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RISK, SELF-PROTECTION AND EX ANTE ECONOMIC VALUE - SOME POSITIVE RESULTS

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Public concern has increased over a variety of health risks such as those arising from exposure to pesticides in food, radiation from natural sources such as radon and from man-made sources, and a variety of natural and artificial carcinogens. In many of these cases, the health outcome experienced by an individual is determined by an interaction between individual actions, ambient levels of risk and purely stochastic factors. For example, exposure to asbestos engenders a risk of cancer which is exacerbated by smoking. However, although these factors affect the probability of developing cancer, the actual outcome for any individual remains random.

Individuals faced with ambient risk must make decisions concerning self-protection and self-insurance. Likely impacts on self-protection decisions are also a critical factor in the analysis of public risk mitigation policy. Lewis and Nickerson [12] present a model of self-insurance against natural disaster in which self-insurance affects the severity of loss. Shogren and Crocker [20] extend this analysis by allowing self-protection to affect both the probability of a given adverse outcome and the severity of the loss associated with that outcome, obtaining primarily negative results.

The analysis of choice in terms of the endogenous probability of various adverse outcomes is a long-standing tradition in the literature on self-insurance and self-protection. However, I shall argue, that, at least in the present case, a more tractable presentation of the problem is obtained if the analysis is conducted in terms of random variables represented as mappings from an exogenous set of states of nature. For any given problem, both representations are available. The choice between them is a matter of analytical and expositional convenience. In terms of an example given by Ehrlich and Becker [6], the probability that lightning strikes your house is exogenous whereas the probability that the house burns down is endogenous. The problem of self-protection may be analyzed in terms of whichever one of these probabilities is most tractable. This is contrary to the claim made by Ehrlich and Becker, but consistent with the arguments of Hirshleifer [10].

In non-stochastic models, such as that of Bartik [2] and two-outcome models such as that of Berger *et al* [3], it may be shown that willingness to pay for risk-reduction may be expressed in terms of the marginal rate of technical substitution between exogenous risk-reduction and self-protection. Shogren and Crocker claim that this result cannot be extended to the general case they consider. Second, they claim, contrary to Kahneman and Tversky [8] and others, that, even with convex dose-response requirements, EU does not require that individuals display increasing marginal compensating variation for risk exposure as the level of risk increases. Third, they claim that the effect of a change in risk on self-protection expenditures is, in general, ambiguous. In each case, they present restrictive conditions under which the results can be obtained.

The main contention of the present paper is that positive results can be obtained more generally. I present necessary and sufficient conditions under which the results of Berger et al may be extended to the general case, and sufficient conditions under which individuals will require increasing marginal compensating variation and will increase self-protection expenditures in response to an increase in risk.

Two additional conditions, not considered by Shogren and Crocker, are necessary to obtain these results. The first is the standard assumption that preferences display decreasing absolute risk aversion. The second is a separability condition similar to that employed in the non-stochastic case by Bartik, and in the two-state case by Berger et al. In the absence of such separability conditions, the main results for the non-stochastic case break down and the question of extension to the case of uncertainty becomes moot.

From a practical viewpoint, the most important implication of the results derived here is that the standard willingness-to-pay approach to valuing environmental hazards is valid under fairly general conditions. Consider, for example, the problem of valuing reductions in atmospheric lead concentrations. A standard analysis would derive a measure of willingness to pay based on the marginal rate of technical substitution between reductions in ambient risk and private self-protection measures such as chelation of blood. The results presented here show that, provided the private self-protection measure is specific to the hazard in question (as it clearly is in the case of chelation), the standard valuation procedure is valid. Under the same conditions, the marginal increase in self-protection expenditures arising from a given increase in atmospheric lead levels is a lower bound for the associated marginal loss. Observations on self-protection expenditures may therefore be used as a basis for policy analysis. Even for problems in which the separability assumption may not hold, it is shown that self-protection expenditures are an increasing function of ambient risks. Finally, it is shown that individuals will display increasing marginal compensating variation for risk exposure as the level of risk increases. This justifies the use of convex, rather than linear, cost functions.

1. THE MODEL

The model used here is logically identical to that of Shogren and Crocker. However, Shogren and Crocker undertake their analysis in terms of the cumulative distribution function of health outcomes, an approach which generates intractable first-order conditions and expressions for willingness to pay. The analysis here is closer in approach to that of Lewis and Nickerson [12], in that the probability distribution of health outcomes is explicitly derived from an interaction between individual self-protection decisions and an exogenous state of the world. Note that it is not assumed that the individual can observe the state of nature *ex ante* or make contracts contingent, *ex post*, on the occurrence of a given state of nature.

