SOME OBSERVATIONS ON INSURANCE, BANKRUPTCY AND INPUT DEMAND

by

John Quiggin

University of Maryland

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Abstract

The effects of insurance on input demand have been central to the debate over crop insurance policy, and are of interest in assessing the moral hazard problem facing insurers in general. In this paper, the effects of insurance on input demand are characterized and related to a schema by which inputs can be assessed as risk-reducing, risk-increasing or risk-constant. The results are extended to analyze the impact of bankruptcy provisions.

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A major issue in the policy debate over crop insurance has been the likely impact on the demand for particular inputs such as fertilizers and pesticides and the consequences for total output. The problem is also of interest in the analysis of insurance problems in general. Moral hazard problems are usually analyzed without a detailed specification of the set of actions available to the insured person. When insurance takes place in the context of production (including the case of household production) it is possible to specify the set of available actions in terms of input choices.

Most of the relevant issues may be captured in a simple model with only two states of the world, one good and one bad. The two-state model is particularly appropriate for the analysis of input demand under insurance because the effect of insurance is to generate the same return in all 'bad' states (those in which insurance payouts occur). Since variations in income between good states are of secondary interest, the two-state model is appropriate[1].

Intuition about incentive effects (referred to in the insurance context as 'moral hazard') would suggest that insurance should generally reduce input use. This intuition is particularly strong in the case of inputs such as pesticide which may be regarded as risk-reducing , since insurance offers an alternative risk-reduction mechanism. By contrast, when inputs are viewed as risk-increasing, intuition becomes ambiguous. By cushioning the effects of bad outcomes, insurance may encourage risk-taking.

As is shown in this paper, these intuitions are generally valid. Under fairly weak conditions, insurance will lead to a reduction in input use for risk-reducing inputs and those which have no risk effects, an increase in input use for those which are 'strongly' risk-increasing and ambiguous effects for those which are 'weakly' risk-increasing.

[1] The mode of analysis here owes much to the work of Yaari (1965) and subsequent papers by Yaari and others.

The two-state model

Consider a production technology with a single variable input denoted x and a single output commodity y. There are two states of the world a good state 1 and a bad state 2, state i occurring with probability p^i. The technology is represented by two functions f^1, f^2 where f^i represents output in state i, $f^1(x) \geq f^2(x)$ x, and each f^i is concave in x. The market price p and the input price w are assumed to be known with certainty. Thus, profit in state i is given by

$$\pi^i = pf^i(x) - wx$$

The input is defined to be risk-reducing (in a neighborhood of x) if $\partial f^1/\partial x \leq \partial f^2/\partial x$, and risk-constant if $\partial f^1/\partial x = \partial f^2/\partial x$. That is, a risk-reducing input is one in which marginal product is higher in the bad state than in the good state. For example, if the bad state represents severe insect attack, then pesticide would be a risk-reducing input.

Two notions of risk-increasingness are considered. The input is weakly risk-increasing if $\partial f^1/\partial x \geq \partial f^2/\partial x$ and strongly risk-increasing if $\partial f^2/\partial x \leq 0$. If large applications of fertilizer may result in burn damage in low rainfall years, then fertilizer may be regarded as a strongly risk-increasing input, at least for some values of x.

The first stage of the analysis is to consider the equilibrium in the absence of insurance. For risk-neutral producers, the equilibrium condition is simply

$$E[\, p\partial f^i/\partial x - wx] = 0 \qquad (1)$$

and for risk-averse producers

$$E[U'(\pi^i)(\, p\partial f^i/\partial x - wx)] = 0 \qquad (2)$$

A standard argument (eg Sandmo 1971) shows that the level of the control variable will be below the risk-neutral level whenever $\partial f^1/\partial x$ is higher in the good state than in the bad state, and hence is negatively correlated with $U'(\pi^i)$. The following result, then, characterizes the impact of risk aversion.

