COMPARATIVE STATICS FOR RANK-DEPENDENT EXPECTED UTILITY THEORY

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Abstract

Recently, a number of generalizations of the Expected Utility (EU) model have been proposed. In order to make such generalizations useful, it is necessary that they should yield sharp comparative static results, like those obtained using EU theory. In this paper, Rank Dependent Expected Utility (RDEU) theory, a generalization of EU theory based on the concept of probability weighting, is examined. A number of methods of extending results from EU to RDEU are considered. It is shown that a major class of comparative static results can be extended to the RDEU model, but not to the case of general smooth preferences. This is because RDEU maintains the separation between probabilities and utilities which is abandoned in the general case.

Comparative statics for rank-dependent expected utility theory

The expected utility (EU) theory, first put forward by von Neumann and Morgenstern (1944) has proved a powerful tool for the analysis of choice under risk. Problems which have been investigated using this model include portfolio choice (Diamond and Stiglitz 1974), savings decisions (Sandmo 1970), labor supply (Newbery and Stiglitz 1981) and the theory of the firm (Sandmo 1971, Coes 1977).

In the last decade, alternative theories of choice under uncertainty have been developed which display properties of transitivity and preservation of stochastic dominance. Machina (1982) has developed a very general theory of ‘smooth’ non-expected utility preferences over probability distributions. A special case[1] of Machina's model, which maintains some independence properties, is the rank-dependent expected utility (RDEU) model (Quiggin 1981, 1982a, Allais 1987,Yaari 1987, see also Gilboa 1987, Schmeidler 1989 and Segal 1989). These generalized theories have been shown to be capable of accounting for most of the well-known violations of EU theory including the Allais paradox (Quiggin 1985, Segal 1987), the common ratio effect and the preference reversal effect (Karni and Safra 1987).

However, this increase in scope is of little use if generalized models are unable to generate sharp comparative static results of the type which have made EU theory such a powerful tool of analysis. To paraphrase Tobin’s challenge to EU theorists critical of the implausible restrictions imposed by mean-variance theory

> “its critics owe us more than demonstrations that it rests on restrictive assumptions. They need to show us how a more general and less vulnerable approach will yield the kind of comparative static results economists are interested in.”

[1] The RDEU model is not smooth, and thus is not, strictly speak a special case of Machina's model. However, as is shown by Chew, Karni and Safra (1987) RDEU is 'smooth enough' to permit the use of Machina's elegant theoretical apparatus.

It is important to examine the extent to which the results already derived in the analysis of the choice under uncertainty can be extended to generalized models. This problem has been examined for general smooth preferences by a number of writers including Machina (1982, 1989) and Chew, Epstein and Zilcha (1988). Machina (1989) shows that results from problems of ‘temporal risk’ can be carried over directly to the case of generalized smooth preferences. The approach used is the path integral method used in Machina (1982) to show that local properties of preferences carry over to global preferences. However, this approach relies on separation conditions which are not satisfied in many comparative static problems.

In this paper, an alternative approach to the problem of extending EU results is put forward. The central observation is that RDEU theory may be regarded as “expected utility with respect to a transformed probability distribution.” Under appropriate conditions, a change in the original probability distribution, such as a shift to the right, will result in a similar change in the transformed probability distribution. Hence, EU comparative static results carry over directly to RDEU. The only requirement for extension of a given comparative static result is the derivation of conditions on the transformation function under which the relevant change in the underlying probability distribution is preserved. In most cases, these conditions are quite weak. In particular, important comparative static results can be extended without the assumption of risk-aversion.

The approach used here could be employed to derive comparative static results for other generalized models based on transformations of probability distributions. Examples include prospect theory (Kahneman and Tversky 1979), the modified version of prospect theory incorporating a rank-dependent probability transformation (Tversky and Kahneman 1990) and prospective reference theory (Viscusi 1989). More generally, it would be possible to combine this approach with models of phenomena such as framing and coding, which have thus far proved difficult to incorporate into models of economic behavior.

1. The Correspondence Principle

RDEU theory deals with individual preferences over a set X of outcomes and an associated set Θ of prospects, or random variables, taking values in X. For the purposes of this article, X will be assumed to be an interval, and may be interpreted in terms of income, profit or wealth levels. A random variable θ Θ may be regarded as a measurable function from some σ-field of events, Ω, to X, and characterized by its cumulative distribution function, F_θ:x→ $\Pr\{\theta \leq x\}$. The degenerate random variable yielding outcome c with probability 1 will be denoted δ_c. It is possible, without loss of generality, to define Ω so that it can be ordered from worst to best states of the world. That is, if ω_1 Ú ω_2, then $\theta(\omega_1) \leq \theta(\omega_2)$ for all random variables θ.

