Supply Response under Proportional Profits Taxation

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A striking result in the theory of the competitive firm under certainty is the proposition that a proportional profits tax (with full offsets for losses) will have no impact on optimal output. The non-distortionary properties of such a tax have been the basis of a large literature, incorporating attempts to devise a real tax which approximates the theoretical ideal as closely as possible (e.g., Brown).

This result does not apply under uncertainty. In the first Expected Utility analysis of the firm under uncertainty, Sandmo claimed that a proportional profits tax would increase or decrease output according to whether the coefficient of relative risk-aversion is increasing or decreasing. Katz pointed out that this result was erroneous and proposed several complex conditions involving both absolute and relative risk-aversion. In their recent survey of the theory of the firm under uncertainty, Robison and Barry use a mean-variance approximation to the expected utility (EU) model to examine this question and conclude that the impact of a proportional profits tax is ambiguous, except in the restrictive special case of constant absolute risk aversion.

The competitive optimal output under certainty is only defined when returns to scale are, at least eventually, decreasing. This situation does not, however, apply, under uncertainty. Quiggin (1982a) shows that, under weak assumptions, a competitive optimal output will exist under constant or increasing returns to scale[1]. This analysis is particularly important in relation to agriculture, since many studies have found evidence of increasing returns to scale (). The most usual result is the existence of an L-shaped cost curve, with an initial region of increasing returns followed eventually by constant returns (). Risk-aversion is a promising explanation for the observed existence of competitive firms of finite size.

The object of this note is to show that, under constant or increasing returns to scale, a proportional profits tax will yield an unambiguous expansion in output. The same result is shown to hold for the more general Rank-Dependent Expected Utility (RDEU) model (Quiggin 1982b, Yaari). The result does not generalize to other general models such as the smooth preferences model of Machina.

The seemingly perverse supply response derived here may seem less surprising when it is considered in the light of the debate over the treatment of risk in public projects, following the work of Arrow and Lind. In this literature, the analysis is cast in terms of discount rates. However, there is a direct relationship to the problem of the firm under uncertainty, because the more the discount rate is increased to take account of risk, the lower is the level of investment and output. One of the themes in that literature is that a proportionate corporate profits tax makes the government an effective partner in private projects, bearing a share of the risk proportionate to the tax rate (Mayshar). This relationship implies that the discount rate for such projects should be lowered, that is, that the number of projects undertaken should be larger than it would be for the same market rate of interest in the absence of a corporate tax rate. Thus, at least in a partial equilibrium sense, this analysis leads us to expect a positive supply response to a proportional profits tax. A similar analysis applies to risk-sharing problems such as those of share-cropping and franchise contracts.

The positive supply response result may also be motivated by considering the case of constant returns to scale. Under constant returns to scale, for any realization of the random price variable p, profit is linear in output α. It is well-known (Sandmo 1971, Quiggin 1982a) that, whereas output under certainty is indeterminate in this case, a risk-averse firm under uncertainty will have a finite optimal output. Denote this output α^* and let the associated distribution of profits be $F(\pi|_{\alpha^*})$. Now suppose a tax is imposed at a rate of 50%. By the linearity property of constant results to scale technology, the distribution $F(\pi/2|_{2\alpha^*})$ obtained at an output of $2\alpha^*$ is identical to the original optimum $F(\pi|_{\alpha^*})$, and is the new optimal distribution.

The economic control problem

The positive supply response result will be proved in the context of a general economic control problem which includes as special cases the problem of the firm under price or yield uncertainty. The firm is required to solve the maximization

(1) $\mathrm{Max}_{\alpha}\ E[U(\pi(\theta, \alpha))]$

where θ is an economically relevant random variable, such as price or yield, and α is a control variable, interpreted here as (planned) output. The profit function π determines the level of profits for given choices of θ and α. In EU theory, the function *U* is a von Neumann-Morgenstern utility function, here assumed to be concave. It will be assumed that