The individual observes a level of exogenous ambient risk $r \in [\underline{r}, \bar{r}]$, and chooses a level of self-protection $s \in [\underline{s}, \bar{s}]$. The values of r and s determine a density function, $f(h; s, r)$, and a corresponding cumulative distribution function, $F(h; s, r)$, over a continuum H of health outcomes. Since the density is defined for every $h \in H$, $F(\cdot\ ; s, r)$ is a continuous, monotone increasing mapping from H into [0, 1]. Further Shogren and Crocker assume that $F(\cdot\ ; s, r)$ is twice differentiable in s and r, that $F_s \leq 0$ and $F_r \geq 0$.

It should be noted that an increase in r does not necessarily involve an increase in risk in the sense used by the decision-theoretic literature. In the decision-theoretic literature, risk simply refers to unpredictable variation about the mean outcome. In the health risk literature, the term is employed in a manner closer to its standard English usage, to refer specifically to the possibility of adverse outcomes. Suppose, for example, there are only two possible outcomes, death and survival. If the probability of death is initially 0.5 and rises to 0.9, it would be natural in the health context to refer to an increase in risk. However, in decision-theoretic usage, this would be a reduction in the desirability of the mean outcome, combined with a reduction in risk. I will use the term 'increase in ambient risk' to refer to increases in r, and 'increasing uncertainty' to refer to the decision-theoretic concept usually labelled 'increasing risk'.

To any cumulative distribution function F we may associate a random variable as follows. Consider the σ-field $\Omega = [0, 1]$ with the associated probability measure ρ (the usual Lebesgue measure). Ω will be interpreted as a set of states of the world. States of the world are elements ω of Ω and events E are measurable subsets of Ω..

The individual's health outcome h is determined by a mapping $h(\omega; s, r)$ into a health outcome set H. For convenience, it will be will assumed that the values of ω [0, 1] are ordered from worst to best, so that $h(\cdot\ ; s, r)$ is increasing in its argument ω for all s, r. With this convention, the random variable h is related to the cumulative distribution function F and density function f as follows.

$$F((h(\omega); s, r) = \omega \tag{1}$$

$$\partial F/\partial s = (\partial h/\partial s) / (\partial h/\partial \omega)$$

Thus, any results on *h* have a direct interpretation in terms of *F* and *vice versa.* Although the analytical tools used here differ from those of Shogren and Crocker, the models are logically identical. Since this claim is critical for comparison between the results of Shogren and Crocker and those derived here, it will be argued in a little more detail.

As in Shogren and Crocker, self-protection is modelled as affecting the severity of a given health outcome *h*, represented by a function of the form $C(h, r, s)$. The individual's welfare outcome is determined by the severity of the health outcome $C(h, r, s)$, and by net wealth, *W*. The price of self-protection is normalized to unity so that

$$W = W_0 - s$$

where W_0 is initial wealth.

The individual's utility is given, in general, by a function of the form $U(W, h, r, s)$. *U* is assumed to be a concave von Neumann-Morgenstern utility function. In the case examined by Shogren and Crocker,

$$U=U(W_0 - C(h, r, s) - s) \tag{2}$$

where *C* is the cost of health care expenditure. In this model, the effects of adverse health outcomes can be captured completely in monetary terms. Hence, this model will be referred to as a 'monetary loss' model.

The individual's problem is to choose $s^* \in S$, so as to maximize E[*U*], yielding the first-order condition

$$\mathrm{E}[U'(W)] = -\mathrm{E}[U'(W)C'(s)] + \int_\Omega U'(W)\partial C/\partial h \, \partial h/\partial s \ d\omega$$

A change of variable, using equations (1), yields Shogren and Crocker's corresponding equation (6). This establishes the logical identity between the model employed here and that of Shogren and Crocker.