Preferences over Θ are given by an RDEU functional V defined by a utility function U:X →   and a probability weighting function q:[0, 1] → [0, 1]. The function q may be composed with a cumulative distribution function F to yield a weighting function $H = q°F$, which possesses the usual properties of a cumulative distribution function. The RDEU functional is defined as

$$(1) \qquad V(\theta) = \int_\Omega U(x)\, \mathrm{d}H(\omega) = \int_X U(x)\, dq(F_\theta(x))$$

There are three main methods of extending results from EU to RDEU. First, a number of results can be extended simply by repeating the EU proof and imposing conditions on q where appropriate. In Quiggin (1986), this approach is used to generalize the main results of the EU theory of the firm under price uncertainty.

The second extension method relies on the fact that an analogue to Machina's local utility function can be defined for this model[2]. Following Machina's analysis it is natural to define the local utility function as

$$(2) \qquad U_F(x) = \int_0^x U'(\theta)\, q'(F(\theta))\, d\theta$$

[2] It is assumed here that q is differentiable. This assumption is not required for the main results in this paper.

Machina (1989) has shown that for an important family of economically significant control problems, the EU analysis can be satisfactorily generalized in terms of properties of the local utility function. This class of control problems is characterised by an objective function in which the control variable is separable from the random variable of economic interest. This is a fairly restrictive condition, which excludes most of the problems mentioned in the introduction. For example, in the theory of portfolio choice, the risky rate of return and the control variable (the amount allocated to the risky asset) interact to determine the distribution of total returns. The separation property applies to situations of temporal choice such as those examined in Machina (1984).

The use of the local utility function associated with an RDEU functional provides an elegant method of extending a range of EU results. It does not, however, exploit the specific features of the RDEU model, most notably the separation between probabilities and utilities inherent in (1). A third method of generalization exploits the fact that RDEU may be interpreted as maximizing Expected Utility with respect to the transformed cumulative distribution function $q{\circ}F$. Hence if the function q preserves a partial ordering, such as first or second stochastic dominance, on the space D of cumulative distribution functions, then any EU results concerning that ordering will carry over to the RDEU context.

This correspondence principle may be stated as follows. Let R be a relation on D, let $\boldsymbol{U}$ be a family of utility functions. Then a typical comparative static result states that if the movement from the initial to the final distribution of some relevant random variable satisfies the relation R, all Expected Utility maximizers with $U \quad \boldsymbol{U}$ will alter their optimal action in a particular way *e.g.* by increasing the value of some control variable.

Now let

$$(3) \qquad Q^*(R) = \{q \quad \mathsf{Q}: F_1 \, R \, F_2 \quad q{\circ}F_1 \, R \, q{\circ}F_2\}$$

Then the correspondence principle states that, provided the outcomes from every action are better, the higher is the realization of the relevant random variable, the conclusion of the comparative static result will be true for RDEU maximizers whenever $U \quad \boldsymbol{U}$ and $q \quad Q^*(R)$.

Because this approach will be central to most of the results derived here, a number of examples are worth considering.

Example 1: Let R = { (F_1, F_2) : F_1 first stochastically dominates F_2}.

Then $Q^*(R)$ = Q. The fact that RDEU preserves first stochastic dominance was proved by Quiggin (1982).

Example 2: Let R = { (F_1, F_2) : $\delta > 0$, $F_1(x) = F_2(x-\delta)$}.

Then $Q^*(R)$ = Q. This special case of first stochastic dominance corresponds to a rightward translation of the entire cumulative distribution function

Example 3: Let R = { (F_1, F_2) : F_1 second stochastically dominates F_2}

Then $Q^*(R)$ = {q Q: q concave}. This is proved by Chew, Karni and Safra (1987) using the local utility function method, but a direct proof is straightforward**.**

Example 4: Let R = { (F_1, F_2) : F_1 second stochastically dominates F_2, F_1 and F_2 are symmetric distributions}

Then q $Q^*(R)$ if q is symmetric; that is $q(p) = 1- q(1-p)$ and q is concave on [0, 1/2]. (Quiggin 1982a)

Example 5: Let R = { (F_1, F_2) : $F_1 = \delta_c$, E[F_2] ≤ c }.