Observation 1: *Let the technology be defined as above. Then for a risk-averse producer* *input demand is above, (equal to, below) the level for a risk neutral producer when the input is risk-reducing (risk-constant, risk increasing).*

Now consider the impact of crop insurance. In this analysis, the demand for insurance is not considered. The object is simply to examine the impact on input demand if insurance is selected by or imposed on, the producer. The insurance scheme under consideration guarantees a minimum revenue level py^* and imposes a premium c. If the bad state occurs a payout of $p(y^*-y)$ is made[2]. The producer's profit is now given by a functions π^* where

$$\pi^{1*} = p f^1(x) - wx - c$$

and

$$\pi^{2*} = py^* - wx - c$$

The marginal return to the input is now given by

$$\partial\pi^{1*}/\partial x = \partial\pi^1/\partial x = p\, \partial f^1/\partial x - w$$

$$\partial\pi^{2*}/\partial x = -w$$

First consider the impact of insurance for a risk-neutral producer. Equation (1) is replaced by

$$E[\partial\pi^*/\partial x] = 0 \qquad (1^*)$$

If the input is strongly risk-increasing,

$$E[\partial\pi^*/\partial x] \geq E[\partial\pi/\partial x]$$

and otherwise

$$E[\partial\pi^*/\partial x] < E[\partial\pi/\partial x]$$

[2] This insurance scheme corresponds most closely to hail insurance where losses are paid contingent on the occurrence of a stated event (eg a hailstorm). Public policy has been centered on 'multiple peril' crop insurance in which losses are covered unless there is clear evidence of moral hazard.

By the concavity of f^1, this yields

Observation 2: *For a risk-neutral producer, insurance will increase input demand if the input is strongly risk-increasing and reduce it otherwise.*

This is the standard moral hazard problem which affects all insurance contracts. The insured person has an incentive to undertake actions which increase losses in adverse states of the world and to refrain from actions which would mitigate losses in those states of the world.[3]

Now consider the situation of a risk-averse producer. It is useful to observe that the presence of insurance means the marginal return to the input is lower in the bad state than the good state whenever the marginal product is positive in the good state, and this will always be true when (1*) is satisfied. That is, under insurance, the input is effectively risk-increasing. Hence, from Observation 1, it follows that

Observation 3: *For a risk-averse producer, input demand under insurance will be below the risk-neutral level.*

From Observations 1 and 3 , it follows that for risk-reducing or risk-constant inputs, the effect of insurance is to move the risk-averse producers from an input use level at or above the risk-neutral level to one below the risk-neutral level. At the same time, from Observation 2, the risk-neutral input level is falling. Hence

Observation 4: *For a risk-averse producer, insurance will reduce input demand whenever the input is risk-reducing or risk-constant.*

In order to deal with the case of risk-increasing inputs, it is necessary to derive the analog of (2)

$$E[U'(\pi^*)\, \partial\pi^*/\partial x] = 0 \qquad (2^*)$$

A continuous shift from the original position to the insurance position may be represented in terms of a parameter λ, such that returns are given by $\lambda\pi + (1-\lambda)\pi^*$. Implicit differentiation with respect to λ yields

[3] In two-state models, moral hazard is sometimes represented in terms of actions which increase the probability of the insured state occurring. From the point of view of standard models of choice under uncertainty this represents a confusion between states, acts and consequences. States should be defined so that their probability of occurrence is independent of the actions of the decision-maker. In a model with more than two states, moral hazard may arise when there are states in which some actions will lead to an insurance payout and others will not, so that the probability of payout (but not the probability of any given state) is dependent on the actions of the insured.

$$\partial x/\partial \lambda = E[U'(\pi^{i}*)\ \partial^2\pi^{i}*/\partial x\partial\lambda] + E[U''(\pi^{i}*)\ \partial\pi^{i}*/\partial x\ \partial\pi^{i}*/\partial\lambda]$$

$$= p_2\ U'(\pi^2*)(-\partial f^2/\partial x) + p_1 U''(\pi^1*)(\ p\ \partial f^1/\partial x - w)(-c) + p_2\ U''(\pi^2*)(-w)\ (py*-pf^2(y)) \quad (3)$$

For a strongly risk-increasing input, all of these terms are positive.

