\phi(x, x) = 0$$

Then, maximization of E[v*] will be equivalent to maximization of E[$v(x_{jn}, x_{in}) - v(x_{in}, x_{jn})$] if

$$\phi(x_{jn}, x_{in}) = v(x_{jn}, x_{in}) - v(x_{in}, x_{jn})$$

This yields a complete specification for ϕ, and hence for v.

The function ϕ is well–defined, not only for finite choice sets, but also for compact sets A when the outcomes x_{ij} are determined by a continuous function on A. This is convenient for many economic applications where the choice consists of selecting a value for some control variable which may range over an interval.

The unique extension derived here depends on **ISDA** and also on the assumption of separability across states, which has been an explicit feature of previous presentations of regret theory. The **ISDA** assumption seems normatively compelling. However, the assumption of separability across states is open to the many of the same criticisms as the corresponding assumption in EU theory, namely the independence axiom (see Machina 1983 for a summary and Camerer 1989 for more recent evidence). The force of some of these criticisms is reduced by the fact that regret theory offers explanations of many of the observed violations of EU theory, such as the Allais problem, that have generally been regarded as counter–examples to the EU independence axiom.

Nevertheless, it would be possible to design general models, incorporating regret phenomena in which the independence assumption did not apply. It seems likely however, that any plausible model of this kind would not satisfy the independence assumption in the two–outcome case. Hence the standard regret–theoretic functional (2) for the two–outcome case would be only a special case of the model. Thus, it seems reasonable to claim that the function derived here is a natural extension of (2) to the case of multiple alternatives.

A particularly tractable functional form arises when the function ϕ is multiplicatively separable, so that

$$\phi\,(x_{in}, M_n\{\boldsymbol{x}\}) = u(x_{in})\ \psi(M_n\{\boldsymbol{x}\})$$

In this case the regret associated with the pair x_{in}, x_{jn} is simply

$$v^*(x_{in}, x_{jn}) = \psi(M_n\{\boldsymbol{x}\})(u(x_{in}) - u(x_{jn})) \tag{7}$$

Thus, the effect of regret is simply to attach different weights to the states of the world. If, as would normally be expected, ψ is increasing in its arguments, states with a high potential for regret are weighted more heavily, relative to their subjective probability, than states with little potential for regret.

Now consider the case when all prospects are comonotonic. That is, for any states n, m and prospects A_i, A_j, $A_{in} \geq A_{im} \Rightarrow A_{jn} \geq A_{jm}$. This assumption is a natural one where there is a scalar choice variable and uncertainty arises from a single random variable with values which may be naturally ranked from worst to best. Examples include portfolio problems with one risky asset (and no short selling), the single product firm under output price uncertainty and labor supply with an uncertain wage.

In this case, (7) yields preferences equivalent to the rank–dependent expected utility (RDEU) model (Quiggin 1982, Yaari 1987). The probability weight attached to state n is simply $\psi(M_n\{\boldsymbol{x}\})$. Comonotonicity ensures that rank-ordering is the same for all x_{in} and x_{jn}. For the comparative static problems listed here, Quiggin (1991) has shown that the standard results of EU theory carry across to RDEU in a natural fashion.

These results provide some useful insights into the general comparative static properties of regret theory. It is apparent from the examples given by Loomes and

Sugden (1982) that regret phenomena are most interesting when prospects are not generally comonotonic. Given local comonotonicity, however, local comparative statics will be comparatively well–behaved. The major implication of regret theory is the possibility of multiple local optima.

4. Generalized regret dominance

The idea that preferences should not be manipulable by the introduction of alternatives which will never be selected suggests consideration of a stronger requirement. Suppose that the manipulator knows that the victim has regret–theoretic preferences. If it is known that a given alternative will never be chosen by such an individual, it may be introduced into the choice set without risk to the manipulator. It is straightforward to show that statewise dominated alternatives will never be selected. As noted above, Quiggin (1989) shows that, for the case of two–alternatives, a stronger stochastic dominance condition, referred to as regret–dominance, applies. A_i regret–dominates A_j (that is, A_i is preferred to A_j for all regret–theoretic preference functionals) if and only if there exists a permutation ρ of the set of states such that

(i) $\rho(\rho(n)) = n$; and

(ii) $x_{i\rho(n)} \geq x_{jn} \quad \forall\, n$

The general regret–theoretic form (6) does not preserve regret–dominance in the sense of Quiggin (1989). In order to define a notion of stochastic dominance for the general case, an additional condition is needed. The states n and $\rho(n)$ must be comparable with respect to the prospects A_i and A_j, in the sense that the utility function ϕ_M is either the same in both states or that the dominating prospect A_i supplies a preferred maximal outcome. More precisely, it is required that either

(iiia) $M_n\{\boldsymbol{x}\} = M_{\rho(n)}\{\boldsymbol{x}\}$ or

(iiib) $x_{i\rho(n)} = M_{\rho(n)}\{\boldsymbol{x}\} \geq M_n\{\boldsymbol{x}\}$

This condition will be referred to as maximum–comparability. Then

Proposition 2: Suppose there exists a permutation ρ of the set of states satisfying conditions (i)–(iii). Then A_i is preferred to A_j for all regret–theoretic functionals.