Then $Q^*(R)$ = {q Q: $q(p) \geq p$ p}. This result is well-known and appears to have been first derived by Yaari (1987).

In this example, F_1 is a certain outcome, and F_2 is a risky variable having the same or lower mean. Preference for F_1 in situations of this kind is equivalent to the requirement for a positive risk premium. While most of the literature on generalizations of EU has concentrated on the definition of risk-aversion as aversion to mean preserving spreads, this relationship gives a more fundamental definition of risk-aversion. As was first noted by Machina (1982, 1983) the EU equivalence between preference for a certain outcome and aversion to mean preserving spreads does not carry over to generalized models.

For the final example, it is useful to consider a notion dual to the class $Q^*(R)$ derived in

Example 5. The idea of a monotone spread, developed in Quiggin (1988), provides the definition of a dual class. It is convenient to express the monotone spread concept in terms of random variables rather than cumulative distribution functions. θ_2 is derived from θ_1 by a mean-preserving monotone spread if there is a smooth path φ: $[0,1] \to \Theta$ such that

(i) $\varphi(\lambda) = (1-\lambda)\theta_1+\lambda\theta_2$

(ii) $\partial/\partial\lambda\ E[\varphi(\lambda)] = 0 \quad \lambda \quad [0,1]$

(iii) ω_1 ¿ $\omega_2 \Leftrightarrow \partial/\partial\lambda\ \varphi(\lambda)(\omega_1) \geq \partial/\partial\lambda\ \varphi(\lambda)(\omega_2) \quad \omega_1, \omega_2 \quad \Omega, \lambda \quad [0,1]$

Example 6: Let $R = \{(F_1, F_2)$: F_2 is derived from F_1 by a monotone spread, $E[F_2] \leq E[F_1]\}$ Then $Q^*(R) = \{q \quad Q: q(p) \geq p \quad p\}$. Proof Quiggin (1988).

The class of monotone spread relationships include multiplicative spreads, and situations where F_1 is a obtained from F_2 by truncating the tails, among others. Whereas only fairly weak results have been obtained for comparative static responses to mean-preserving spreads, strong results have been obtained for several categories of monotone spreads. In this paper, these results will be generalized to cover the entire class of monotone spreads.

The correspondence principle presented here covers the main class of comparative static problems. There are a number of related problems in EU theory which may be tackled in a similar manner. The most important are the Diamond-Stiglitz (1974) results on increasing risk aversion and on mean-utility preserving increases in risk. Because the RDEU functional incorporates both a utility function and a probability transformation function, it is necessary to consider two separate cases on increases in risk-aversion. The first is an increase in the concavity of the utility function. Given the interpretation of RDEU maximization as EU maximization with respect to a transformed cumulative distribution function, it is apparent that EU results for an increase in the concavity of the utility function will carry over directly to the RDEU case. The second idea of an increase in the risk-aversion of the RDEU functional arises from a change in the probability transformation function. Examples 2 and 4 suggest two alternative definitions for an increase in risk-aversion. The first is a concave transformation of q and the second is a increase in $q(p)$ for all p. The implications of these two changes will be considered briefly below.

2. The economic control problem

A wide range of maximization problems have been considered in the EU theory of choice under uncertainty. Many of these can be fitted into the following general framework, which is a slight generalization of that treated by Feder (1977):

$$(4) \qquad \text{Max}_{\alpha}\ V = \text{E}[U(\phi(\theta, \alpha, W_0))]$$

where U is a von Neumann-Morgenstern utility function, α is a control variable (assumed to take positive values), θ is an economically relevant random variable with cumulative distribution function F, W_0 is initial wealth (here taken to be non-stochastic), and ϕ is a function mapping actions and realizations of θ into outcomes, normally taken to be wealth levels. Two examples are the two-asset (one safe asset, one risky asset) portfolio choice model and the theory of the firm under uncertainty. In the first case, α is the quantity of the risky asset, θ is the rate of return on the risky asset, and ϕ is given by

$$(5) \qquad \phi(\theta, \alpha, W_0) = \alpha\theta + (W_0 - \alpha)\theta_0$$

where θ_0 is the rate of return on the safe asset. For the theory of the firm under uncertainty, α is firm output, θ is output price, W_0 is net assets after fixed costs have been met, and ϕ is a profit function of the form

$$(6) \qquad \phi(\theta, \alpha, W_0) = \alpha\theta - C(\alpha) + W_0$$

where C is a variable cost function. Clearly (6) is a generalization of (5). It may be modified to cover yield uncertainty, input price uncertainty etc.