In addition to the problems analyzed by Shogren and Crocker, results will be derived on the effects of an exogenous increase in uncertainty. In order to develop a notion of increasing uncertainty in the present model, it is necessary to employ a state space approach in which the stochastic element is explicitly represented by a random variable. Unobservable[1] risk factors affecting health are represented by $\theta(\omega)$, where θ is a real-valued random variable (that is, a measurable mapping form the state space Ω to) representing the individual health implications of the state of the world ω. An increase in uncertainty may be now be regarded as a mean preserving spread in the distribution of the random variable $\theta(\omega)$. The best-known definition of a mean preserving spread is due to Rothschild and Stiglitz [17]. A slightly more restrictive notion will be used below.

The combined impact of the individual's actions on the probability and severity of loss may be captured by the function.

$$\phi(s, \theta, W_0; r) = W_0 - C(h(\theta, r, s), r, s) - s.$$

It is convenient to conduct the analysis in terms of the combined impact, given by ϕ, and then to derive implications concerning probability and severity effects, given separately by h and C.

For given r, the individual's problem is a special case of the general maximization problem

$$\mathrm{Max}_s \; \mathrm{E}[U(\phi(s, \theta, W_0; r))] \tag{3}$$

with first order condition (when ϕ is differentiable with respect to s)

$$E[U'(\phi)\partial\phi/\partial s] = 0$$

[1] The critical requirement is that θ be unobservable *ex ante*. The individual may be able to observe θ *ex post*, provided that this information can not be verifiably communicated to another party, and thereby made the contingent variable for an insurance contract. I am indebted to a referee for pointing out an ambiguity on this point in a previous draft.

For the specific case modelled here, we have

$$\partial\phi/\partial s = -\,(1 + \partial C/\partial s + \partial C/\partial h\ \partial h/\partial s)$$

so that the first-order condition becomes

$$E[U'(\phi)\ (1 + \partial C/\partial s + \partial C/\partial h\ \partial h/\partial s)] = 0 \tag{4}$$

Shogren and Crocker claim (Proposition 3) that the effect of a change in the ambient risk r on the optimal level of self-protection expenditure s^* is, in general ambiguous. In order to show that, under plausible conditions $\partial s^*/\partial r$ will be positive, it is necessary to use some results from the literature on comparative statics under uncertainty.

2. COMPARATIVE STATICS

The comparative static properties of several versions of the general problem (3) have been studied by a number of writers (Feder [7], Meyer and Ormiston [13], Quiggin [15]). These results generalize those found previously for particular applications such as the theory of the firm under price uncertainty (Sandmo [18]). Lewis and Nickerson [12] treat this general problem in the context of self-protection against natural disaster.

In all of these papers, it is assumed that U displays decreasing absolute risk aversion. This assumption is standard in the literature on choice under uncertainty, to the point where most recent writers invoke it without discussion. However, since decreasing absolute risk aversion is not assumed by Shogren and Crocker, a review of the justification for this assumption may be timely.

There are two main reasons why the decreasing absolute risk aversion assumption is so widely used. The first, based on introspection, is its plausibility as a characteristic of preferences. It is natural to suppose that someone with a net wealth of say $100 000 might be willing to take a bet of $5000 at favorable odds, but that the same person would refuse such a bet if their net wealth were $5000. The second is that decreasing absolute risk aversion, or some closely related condition, is required to extend a number of basic comparative static results under certainty to the (Expected Utility) theory of choice under uncertainty. For example, Sandmo, shows that given decreasing absolute risk aversion, supply curves slope upwards for competitive firms under price uncertainty.

Decreasing absolute risk aversion is difficult to test empirically. It is usually impossible to impose changes in base wealth in experimental settings, and quite difficult to infer complex properties of risk preferences from observed market behavior. Those studies that have attacked the problem have generally supported the decreasing absolute risk aversion hypothesis (Bardsley and Harris [1], Binswanger [4], Hamal and Anderson [9]).

Perhaps the strongest empirical evidence for decreasing absolute risk aversion comes from the successful use of constant relative risk aversion (that is, logarithmic and power) utility functions in a wide range of empirical work, including studies of intertemporal choice as well as choice under uncertainty. Most studies have found evidence consistent with constant relative risk aversion but not with constant absolute risk aversion (Pope and Just [14]). The only functional form characterized by increasing absolute risk aversion that has had any significant use has been the quadratic, which has generally performed very badly (Borch [5]).