Proof: The proof is essentially identical to that of the main proposition in Quiggin (1989).

Given that a regret–dominated alternative will never be selected, it is natural to consider a modification of **ISDA** in which statewise dominance is replaced by regret dominance.

Irrelevance of regret–dominated alternatives (IRDA): Let A_i be preferred to A_j given the set of alternatives A, and suppose there are some A_r, $A_s \in \mathsf{A}$ such that A_r regret–dominates A_s in the sense of Proposition 2. Then A_i is preferred to A_j given the set of alternatives $\mathsf{A}^* = \mathsf{A} - \{A_s\}$.

Unfortunately, this restriction is inconsistent with the functional form (6) except in the polar case of an EU functional. Hence there is no proper generalization of the two–action case of regret theory which satisfies **IRDA**. As a corollary, there is no generalization in which first stochastically dominated alternatives are irrelevant.

Proposition 3: Assume that **IRDA** holds. Then v is of the form

$$v(x_{ji},\ (\{x_{kj}\}\ ) = U(x_{ji}) \qquad (8)$$

Proof: IRDA $\Rightarrow$ **ISDA**, so v must take the general form (3). It is sufficient to consider the case of three equiprobable states.

For any given v, and $x < x' < x''$ choose $\delta > 0$ and

$A_1 = (x, x'', x'), A_2 = (x', x'+\gamma, x'), A_3 = (x'', x, x'-\delta)$

where γ is chosen such that in a pairwise choice $A_1 \sim A_2$

The fact that $A_1 \sim A_2$ means

$$\phi(x, x') + \phi(x'', x'') = \phi(x', x') + \phi(x'+\gamma, x'')$$

A_1 regret–dominates A_3. Hence **IRDA** implies

$$\phi(x, x'') + \phi(x'', x'') = \phi(x', x'') + \phi(x'+\gamma, x'')$$

Hence

$$\phi(x, x') - \phi(x, x'') = \phi(x', x') - \phi(x', x'') = -\phi(x', x'')$$

This is true for any $x < x' < x''$. Hence ϕ must be additively separable in its arguments and by cancellation, must take the form

$$\phi(x, x') = U(x) - U(x')$$

Thus, v must be independent of its second argument ■

The fact that there is no generalised form of regret theory which satisfies **IRDA** and preserves regret–dominance reduces the attractiveness of the model. However, the opportunities for manipulation are not as great as in the case of **ISDA**, where preferences can be manipulated by offers of prospects which have negative or zero returns in every state, and hence can never cost the manipulator anything. In the absence of **IRDA**, the manipulator can make an offer which will not be accepted by any agent with regret–theoretic preferences, but this offer must involve positive payouts in some states. Hence, unless the manipulator is certain that the agent's preferences may be described by regret theory, the offer is not without risk.

This problem is not peculiar to regret theory, but applies to any resolution of non–transitive pairwise preferences through a procedure to pick the most preferred element from a set of prospects. With sufficient knowledge of preferences, anyone whose preferences over pairwise choices depend on the other elements of the choice set may be manipulated, in a fashion similar to that outlined here, by the offer of prospects that will not be chosen but will affect preferences. The special normative status of **ISDA** arises because it requires no knowledge of preferences and no risk–taking on the part of the manipulator.

6. Concluding comments

Regret theory has hitherto been in a somewhat anomalous position in the theory of choice under uncertainty. It has performed quite well in experimental tests. However, the absence of a fully developed model of choice over general sets of alternatives has precluded significant use in economic applications, and has raised doubts about the general validity of the theory.

The general version of regret theory offered here may allay these concerns. While remaining consistent with the intuition behind the original version, it removes the restriction to pairwise (or even finite) choice sets. This permits a comparison of regret theory with alternative approaches, not merely in laboratory settings, but in application to real–world economic problems.

On the other hand, the generalization offered here is the only extension of regret theory which satisfies the very weak **ISDA** rationality criterion. If this extension is found to be unsatisfactory, regret theory must be abandoned or radically modified, at least as a normative model. The failure to meet the stronger **IRDA** criterion provides some grounds for normative objections to the general model proposed here, but these objections do not appear fatal.

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