The first-order condition for the general model (4) is

$$(7) \qquad \partial V/\partial\alpha = \text{E}[U'(\phi(\theta, \alpha, W_0))\ \partial U/\partial\phi\ \partial\phi/\partial\alpha] = 0$$

and the second-order condition is

$$(8) \qquad \partial^2 V/\partial\alpha^2 < 0$$

If the second-order condition is satisfied globally, then there will exist a unique optimum. However, in many of the problems to be examined here, it is unlikely that (8) will hold globally. It is, therefore, useful to consider an alternative condition which guarantees the existence (although not the uniqueness) of a finite optimum

(9) $$\lim_{\alpha \to \infty} \partial V/\partial\alpha = U_1 \int_{\phi_\alpha > 0}^{\infty} \phi_\alpha \, dF(\theta) - U_2 \int_{\phi_\alpha \le 0}^{\infty} \phi_\alpha \, dF(\theta) \text{ exists and is less than 0.}$$

Versions of this condition have been examined for the problem of portfolio choice by Bertsekas (1974) and for the firm under uncertainty by Quiggin (1982b).

Now let λ be a parameter which affects the value of W_0. Then, differentiating (7) with respect to λ yields

(10) $$\partial^2 V/\partial\alpha^2 \, \partial\alpha/\partial\lambda + \partial^2 V/\partial\alpha\partial W_0 \, \partial W_0/\partial\lambda = 0$$

By (8), $\partial\alpha/\partial\lambda$ will have the same sign as $\partial^2 V/\partial\alpha\partial W_0 \, \partial W_0/\partial\lambda$. Thus, the problem for comparative static analysis is to sign this term.

Similar considerations apply to a change in the distribution of θ from F_1 to F_2. The distribution of θ may be represented by $\lambda F_2 + (1-\lambda)F_1$ and the change in distribution may be represented by an increase in λ from 0 to 1. We have

(11) $$\partial^2 V/\partial\alpha^2 \, \partial\alpha/\partial\lambda + E[\, \partial^2 V/\partial\alpha\partial\theta \, \partial\theta/\partial\lambda] = 0$$

This is identical to (10) except that, since $\partial\theta/\partial\lambda$ and $\partial^2 V/\partial\alpha\partial\theta$ are stochastic, it is necessary to replace the second term with an expectation. Once again, the problem is to sign the second term in the LHS. Numerous results of this kind have been derived in the EU literature. In the present paper, these will be presented in a systematic fashion and, in some cases, generalized. To obtain results for RDEU theory, the major remaining requirements are to demonstrate that the correspondence principle derived above is applicable and to describe the conditions for the existence of a finite optimum. With these tasks accomplished, conditions for the extension of EU results to RDEU may be derived easily.

In order to obtain sharp comparative statics results, it is necessary to impose some conditions on the function ϕ. The following assumptions are sufficiently general to cover most cases of interest

(Φ.1) $\partial\phi/\partial\theta > 0$,

(Φ.2) $\partial\phi/\partial W > 0$

(Φ.3) $\partial^2\phi/\partial\theta\partial\alpha \geq 0$

(Φ.4) $\partial^2\phi/\partial W\partial\alpha \geq 0$

(Φ.5) $\partial^2\phi/\partial\theta^2 = 0$

Assumptions (Φ.1) and (Φ.2) are natural for most of the problems under consideration here. As Diamond and Stiglitz (1974) observe, (Φ.1) rules out short selling in portfolio problems. This is consistent with the assumption, noted above, that α is confined to positive values. Assumption (Φ.3) is natural if increases in α reflect increases in risk-taking. The assumption is then that the better the outcome of the risky variable, the more desirable (*ex post*) risk-taking will be (Φ.4) is a similar assumption, though in most cases, the inequality may be replaced by an equality. It states that increases in initial wealth will have a non-negative effect on the benefits of risk-taking. Finally (Φ.5) states that the outcomes are linear in the risky variable. This assumption holds for the examples presented above. Moreover, in problems where it is not satisfied, (for example if the firm problem is stated in terms of uncertain demand rather than uncertain price) an appropriate monotonic transformation of the random variable can often be used to impose linearity. It will be assumed throughout that the utility function displays decreasing absolute risk-aversion and hence that $U'''\geq 0$.

Proposition 1 is a summary of a number of major results of EU theory.