The following assumptions have been found useful for the case when ϕ is twice differentiable in each of its arguments

(A.1) $\partial\phi/\partial\theta < 0$,

(A.2) $\partial\phi/\partial W_0 > 0$

(A.3) $\partial^2\phi/\partial\theta\partial s \geq 0$

(A.4) $\partial^2\phi/\partial W\partial s \geq 0$

(A.5) $\partial^2\phi/\partial\theta^2 = 0$

In the present context, only assumption (A.3) is problematic. It states that, the worse is the random state of the world, the higher is the marginal return to self-protection. For example, suppose r is the observed level of ambient risk, and ω consists of unobserved risk factors which increase the likelihood of an adverse health outcome. If the self-protection activity has a higher marginal return when risks are high, this condition will be satisfied.

In most of the economic applications of the general control model (eg that of the firm under uncertainty) (A.3) is trivially true. However, in the context of the self-insurance and self-protection problem, there are plausible cases in which (A.3) will not hold. These cases arise when the random variable θ represents possible occurrences which may reduce the effectiveness of self-protection. For example, if the hazard is pesticide contamination in food, the self-protection activity is some form of screening or monitoring, and θ represents the frequency of screening failures, we would expected (A.3*) $\partial^2\phi/\partial\theta\partial s < 0$. In the problem representation given here, self-protection is equivalent to a reduction in uncertainty when (A.3) holds. When (A.3*) holds, self-protection is equivalent to an increase in uncertainty, since its effects are greatest in good states of the world.

Lewis and Nickerson [12] appear to be the first authors to consider the role of an assumption equivalent to (A.3). In their model of self-insurance against natural catastrophes, such as earthquakes and floods, they demonstrate the crucial rôle of this assumption. Lewis and Nickerson suggest that ordinary expenditures on home improvement may be regarded as risk-increasing in this context, while measures aimed at mitigating earthquake (or other natural catastrophe) damage are risk-reducing. Their main concern is with implications of self-insurance behavior for the costs and effectiveness of public disaster relief. However, a number of their comparative static results prove useful for the present problem.

The difference with the present treatment of (A.3) is mainly one of interpretation. The central point is that expenditures which satisfy (A.3*) may mitigate the effects of the ambient risk r, even though they increase uncertainty. Thus, in terms of the distinction presented above, expenditures which satisfy (A.3) will be described as 'uncertainty-reducing' and those which satisfy (A.3*) as 'uncertainty increasing.'

It is desirable to obtain results on the effect of an exogenous increase in uncertainty. The most commonly used definition, due to Rothschild and Stiglitz [17], has proved intractable in comparative static analysis. More recently attention has focused on a more restrictive definition, referred to by Quiggin [15] as a monotone spread, and by Meyer and Ormiston [13] as a deterministic transformation.

The class of monotone spread relationships include multiplicative spreads, and situations where one variable is obtained from another by truncating the tails. More formally, if θ is a random variable and $f(\theta, \lambda)$ is a function with $\partial f/\partial\theta \geq 0$, an increase in the exogenous variable λ is a monotone spread in the random variable $f(\theta, \lambda)$ if $f_{\theta\lambda} \geq 0$, and $\partial/\partial\lambda\ E[f(\theta, \lambda)] = 0$. Whereas only fairly weak results have been obtained for comparative static responses to mean-preserving spreads, strong results have been obtained monotone spreads.

Proposition 1 is a summary of a number of major results of EU theory.

Proposition 1:

(a) The optimal level of s, denoted s^*, will be higher than the level which maximizes expected wealth.

(b) Let θ_1 be the initial random variable and θ_2 the final random variable. Given the assumptions above, the following are sufficient conditions for an increase in s^*.

(i) $\theta_2 = \theta_1 + \delta$ for some $\delta > 0$

(ii) θ_2 is derived from θ_1 by a monotone spread

(c) An reduction in W_0 leads to a increase in s^*.

(These results are reversed when A.3* holds).

Proof: All of the results except b(ii) are well-known (see, for example, Feder [7]). Lewis and Nickerson prove several of these results in the self-insurance context, including (c) and, for the case of a uniform distribution, (b(ii)). The general result for monotone spreads is derived by Quiggin [15] and, independently, by Meyer and Ormiston [13]. The formulation of the control problem presented here follows Quiggin [15]. An appendix giving proofs for this formulation is available from the author. n

The intuition behind these results is straightforward. Proposition 1(a) is a natural consequence of the fact that under (A.3), self-protection is uncertainty-reducing and under (A.3) self-protection is uncertainty-increasing. Proposition 1(c) follows similarly, given decreasing absolute risk aversion. In Proposition 1(b), the shifts in the distribution of θ have two effects. The first is to increase the average marginal return to self-protection, and the second is to reduce wealth (or in the case of b(ii) increase the riskiness of wealth) thereby increasing effective risk-aversion and raising the optimal level of s. Since these effects reinforce each other the result follows. Note that, if decreasing absolute risk aversion is not assumed, these results (except for (a)) are ambiguous.