(A.1) $\partial\pi/\partial\theta > 0$,

(A.2) $\partial^2\pi/\partial\theta\partial\alpha \geq 0$

These conditions will be fulfilled in all the standard versions of the control problem. The first ensures that increasing values of the random variable (eg output price) increase profits. This condition is a natural one in most versions of the problem. The second condition ensures that increasing values of the random variable increase the marginal return to the control variable (eg output). For the firm under output price uncertainty, we have

(2) $\pi(\theta, \alpha) = \theta\alpha - C(\alpha)$

and (A.2) is clearly satisfied. Many more general forms of the profit function will also satisfy (A.2)

The first-order condition for an interior optimum for the problem (1) is

(3) $E[\ \partial U/\partial\pi\ \partial\pi/\partial\alpha] = 0$

and the second-order condition is

(4) $D = E[\ \partial^2 U/\partial\pi^2\ (\partial\pi/\partial\alpha)^2 + \partial U/\partial\pi\ \partial^2\pi/\partial\alpha^2] < 0$

Given the assumed concavity of *U*, the second-order condition will always be satisfied if π is concave or linear in α.

It remains to be shown that an interior solution will exist. The decision-maker must be sufficiently risk-averse to permit the existence of a finite optimal output, while production must be sufficiently attractive that the optimal output is non-zero. Quiggin (1982a) derives necessary and sufficient condition for the existence of a finite optimum. It is assumed that there is, in the limit, a positive probability that increases in the control variable will reduce profit. That is

$$\lim_{\alpha \to \infty} \Pr\{\partial\pi/\partial\alpha \leq 0\} > 0$$

Given this ‘no easy money’ assumption, a fairly weak sufficient condition on the utility condition is $\lim_{\pi \to \infty} U'(\pi) = 0$. This condition is satisfied by all the constant relative risk-aversion functions, and hence by any function in which the coefficient of relative risk-aversion is bounded away from zero.

The second requirement is that the optimal output should be positive. A sufficient condition may be derived from the observation, that, for 'small' risks, the decision-maker will be effectively risk-neutral. Thus, if $E[\partial\pi/\partial\alpha]$ is positive in a neighborhood of $\alpha = 0$, a positive level of output will be chosen.

Now let a proportional profit tax, with full rebates for losses, be imposed at a rate τ. Then the problem becomes

(5) $\quad \mathrm{Max}_{\alpha}\ E[U(\pi_0(\theta, \alpha))]$

where $\pi_0 = (1-\tau)\,\pi$

with first and second order conditions

(6) $\quad E[\,\partial U/\partial\pi_0\ \partial\pi_0/\partial\alpha] = 0$

and

(7) $\quad D = E[\,\partial^2 U/\partial\pi_0^{\,2}\ (\partial\pi_0/\partial\alpha)^2 + \partial U/\partial\pi_0 \partial^2\pi_0/\partial\alpha^2] < 0$

Implicit differentiation of (6) with respect to τ yields

(8) $\quad \partial\alpha^*/\partial\tau = (1/D)\ E[\,\pi\ \partial^2 U/\partial\pi_0^{\,2}\ \partial\pi/\partial\alpha]$

where, as before, α^* denotes the optimum level of α.

By (A.1), for any α, there is a unique value $\tilde{\theta}$ such that $\theta \geq \tilde{\theta} \Leftrightarrow \partial\pi/\partial\alpha \geq 0$. Let $\tilde{\pi}$ be the value of $\tilde{\pi}$ given $\theta = \tilde{\theta}$.

Then

(9) $\partial\alpha^*/\partial\tau = (1/D)\{ E[(\pi-\widetilde{\pi})\, \partial\pi/\partial\alpha\, \partial^2 U/\partial\pi_0^{\,2}] + \widetilde{\pi}\; E[\partial\pi/\partial\alpha \partial^2 U/\partial\pi_0^{\,2}]\}$

The first expectation on the RHS is everywhere negative by the definition of $\widetilde{\pi}$, since $(\pi-\widetilde{\pi})$ has the same sign as $\partial\pi/\partial\alpha$ for every value of θ. The second expectation may be shown to be positive as follows. Observe that $\partial^2 U/\partial\pi_0^{\,2} = - A\, \partial U/\partial\pi_0$, where A is the coefficient of absolute risk-aversion. Thus

(10) $E[\partial\pi/\partial\alpha \partial^2 U/\partial\pi_0^{\,2}] = - E[A\, \partial\pi/\partial\alpha \partial U/\partial\pi]$

From the first order condition (6), the second expectation in the RHS of (9) will be zero whenever A is constant. This yields the result, derived by Robison and Barry, that $\partial\alpha^*/\partial\tau \geq 0$ under constant absolute risk aversion.