Proposition 1:

(a) Let F_1 be the initial distribution and F_2 the final distribution, and let θ_1 and θ_2 be the corresponding random variables. Given the assumptions above, the following are sufficient conditions for an increase in α^*.

(i) $\theta_2 = \theta_1 + \delta$ for some $\delta > 0$

(ii) $F_2 = \chi_c$, $E[F_1] \leq c$

(iii) F_1 is derived from F_2 by a monotone spread, $E[F_2] \geq E[F_1]$

(*b*) An increase in W_0 leads to an increase in α^*

Proof: All of the results except a(iii) are well-known (see, for example, Feder 1977). The result for monotone spreads is derived by Quiggin (1988) and independently derived by Meyer and Ormiston (1989) who used the term deterministic transformations. The formulation of the control problem presented here is slightly more general than that of the authors cited. An appendix giving proofs for this formulation is available from the author. n

The result in part (iii) appears to be the most general that can be obtained for increases in risk. The notion of mean-preserving spreads has generally proved intractable in comparative static problems of this kind. The use of the monotone spread definition of an increase in risk in part (iii) generalizes a number of results in the literature.including those of Rothschild and Stiglitz (1971b), Coes (1977) and Meyer and Ormiston (1983).

Machina (1989) considers objective functions of the general form $E[U(\theta, \alpha)]$. He observes that if $U_{\alpha\theta} > 0$ α, θ, a first stochastically dominating shift in F will lead to an increase in α^*. Similarly if $U_{\alpha\theta\theta} < 0$ α, θ, an increase in risk in the sense of Rothschild and Stiglitz will lead to a reduction in α^*, and analogous results hold for higher orders of stochastic dominance. It should be noted, however, that the conditions $U_{\alpha\theta} > 0$ and $U_{\alpha\theta\theta} < 0$ have not proved very tractable in the context of models of the form (4) (Rothschild and Stiglitz 1971).

IV. Extension to RDEU

It is fairly straightforward to obtain a formal extension of the results derived in the previous section to the RDEU case. Results are given for a generalized problem including (4) as a special case.

(12) $$\text{Max}_{\alpha}\ V = \int_X U(x)dq(F(x))$$

where

$$x = \phi(\theta, \alpha, W_0) \quad \text{and}$$

$$F(x_0) = \Pr\{\phi(\theta, \alpha, W_0) \leq x_0\}$$

First, it is desirable to derive conditions for the existence of a finite optimum. In order to carry the EU existence results over to the RDEU case, it is necessary to impose a continuity condition, namely that $q'(p)$ be bounded. If this condition is not satisfied, then it is possible to design a sequence of lottery tickets with constant expected value and unbounded certainty equivalent. One example of a weighting function which does not satisfy this condition is that proposed by Karmarkar (1978), namely

(13) $q(p) = p^{\alpha}/[p^{\alpha} + (1-p)^{\alpha}], 0 < \alpha \leq 1$

In this case, as p approaches zero or unity, $q'(p)$ increases without bound. If functions of this kind are excluded, it is possible to prove

Proposition 2: Let $q'(p)$ be bounded and define

(14) $$K = \lim_{\alpha \to \infty} \partial V/\partial\alpha = U_1 \int_{\phi_\alpha > 0}^{\infty} \phi_\alpha^{\bullet}\, q'(F(\theta)) - U_2 \int_{\phi_\alpha \leq 0}^{\infty} \phi_\alpha^{\infty}\, q'(F(\theta))$$

Then a finite optimum will exist if K < 0 and not if $K > 0$

Proof: The proof is a modification of the proof for the EU result presented by Bertsekas (1974) and Quiggin (1982b). It is available in an Appendix from the author.

The critical task is to obtain a formal statement of the Correspondence Principle for the control problems (4) and (12). In order for the principle to apply, it is necessary that ϕ be strictly monotone increasing in θ.

Correspondence Principle: Let R be a relation on D, let ***U*** be a family of utility functions, and let q $Q^*(R)$. Suppose that whenever (F_1, F_2) R, U ***U*** a shift from F_1 to F_2 in the distribution of θ leads to an increase (decrease) in the optimal value for α in a problem of the form (4). Then, provided ϕ is strictly monotone increasing in θ, a shift from F_1 to F_2 in the distribution of θ leads to an increase (decrease) in the optimal value for α in the corresponding problem of the form (12).