These results have been concerned with the effect of exogenous changes in the distribution of θ. However, it is apparent that changes in the value of r will normally have effects similar to those of changes in the distribution of θ. For example, if the total risk is a function of $(r+\theta)$, the sum of the observed ambient risk and the unobserved risk variable, the effects of changes in r follow directly from Proposition 1b(ii). Provided A.3 holds, $\partial r/\partial s$ will be positive in this case.

We will impose the following assumptions on r.

(R.1) $\partial\phi/\partial r \leq 0$

(R.2) $\partial^2\phi/\partial r\partial s \geq 0$

(R.3) $\partial^2\phi/\partial\theta\partial r \leq 0$

These conditions will be satisfied provided the corresponding conditions apply to h and C separately. Only (R.2) requires comment. This assumption requires that the marginal return to self-protection increases with increasing risk. In the notation of

Shogren and Crocker, this condition will be satisfied provided $F_{sr} \leq 0$ and $C_{sr} \leq 0$ which will be true under strong non-convexity. The sufficient conditions derived by Shogren and Crocker (Corollary 3) are special cases of those presented here.

Under these assumptions, an increase in r will generate

(i) A reduction in wealth in every state of the world

(ii) An increase in the marginal return to self-protection

(iii) A monotone spread in the riskiness of outcomes

From Proposition 1, each of these effects calls forth an increase in s^*, the optimal level of self protection. Thus, we have

Proposition 2: Let (A.1-5) and (R.1-3) hold and assume that U displays decreasing absolute risk aversion. Then $\partial s^*/\partial r \geq 0$.n

This result is contrary to Shogren and Crocker (Proposition 3), as it applies to strong non-convexity. In order to see why Shogren and Crocker obtain a negative result here, it is useful to examine their equation (13) derived from implicit differentiation of the first-order conditions. This contains four terms, two of which are positive and two of which are *prima facie* ambiguous. Shogren and Crocker assert that these terms cannot be signed without prior information. The expressions in Shogren and Crocker's equation are, indeed, intractable. However, the corresponding expressions derived from implicit differentiation of the first-order conditions for the state space representation (3) may be signed using the weak assumption of decreasing absolute risk aversion. A classic example of this procedure is Sandmo's [18] proof that an increase in mean price will always generate an increase in supply for a competitive firm under price uncertainty.

3. WILLINGNESS TO PAY FOR REDUCED AMBIENT RISK

A standard result from the literature on nonstochastic health risk is that willingness to pay for reduced ambient risk is determined by the marginal rate of technical substitution between exogenous risk-reduction and self-protection. Shogren and Crocker claim (Proposition 1) that this result never holds when H is a continuum. This negative result is claimed to hold even when self-protection influences only the probability $F(h)$ and not the severity $C(h)$ or vice versa. They further present (Corollary 1) a list of instances in which the result holds. In each of these instances the health outcome space H is discrete. Using the state space approach, it is possible to derive necessary and sufficient conditions under which the standard result holds for the case when H is a continuum and self-protection affects both probability and severity of health outcomes.

With a slight abuse of notation, we may use the symbol ϕ to denote the function $\phi(s, r, \theta, W_0)$, where r is now taken as an argument of the function, rather than determining a specific function $\phi(\cdot\,, \cdot, \,\cdot\,; r)$ as above. Suppose that the pair (s, r) is separable from θ in this function, so that we may write $\phi(s, r, \omega, W_0) = \upsilon(\gamma(s,r), \theta, W_0)$[2] for some functions γ, υ. In this representation, reduction of ambient risk has precisely the same effect as a an increase in self-protection in the proportion $-(\partial\gamma/\partial r)/(\partial\gamma/\partial s)$. A sufficient condition for separability of ϕ is that each of the functions h, C be representable as functions of $\gamma(s, r)$ and θ

We may now derive

Proposition 3: Willingness-to-pay for risk-reduction will be independent of risk attitudes if and only if (s, r) is separable from θ in ϕ.