Observation 5: *For a risk-averse producer, insurance will increase input demand whenever the input is strongly risk-increasing.*

We are left with an ambiguous result for the remaining class of risk-increasing inputs. For these inputs, the moral hazard effect, given by the first term in the RHS of (3) would lead to a reduction in input use, but the risk effects, given by the other two terms, are positive, reflecting the fact that insurance reduces the riskiness of income and encourages more use of risk-increasing inputs. As $\partial f^2/\partial x$ rises, the moral hazard effect will gradually come to predominate. The results may be summarized in the diagram below (Figure 1).

The diagram may be understood as follows. The vertical axis, denoted Δx, shows the change in input demand associated with insurance. The horizontal axis shows $\partial f^2/\partial x$, marginal product in the bad state. In any equilibrium, the marginal product must be less than or equal to w in one state and greater than or equal to w in the other. Hence the input is risk-reducing (constant, increasing) as $\partial f^2/\partial x > (=, <)\ w$. By definition the input is strongly risk-increasing when $\partial f^2/\partial x \leq 0$. Thus the crossing point for risk-averse producers must lie in the range $[0,w]$. By Observations 1 and 3, the insurance effects for risk-neutral and risk-averse producers must also intersect somewhere in this interval, as shown.

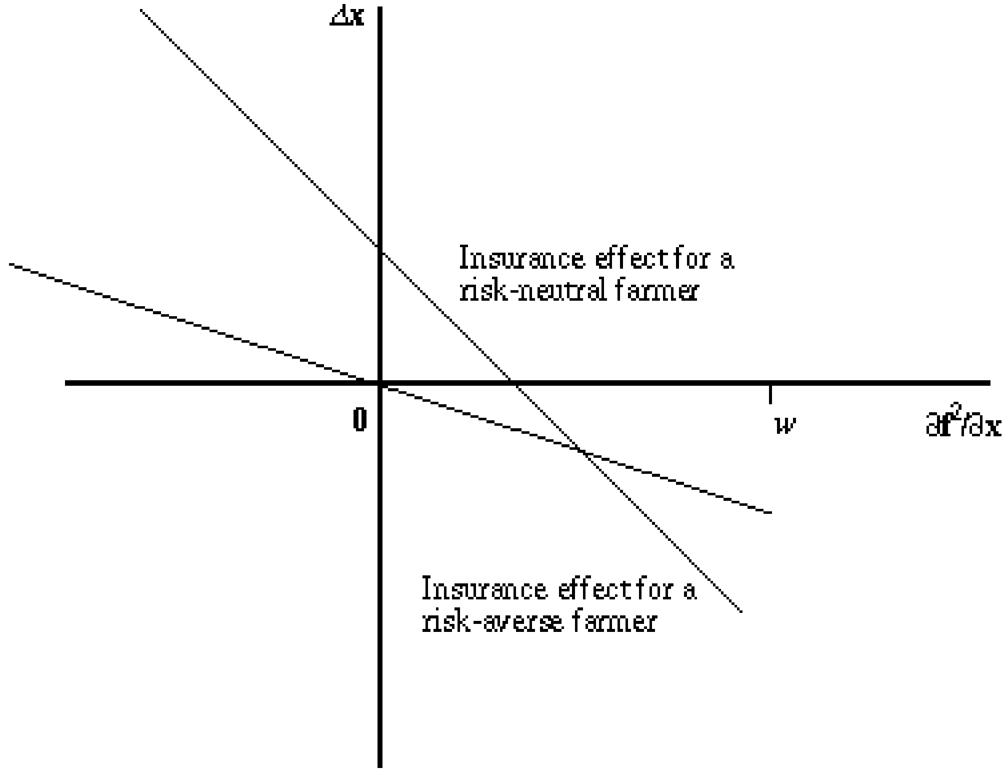

Figure 1: *Effects of insurance for risk-averse and risk-neutral farmers*

In order to determine the crossing points, is useful to consider a range of insurance contracts, beginning with the 'zero contract' in which $c = 0$ and $y^*= f^2(y)$, where f^2 is evaluated at the initial level of input use. A higher level of insurance corresponds to an increase in c and y^*.[4] Under the zero contract, the second and third terms in the RHS of (3) are zero, and the input demand effect of insurance is unambiguously negative whenever $\partial f^2/\partial x$ is positive. As the level of insurance rises, both of these positive terms increase and the crossover point rises.