Proof: By Φ.1, ϕ is strictly monotone increasing in θ. Hence, for any θ_0 such that $\phi(\theta_0, \alpha, W_0) = x_0$, $F(\theta_0) = G(x_0)$. So q $Q^*(R)$ (G_1, G_2) R. Hence a shift from G_1 to G_2 leads to an increase (decrease) in the optimal value for α in a problem of the form (4). But these are precisely the problems of the form (12) corresponding to a shift from F_1 to F_2 .n

The main result of this paper is:

Proposition 2:

(a) Let F_1 be the initial distribution of z and F_2 be the final distribution, and let θ_1 and θ_2 be the corresponding random variables. Let the assumptions on U be the same as for Proposition 3. Then the following are sufficient conditions for an increase in α^*.

(i) $\theta_2 = \theta_1 + \delta$ for some $\delta > 0$, $q \quad Q$

(ii) $F_2 = \delta_c$, $E[F_1] \leq c$, $q(p) \geq p \quad p$

(iii) F_1 is derived from F_2 by a monotone spread, $E[F_2] \geq E[F_1]$, $q(p) \geq p \quad p$

(*b*) An increase in W_0 leads to an increase in α^*, $q \quad Q$

Proof: The result follows immediately from Proposition 1 and the Correspondence Principle. n

The economic interpretation of this extended result has a number of interesting features. In particular, the results suggest a re-interpretation of notions of risk-aversion and risk-seeking. It is also of interest to compare these results with those which apply for general smooth preferences. It is noteworthy that none of the results in Proposition 2 requires risk-aversion in the sense proposed by Machina for the smooth model and Chew, Karni and Safra for the RDEU model. The results in parts a(i) and (*b*) do not require risk-aversion even in the weaker sense that a certain outcome is preferred to a risky variable having the same expectation.

Although risk-aversion is the predominant form of behavior for both firms and individuals one of the main reasons for the development of RDEU theory was the desire to resolve the paradox (in terms of EU theory)[3] of simultaneous gambling and insurance undertaken by many, perhaps most, people.

The main case of interest where risk aversion does not apply arises with an 'S-shaped' function q, and a distribution for the variable θ which is skewed to the right. An example is

[3] Machina (1982, 1983) and Quiggin (1990) discuss a number of attempts to resolve this paradox within the EU framework, notable including that of Friedman and Savage (1948) and indicate some of the problems these approaches have encountered.

that of a speculative asset which will probably yield a zero return, but which will yield a high return if the associated venture is successful. An RDEU-maximizing investor may purchase such an asset even if expected profits are negative. Similar examples apply in the theory of the firm and in the theory of labor supply for high-risk professions such as acting and politics.

The EU results noted by Machina (1989) may also be generalized using the Correspondence Principle. All monotonic functions q preserve first stochastically dominating shifts in F and all concave functions preserve second stochastically dominating shifts. As has already been observed, an alternative approach to generalizing these results may be followed using the fact that the RDEU model is essentially a special case of Machina's model. Hence Machina's (1989) results apply to RDEU. However, there are some difficulties in interpretation here. In Machina's (1982) presentation of the general model, the local utility function is of the form $U(\cdot\,;\,F)$ where F is the cumulative distribution function for income (or wealth), the varaible of ultimate interest. In Machina (1989), F is the distribution of the random variable θ, so that the lcoal utility function will not in general, be the same for different values of α and F, even if they yield the same final distribution of wealth. The generalization based on the Correspondence Principle does not encounter these difficulties.

VI. Concluding Comments

Rank dependent utility theory is a generalization of Expected Utility theory which does not rely on the axiom of Independence of unrelated outcomes and hence overcomes many of the criticisms of Allais and others. The results given in this paper indicate that this generalization is achieved without a significant loss of power. RDEU theory can generate results in areas such as portfolio theory and the theory of the firm under uncertainty which are just as sharp as those derived from EU theory. By contrast, no straightforward extension of these results can be obtained for the case of generalized smooth preferences. The fact that results of this kind can be carried over to RDEU theory should increase our confidence in the validity of much of the analysis using EU theory which has been performed in the last two decades, since it is likely that most of these results do not rely crucially on the independence axiom.

It is however, important to note that while the behavior of risk-averse individuals will be very similar in both EU theory and RDEU theory, RDEU theory does not give the same level of support to the presumption that most rational people will be globally risk-averse. Whereas the behavior of an EU risk-lover seems very odd indeed, the behavior of an individual whose preferences are described by an RDEU functional with a concave outcome utility function and an S-shaped probability weighting function seems quite plausible. Such an individual will display risk-aversion except when confronted with probability distributions which are skewed to the right.

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