For the more plausible case of decreasing absolute risk aversion, we may proceed as follows. Let $\widetilde{A}$ be the value of A when $\theta = \widetilde{\theta}$. Since A is monotone decreasing, while $\partial\pi/\partial\alpha \partial U/\partial\pi$ is positive if and only if $\theta \geq \widetilde{\theta}$,

(11) $E[A\, \partial\pi/\partial\alpha \partial U/\partial\pi] \leq \widetilde{A}\; E[\partial\pi/\partial\alpha \partial U/\partial\pi]$

$= 0$, by (6).

Thus if $\widetilde{\pi}$ is negative, the entire term in curly brackets in equation (9) is negative. Then, by (8), $\partial\alpha^*/\partial\tau \geq 0$.

When the control problem is that of the competitive firm under output price uncertainty, θ represents price and the definition of $\widetilde{\theta}$ implies $\widetilde{\theta}$ is equal to marginal cost. Profit is equal to output multiplied by price less average cost. Thus, the sign of $\widetilde{\pi}$ depends on whether MC < AC. Under constant returns to scale, MC = AC and $\widetilde{\pi} = 0$. Under increasing returns to scale, MC < AC and $\widetilde{\pi} < 0$.

The discussion above yields

Proposition 1: For the competitive firm under output price uncertainty, given decreasing absolute risk aversion, a sufficient condition for a positive supply response to an increase in the rate of proportional profit taxation is that returns to scale should be non-decreasing.

For a U-shaped cost curve displaying eventually decreasing returns, the same argument establishes that there will be a positive supply response whenever the initial optimum is in the region of the curve characterised by declining or flat average costs; that is, whenever output is at or below the point at which average costs are minimized. Such a production point can never be optimal for the competitive firm in the absence of uncertainty, but it is perfectly feasible when uncertainty is present.

The extension to RDEU preferences is straightforward. Maximization of an RDEU functional is equivalent to maximization of Expected Utility with respect to a transformed probability distribution. For a problem such as that of a proportional profits tax, therefore, the EU result carries over directly. This technique does not work for the general smooth preferences modelled by Machina. The change in the distribution of income associated with a proportional profits tax will lead to a change in the local utility function with respect to which the first order condition (6) applies, and this ensures that comparative static results will be ambiguous.

Concluding comments

The existence of a tax which calls forward a positive supply response might seem like the discovery of a fiscal Philosopher's Stone[2]. However, the analysis presented here does not apply to ordinary forms of capital taxation such as corporate income taxes. The tax base is pure economic profit, that is, above-normal returns to capital and the tax must include full offsets for firms with below-normal returns to capital. Hence, if a proportional profits tax of the type analyzed here were applied across the board, the net revenue raised would be unlikely to be large, and might not even be positive.

The appropriate rôle for taxes of this kind arises when the tax base is some form of economic rent. As noted in the introduction, proportional profits taxes have been proposed as a means of taxing the rent associated with mineral deposits. A similar approach might be used as a means of financing agricultural developments such as irrigation schemes, particularly where the returns to farmers are very risky. Rather than selling water rights at market prices, governments could allocate them at low cost and impose a proportional levy on (pure economic) profit. In both mining and agricultural contexts, however, numerous practical difficulties must be overcome before a tax of this kind could be implemented.

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[1] The term returns to scale is sometimes used in the specific sense of the cost structure in relation to a proportionate expansion in all inputs, and distinguished from the case, referred to as returns to size, where the least-cost input mix varies as output increases. In this analysis, as in most of the literature on the firm under uncertainty, the term returns to scale will be used in the general sense including returns to size.

[2] The discovery of the Philosopher's Stone, which would transmute base metals into gold, was a key research objective of the medieval alchemists.