Proof: Sufficiency: It is immediate that the compensating variation is given by

$$\partial W/\partial r = -\,(\partial\gamma/\partial r)/(\partial\gamma/\partial s).$$

More formally we may observe

$$dW/dr = \left(\int_\Omega U'(\phi)\partial\phi/\partial r\, d\omega\right) / \left(\int_\Omega U'(\phi)\partial\phi/\partial s\, d\omega\right) \qquad (5)$$

$$= \left(\int_\Omega U'(\phi)\partial\phi/\partial\gamma\ \partial\gamma/\partial r\, d\omega\right) / \left(\int_\Omega U'(\phi)\partial\phi/\partial\gamma\ \partial\gamma/\partial s\, d\omega\right)$$

[2] Strictly speaking, this equality is required only to hold except on a set of measure zero.

$$= (\partial\gamma/\partial r)/(\partial\gamma/\partial s)$$

Necessity: For the case when U is linear, we have

$$dW/dr = \left(\int_\Omega \partial\phi/\partial r\, d\omega\right) / \left(\int_\Omega \partial\phi/\partial s\, d\omega\right) \quad (6)$$

Denote the RHS of (6) by ζ. If separability does not hold, then consider the family of subsets of $\Omega = [0, 1]$ of the form $[0, \omega]$. There must be some subset Ω^* of this form on which either $(\partial\phi/\partial r)/\partial\phi/\partial s) \geq \zeta$, with strict inequality on a set of positive measure, or $(\partial\phi/\partial r)/(\partial\phi/\partial s) \leq \zeta$, with strict inequality on a set of positive measure. By making U sufficiently concave, it is possible to ensure that (5) is dominated by its value on Ω^* and is therefore not equal to ζ. The value of dW/dr must, therefore, depend on the utility function U ∩.

This separability assumption is a natural one to make in a situation where private self-protection is a perfect substitute for public action to reduce ambient risk. Most of the examples proposed by Shogren and Crocker may be characterized in this fashion. For example, suppose that the ambient risk r under consideration relates to pollution of municipal water supplies. The level of r may be affected by a range of public policies. Individuals may undertake self-protection by installing filtration systems, or by obtaining water from other sources. The total exposure to pollution from drinking water will be a function $\phi(s, r)$. The probability of a given health outcome h and the severity C will be determined by an interaction between $\phi(s, r)$, the exposure to water pollution, and the random variable θ which may be taken to capture all other stochastic events affecting health. Thus, separability is consistent with a situation in which individual actions affect both the probability of an adverse health outcome and the severity of that outcome. A separability assumption similarly applies to chelation of children with high blood lead concentrations and the use of sunscreen as a protection against UV radiation. In all of these cases, the self-protection activity is directed specifically at mitigating a given ambient risk. The benefits of public action to reduce the ambient risk (e.g. by cleaning up water supplies, banning lead in gasoline, protecting the ozone layer) will be equal, at the margin, to the costs of the private self-protection activities they displace.

An example where the separability assumption is unlikely to apply is that of reducing physical activities in response to air pollution. The reduction in physical activity will have health effects, such as increased susceptibility to heart disease, which are related to the general random variable θ, rather than the ambient level of air pollution.

There is a more fundamental reason for adopting the separability assumption in the present paper. Much of the standard analysis under certainty, such as the result that self-protection expenditures are a lower bound for the benefits of risk-reduction (see Proposition 4), depends on the assumption that self-protection is a substitute for public risk-reduction. Self-protection measures which are not specific to a given risk, such as reductions in physical activity, are likely to interact in a complex fashion with other elements of the individual's consumption bundle. This point is discussed in detail by Bartik [1] and, for the two-state case by Berger et al [3].

The central concern of Shogren and Crocker (and of the present paper) is whether the results of analysis from the certainty case are valid for risk-averse EU maximizers under uncertainty. If separability does not hold, the standard results for the certainty case break down. Non-separability may explain violations of theoretical predictions derived from the standard analysis, but, if so, there is no need to introduce the question of risk attitudes.

4. SELF-PROTECTION EXPENDITURES AND LOSSES DUE TO AMBIENT RISK

The third negative result claimed by Shogren and Crocker concerns the use of marginal levels of self-protection expenditures as lower-bound estimators for losses due to ambient risk. It follows immediately from (2) that, provided $C \geq 0$ and $r = 0 \Rightarrow C = 0$, total self-protection expenditure is a lower bound for total loss.