The discussion so far has focused on the effects of insurance on input demand. Whenever marginal product is positive in both states, the change in output will have the same sign as the change in input. When marginal product is negative in one state, it is necessary to consider the change in expected output. For a risk-neutral producer, the expected value of marginal product must be positive in the initial equilibrium, and this will also be true for a risk-averse producer if

[4] A noteworthy feature of the analysis is that no assumption has been made concerning the actuarial fairness or otherwise of the insurance scheme. Indeed since the expected return to insurance depends on the value of x, definition of a fair scheme poses some conceptual problems.

the input is risk-constant or risk-increasing. Hence the only problematic case is that of a risk-reducing input with negative marginal product in the good state. In the absence of insurance, a highly risk-averse producer may employ such an input beyond the point where its expected marginal product becomes negative. Thus, the reduction in input use arising from insurance will lead to an increase in expected output.

Bankruptcy

The availability of bankruptcy acts as a form of insurance. There are two main differences between the effects of bankruptcy provisions and those of the crop insurance problem analyzed above. The first is that bankruptcy sets a floor for wealth rather than for output. Hence in states where bankruptcy occurs, it is not the marginal revenue product but the net marginal return to use of an input which is set to zero. This means that bankruptcy states can effectively be excluded from consideration in input decisions. The second is that changes in input choices may affect the probability of bankruptcy, that is, may determine whether or not bankruptcy occurs in certain states of the world. This second effect will not be treated in the two-state setting. Rather, it will be assumed that the bad state is such that bankruptcy is inevitable. While this is a strong assumption, it is not wholly implausible. Many businesses are founded on the basis of a single product or a single major contract, and the failure of the product or the loss of the contract will be sufficient to ensure the bankruptcy of the company. Furthermore, as long as the existence of bankruptcy does not generate discontinuities, the first-order conditions for an optimum are the same whether or not the probability of bankruptcy is exogenous.

The marginal return to the input is now given by

$$\partial\pi^{1*}/\partial x = \partial\pi^{1}/\partial x = p\, \partial f^{1}/\partial x - w$$

$$\partial\pi^{2*}/\partial x = 0$$

By the EU independence axiom, the fact that differences in input choices make no difference to the return in the bankruptcy state means that this state may be ignored. This result also holds for the weaker axiom of ordinal independence (Green and Jullien 1988) which is satisfied by a range of model notably including the rank-dependent expected utility model (Quiggin 1982).

Hence, the equilibrium condition for all producers, whether risk-neutral or risk-averse, becomes simply

$$\partial\pi^1/\partial x = 0$$

and under the provision of bankruptcy, all producers make the same input choices.

Given the concavity of f, this is sufficient to sign the input demand response to the existence of bankruptcy provisions.

Observation 6: *The existence of bankruptcy provisions will increase input demand if the input is risk-increasing and reduce it if the input is risk-reducing.*

One interpretation of this is that the existence of bankruptcy provisions adds a convex segment to the bottom tail of the utility function, thus making the producer more like a risk-seeker.

The General Case

Now turn from the convenient two-state model to the general case. The state space is represented without loss of generality by an interval of the form $\Omega = [0, 1]$, where the lower the value of ω Ω, the worse the state. The production technology is given by a function $f(x, \omega)$ which is increasing in ω and concave in x. In a neighborhood of a given x, the input is defined as risk-reducing (constant, increasing) as $\partial^2 f/\partial x\partial\omega < (=, >) 0$ ω. (Note that, unlike the case of the two-state model, some inputs may not fall into any of these categories, since $\partial^2 f/\partial x\partial\omega$ may change sign for different values of ω.) An input will be referred to as strongly risk-increasing (with respect to ω^0) if it is risk-increasing and $\partial f/\partial x \leq 0$ ω $[0, \omega^0]$. With these definitions, Observation 1 holds in the general case, since $\partial f/\partial x$ is positively correlated with $U'(\pi)$ whenever the input is risk-reducing. Hence we may restate it as

