TESTING BETWEEN ALTERNATIVE MODELS OF CHOICE UNDER UNCERTAINTY - COMMENT

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Journal of Risk and Uncertainty 6(2), 161–4.

Abstract

Battaglio, Kagel and Jiranyakul use experimental tests to compare rank-dependent expected utility (RDEU), regret theory, prospect theory and Machina's generalized smooth preferences model. They conclude that none of these models consistently organize the data. The purpose of this note is to point out that RDEU theory was tested in combination with a hypothesis on the choice of functional form that has been explicitly rejected by the original author of the model (Quiggin 1982, 1987). When the original form of RDEU theory is tested, it performs quite well.

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A number of alternatives to and generalizations of expected utility (EU) theory have been developed. An important reason for dissatisfaction with EU theory was the consistent violation of its predictions in empirical tests. It is unfortunate, then, that most tests of alternative models have failed to produce clear results. The conclusion of Battaglio, Kagel and Jiranyakul (1990, *p*46) is typical

> "Our overall conclusion is that none of the alternatives to expected utility considered here consistently organize the data, so we have a long way to go before having a complete descriptive model of choice under uncertainty."

The models analyzed were rank-dependent expected utility (RDEU), regret theory, prospect theory and Machina's generalized smooth preferences model.

The purpose of this note is to point out that RDEU theory was tested in combination with a hypothesis on the choice of functional form that has been explicitly rejected by the original author of the model (Quiggin 1982, 1987). When the original form of RDEU theory is tested, it performs quite well.

Battaglio et al test RDEU theory in conjunction with the hypothesis of a concave weighting function. As Chew, Karni and Safra (1987) and Yaari (1987) have observed, a concave weighting function (along with a concave or linear utility function) is equivalent to global risk aversion, in the sense that second stochastically dominated prospects will never be preferred. Segal (1987) has shown that this condition is also necessary if the model is to be consistent with Machina's generalized Allais paradox.

Segal's paper was presented as a comment on Quiggin (1982), where the RDEU model was first presented under the name ‘anticipated utility’. In the original presentation, the proposed weighting function f was S-shaped, with a concave segment below $f(1/2) = 1/2$, and a convex segment above 1/2. The restriction that $f(1/2) = 1/2$ was a consequence of the axiomatic system used by Quiggin. More general sets of axioms, removing this restriction, have subsequently been proposed by Chew (1989), Segal (1989) and Wakker (1991). In a reply to Segal, Quiggin (1987) conceded that the imposition of the axiomatic requirement that $f(1/2) = 1/2$ was inappropriate, but claimed (*p*645) that “as a behavioral hypothesis, [the S-shaped function] fits many, and perhaps most, people.”

Two main arguments were given in support of this claim. First, given the evidence of simultaneous gambling and insurance, global risk aversion is an inappropriate assumption (this is confirmed by the results of Battaglio et al) . Second, Machina’s generalization of the Allais paradox cannot be justified on the basis of the available data. Quiggin (1987) showed that an S-shaped weighting function fitted the data (derived from the pioneering work of MacCrimmon and Larson 1979) just as well. Battaglio et al cite Segal's comment, but mention neither the original 1982 paper nor the 1987 reply. They therefore proceed to test RDEU theory on the basis of a hypothesis derived from a competing model, that of Machina.

Given the fairly simple questions used by Battaglio et al, it is straightforward to derive and test the predictions of the original form of RDEU theory. Since all of the money amounts used are small, even in relation to the wealth of the average undergraduate, it seems reasonable to impose a linear utility function. In combination with the assumption that the weighting function is S-shaped, this is sufficient to give unambiguous predictions for the majority of the questions reported by Battaglio et al.

To take an example at random, Question 4' is a choice between ($14, 1) and ($0, 0.3; $20, 0.7). The RDEU evaluations of these prospects, with the utility function

set to the identity, are 14 and 20(1 - f(0.3)) respectively. Since, by hypothesis, f(0.3) > 0.3, the ($14, 1) bet is to be preferred. Similar reasoning shows that, when the payoffs are negative, as in Question 4, (-$20, 0.7; $0, 0.3) should be preferred to (-$14, 1). Note that, as the name rank-dependent EU implies, the weight applied to an outcome depends on its ranking, and not merely on its probability.

For 15 of the 23 choice problems reported by Battaglio et al, the assumptions set out above are sufficient to derive an unambiguous prediction. The remaining 8 problems fall into two groups.

The first group (Questions 7, 7', 8 and 8') are choices of the form (±$18, p), (±$27, 0.8p) where p takes the values 1 (for 7 and 7') and 0.2 (for 8 and 8'). The expected value of the bets is positive, but the effects of RDEU probability weighting tend to counteract this. However, given the empirical estimates reported in Quiggin (1981), where it was suggested $f(0.75) \approx 0.73$, it seems reasonable to propose that the expected value effect will predominate except, perhaps, for very small p.

The second group (Questions 12-15) are choices betweeen a certain outcome x and a fair, even-money bet of the form ($x+\delta$, 0.5; x$-\delta$, 0.5). The RDEU model, with f(1/2) = 1/2 and linear utility, predicts indifference in choices of this kind. It seems reasonable to model this prediction by the hypothesis that the responses were purely random.

Given these assumptions, it is possible to test the predictions of the standard RDEU model against the results of the 23 questions reported in Battaglio et al Tables 1-4. In the 19 cases where a definite prediction was made, the RDEU model is correct in 17. In all 4 even-money bets, the results are consistent, at the 0.05 level, with random choice, as predicted by RDEU. The two failed predictions are

Question 1'

A ($5, .6; $25, .4) vs *B* ($13, 1.0)

and

Question 5'

A(0, 0.8; $12, 0.2) vs *B*(0, 0.88; $20, 0.12)

In Question 1', RDEU predicts preference for *A*, but this was chosen by only 14 subjects, as against 17 for *B*. In Question 5', RDEU predicts preference for *B*, but this was chosen by only 6 subjects, as against 24 for *A*.

The failure on Question 1' does not seem too disturbing. The RDEU model is almost indifferent between the two and the small majority going the wrong way could arise from a chance fluctuation. The results of Question 5' are more difficult to explain. The reported responses are sharply different from those for the almost identical Question 6'. Battaglio et al give a rather confusing discussion of Question 5', since they refer to it as showing risk-loving and being an example of the reflection effect. Neither of these statements are consistent with the reported responses. This question may involve some form of experimental error.

In summary, the pessimism expressed by Battaglio et al is unwarranted, at least for this data set. The RDEU model, in its original form, provides a quite satisfactory description of observed behavior in this case.

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