The marginal question is more difficult. From (2), it is apparent that this question is equivalent to that of whether $C(h(\theta, r, s), r, s^*(r))$ is increasing in r for all values of θ. If C is increasing in r, optimal self-protection only partially offsets the adverse effects of increasing ambient risk, and self-protection expenditures are a lower bound estimate. Even under certainty, it is possible that threshold effects or lumpiness in the self-protection technology could imply that, over some ranges of ambient risk, the optimal level of self-protection could increase rapidly enough to reduce C. Some form of convexity assumption is required to eliminate this possibility.

Assumptions of this kind are most easily specified in the case when ϕ is separable, as in Proposition 3. Turning first to the function $\gamma(s, r)$ specifying the combined impact of ambient risk and self-protection, note that this defines a dual function $\kappa(\gamma^*, r) = \min\{s: \gamma(s, r) = \gamma^*\}$. We wish to derive conditions under which the optimal response to a marginal increase in risk should only partially offset that increase. The natural condition is that $\partial^2\kappa/\partial r\ \partial\gamma^* \geq 0, \quad \gamma^*, r$. That is, the marginal cost of the self-protection expenditures required to achieve a given improvement in the cost outcome in any state of the world increases with the level of ambient risk. If this condition is satisfied, it is apparent that the desired result will hold under certainty.

We may now specify the general control problem as

$$Max_{\gamma}\ \mathrm{E}[U(\upsilon(\gamma, \theta, W_0))] \tag{7}$$

and apply an analysis similar to that of Propositions 1 and 2. Since the effect of an increase in r is to increase the marginal cost of reductions in the net risk γ, decreasing absolute risk aversion is sufficient to ensure that an increase in r will lead to an increase in the optimal value of γ, and hence that the lower bound condition will be satisfied.

Proposition 4: Assume that, (A.1-5) and (R.1-3) hold, that (s, r) is separable from θ in ϕ, and $\partial^2\kappa/\partial r\ \partial\gamma^* \geq 0, \quad \gamma^*, r$. Then the marginal increase in self-protection expenditures arising from a given increase in ambient risk is a lower bound for the marginal loss.

Proof: From the discussion above, this is a corollary of Proposition 1. ∎

5. WILLINGNESS TO PAY FOR RISK REDUCTION

The final set of negative results claimed by Shogren and Crocker (Proposition 2) concern conditions under which willingness to pay for marginal reductions in risk is an increasing function of the ambient risk level. They derive clear-cut results only for the case when there are no severity effects.

A more general approach to the problem may be made using the theory of stochastic dominance. Consider a given level of risk r, for which the optimal level of self-protection is $s^*(r)$. The pair $(r, s^*(r))$ determines the distribution of the random welfare outcome $\phi(\theta, r, s^*(r), W_0)$. The change from the situation when $r = 0$ is given by the random variable

$$\phi^*(\theta, r, s^*(r), W_0) = \phi(\theta, r, s^*(r), W_0) - W_0 \tag{8}$$

For any $\lambda < 1$, the pair $(\lambda r, \lambda s^*(r))$ also determines a random variable $\phi^*(\theta, \lambda r, \lambda s^*(r), W_0)$. When the ambient risk value takes the value λr, this random welfare distribution is available to the individual, although it may not be optimal. Under appropriate convexity conditions, it will be true that $\phi^*(\theta, \lambda r, \lambda s^*(r), W_0)$ second stochastically dominates $\lambda\phi^*(\theta, r, s^*(r), W_0)$ for any choice of r and $\lambda \leq 1$. This result applies with self-protection constrained to the value $\lambda s^*(r)$. *A fortiori*, for any r and λ, $\phi^*(\theta, \lambda r, s^*(\lambda r), W_0)$ second stochastically dominates $\lambda\phi^*(\theta, r, s^*(r), W_0)$. By the concavity of U, the welfare loss Δ associated with the distribution $\lambda\phi^*(\theta, r, s^*(r), W_0)$ is convex in λ. Hence by the second stochastic dominance argument presented here, it is apparent that

$$\Delta(\phi^*(\theta, \lambda r, s^*(\lambda r), W_0)) \leq \lambda\Delta(\phi^*(\theta, r, s^*(r), W_0)) \quad \lambda \leq 1, r.$$

This means that welfare loss is a convex function of λ as λ ranges over the set [0, 1] and hence that the marginal welfare loss is an increasing function of r.