Observation 1*: *Let the technology be defined as above. Then for a risk-averse producer input demand is above, (equal to, below) the level for a risk neutral producer when the input is risk-reducing (risk-constant, risk increasing)*

.An insurance scheme yields a guaranteed revenue py* as before. It is convenient to denote by ω^* the state of the world for which the initial input level x yields $f(x, \omega^*) = y^*$. For a risk-neutral producer the impact of insurance depends only on the sign of $E[\ \partial f/\partial x : \omega\leq\omega^*]$. The important cases are given by

Observation 2*: *For a risk-neutral producer, insurance will increase input demand if the input is strongly risk-increasing with respect to* ω^* *and reduce it whenever* $\partial f/\partial x$ *is everywhere positive.*

The reasoning leading to Observation 3 requires that after insurance, the input should be effectively risk-increasing. This will be true if for $\omega > \omega^*$, $\partial f/\partial x$ is constant or increasing in ω. This condition (referred to henceforth as Condition 1) will trivially be satisfied for risk-constant or risk-increasing inputs. There are also a large class of risk-reducing inputs for which Condition 1 is at least a reasonable approximation. Pesticides provide an example. It seems reasonable to assume that the typical mix of pesticides includes some, aimed at endemic pests, which yield a more-or-less constant return and others which are used to minimize the impact of particular adverse events (unpredictable attacks by particular pests). Thus for those states in which an attack occurs the benefits of pesticides will be greater, the worse the state. Across the remainder of the state space, the benefits will be constant.

Observation 3*: *Assume Condition (1) is satisfied. Then, for a risk-averse producer, input demand under insurance will be below the risk-neutral level.*

Observation 4*: *Assume Condition (1) is satisfied. Then, for a risk-averse producer, insurance will reduce input demand whenever the input is risk-reducing or risk-constant.*

The argument of Observation 5 carries across for inputs which are strongly risk-increasing with respect to ω^* with the additional proviso, implicit in the two-state case, that $\partial f/\partial x$ is positive for all $\omega > \omega^*$.

Observation 5*: *Assume* $\partial f/\partial x$ *is positive for all* $\omega > \omega^*$. *Then, for a risk-averse producer, insurance will increase input demand whenever the input is strongly risk-increasing with respect to* ω^*.

Finally, the results on bankruptcy carry over neatly. For a risk-reducing input the effect of bankruptcy provisions is to eliminate from consideration the states in which net marginal returns are highest. By the concavity of *f*, the optimality conditions can only be restored by a reduction in input use. The converse argument applies for risk-increasing inputs. Observation 6 is thus carried over without modification.

Observation 6*: *The existence of bankruptcy provisions will increase input demand if the input is risk-increasing and reduce it if the input is risk-reducing.*

Note, however, that the optimum is no longer the same regardless of the level of risk-aversion, as in the two-state case. The elimination of a subset of bad states does not change the basic form of the problem, and Observation 1 applies to the solution in the presence of bankruptcy provisions.

Thus the results from the two-state case carry across with comparatively minor restrictions on $f(x, \omega)$. With additional effort, these restrictions could certainly be weakened further, but it would appear that the point of diminishing returns has been reached.

It is also straightforward to relax the assumption of a single variable input. Consider instead a generalized Pope-Just production function of the form

$$\phi(x, \mathbf{z}, \omega) = g(x, \mathbf{z}) + h(x, \omega)$$

Here $\mathbf{z}$ is a vector of risk-constant inputs and has the properties already derived. By optimizing out with respect to $\mathbf{z}$ a production function of the form $f(x, \omega)$ is derived. The input x is risk-reducing for $f(x, \omega)$ if and only if it is risk-reducing for $h(x, \omega)$.

Concluding Comments

There are a number of state-contingent market and public policy instruments which affect states yielding outcomes in the lower tail of some distribution. In addition to the cases of insurance and bankruptcy considered here, these include stock options and minimum guaranteed income schemes. It seems likely that decision problems involving these instruments could be modelled in terms of production and that effects on input demand would be of interest. It would also be useful to extend the analysis presented here to take account of interactions between multiple inputs, all of which affect risk.

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