All that remains is to specify conditions under which $\phi^*(\theta, \lambda r, \lambda s^*(r), W_0)$ second stochastically dominates $\lambda\phi^*(\theta, r, s^*(r), W_0)$. In the separable case, sufficient conditions are given by

(i) $\partial^2\kappa/\partial r\, \partial\gamma^* \geq 0, \quad \gamma^*, r$

(ii) $\upsilon(\gamma, \theta, W_0) = W_0 + \gamma\theta + \bar{C}(\gamma)$

The first condition ensures that $\gamma(\lambda r, \lambda s^*(r)) \leq \lambda\gamma(r, s^*(r))$. By the second condition this means that $\lambda\phi^*(\theta, r, s^*(r), W_0)$ may be derived from $\phi^*(\theta, \lambda r, \lambda s^*(r), W_0)$ by a downward shift and a multiplicative spread about the mean and is therefore second stochastically dominated. This yields

Proposition 5: Assume that (s, r) is separable from θ in ϕ, and $\partial^2\kappa/\partial r\, \partial\gamma^* \geq 0$, γ^*, r, where

$$\upsilon(\gamma, \theta, W_0) = W_0 + \gamma\theta + \bar{C}(\gamma).$$

Then the marginal willingness to pay for a reduction in risk is an increasing function of ambient risk. n

Substantially weaker conditions could be obtained. In particular the term $\gamma\theta$ can be replaced by any risk term which increases at least multiplicatively in γ. This is part of the content of the assumption of strong convexity made by Shogren and Crocker (Corollary 2).

The most restrictive condition used in deriving Propositions 3-5 is the requirement that ϕ be separable in (r, s) and θ. This requirement is necessary and sufficient for Proposition 3, but merely sufficient for Propositions 4 and 5.

It may be useful to consider an alternative possibility - that ϕ is separable in (r, θ)

and s. In this case, the observed ambient risk interacts with the unobserved risk factors to determine an overall risk level $\varphi(r, \theta)$, and self-protection counteracts this overall risk. As an example, suppose that r represents road quality and θ represents random factors, such as the behavior of other drivers, that affect the riskiness of driving. Suppose that the combined impact of these factors may be represented by a function $\varphi(r, \theta)$, and that self-protection reduces the impact of φ. Individuals may undertake self-protection through careful driving, which affects both the probability and severity of accidents. Alternatively, they may choose to make fewer journeys (reducing the probability of accident) or wear seat belts (reducing the severity).

This alternative separability condition is sufficient to ensure that Proposition 4 will hold whenever the optimal self-protection strategy only partially offsets increases in total risk. Proposition 5 will hold when the two risk factors interact in a synergistic fashion and the self-protection technology is convex.

When ϕ is not separable, the problem is more complex, since interactions between three variables must be taken into account. Nevertheless, the results presented above yield a presumption, that when all of these interactions take their most probable forms, the main results from the analysis under certainty will carry over to the expected utility model.

CONCLUDING COMMENTS

Shogren and Crocker conclude that the implications of their work for efforts to value risks to human health and property are “unequivocally negative”. Correspondingly, the implications of the present paper are unequivocally positive. The main assumptions employed in empirical work are valid under quite plausible assumptions. Apart from the standard assumption of decreasing absolute risk aversion, the most important assumption is that of separability between the pair (r, s) and the random element θ. This assumption means that the self-protection activity under consideration is directed specifically at mitigating the effects of the ambient risk factor *r* and has no other health implications. Even when this condition is not satisfied, important positive results can be obtained in place of the negative results claimed by Shogren and Crocker. In particular, the result that increasing ambient risk calls forth increasing self-protection does not depend on separability assumptions, but only on decreasing absolute risk aversion.

Positive results are generally appealing for economists. Their appeal is diminished, however, when they appear to be contradicted by the available empirical evidence. Shogren [19] finds evidence against the prediction, derived above, that individuals will display increasing marginal compensating variation for risk exposure as the level of risk increases. One possible explanation of this result, is that individuals do not satisfy the axioms of Expected Utility. In related work [16], I have shown that this prediction does not hold under more general models of choice under uncertainty. However, the other results derived here carry over to the general case. Since the assumption of increasing marginal compensating variation is not critical in most applied work, it may be that generalized EU models will provide a more robust and satisfactory basis for analysis.

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