Incentives and Standards in Agency Contracts

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Abstract

This paper studies the structure of state-contingent contracts in the presence of moral hazard and multitasking. Necessary and sufficient conditions for the presence of multitasking to lead to fixed payments instead of incentive schemes are identified. It is shown that the primary determinant of whether multitasking leads to higher or lower powered incentives is the role that noncontractible outputs play in helping the agent deal with the production risk associated with the observable and contractible outputs. When the noncontractible outputs are risk substitutes and are socially undesirable, standards are never optimal. If the noncontractible outputs are socially desirable, standards are never optimal if the noncontractible outputs play a risk-complementary role.

1. Introduction

The central tenet of contract theory is that agents respond to incentives in a self-interested manner. A principal who rewards an agent should be prepared for the agent to undertake actions in pursuit of those rewards that may be counter to the principal's own objectives. The literature on compensation systems has long recognized that inappropriately designed incentive schemes can lead to actions by the agent that are, from the principal's perspective, counterproductive (Lawler 1971, Kerr 1975). Similarly, economic regulation of firms and industries often has unintended and unforeseen consequences.

The notion that basing agent remuneration on objective measures of performance can be harmful to other aspects of the principal's interest where

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objective measures of performance are unavailable is central to the multitask moral hazard and performance measurement literatures (Holmström and Milgrom 1991, Baker 1992). Some examples illustrate this point. Holmström and Milgrom (1991) cite the controversy surrounding proposals to link teachers' pay to students' performance on standardized tests. Such teacher "accountability" schemes have been introduced in a number of U.S. states. Advocates argue that such schemes introduce incentives for teachers to undertake increased effort. Opponents counter that they encourage teachers to neglect educational activities that are not objectively measurable in favor of "teaching to the test," that is, of activities designed solely to raise student scores on standardized tests. While the debate on this issue continues, little doubt exists that teachers respond to these incentives,[1] and there is some limited evidence that these schemes affect students' performance (Ladd 1999).

Another example emerges from the observation that most manufacturing firms do not employ piece-rate payment systems. One explanation is that piece-rate rewards can reduce workers' incentives to maintain the quality of the firm's machinery (Alchian and Demsetz 1972, Prendergast 1999).

Chambers and Quiggin (1996) examine an example drawn from a regulatory problem. Both the European Community and the United States have long provided subsidies and price support to farmers. Such support schemes may conflict with environmental objectives. For example, providing farmers with formal or informal crop insurance may encourage riskier production activities, which also degrade the environment in the form of chemical runoff and soil erosion.

In each of these examples, an incentive scheme is based on an observable and objective performance criterion, even though the principal also cares about noncontracted aspects of the agent's performance. Starting with the seminal papers of Holmström and Milgrom (1991) and Baker (1992), theorists have speculated that the existence of multitask concerns may partially explain the tendency of employment contracts to specify fixed wages, or, more generally, the commonly noted presence of muted incentives within firms. In a regulatory context, one might similarly speculate that the frequent tendency toward command and control regulation in place of taxes or subsidies may emerge from multitask concerns.[2]

This paper revisits the issue of multitask agency problems, or multitasking for short, from a state-contingent perspective. It was long thought that the

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[1]Two examples taken from recent newspaper stories illustrate. In June 2000, the *Washington Post* reported two separate incidents involving allegations that teachers and school principals had been involved in assisting students to cheat on standardized examinations (Eggen 2000, Schulte 2000). On a more positive note, the *New York Times* reports on the success of a cash incentive scheme for teachers in North Carolina (Steinberg 2000).

[2]Slade (1996) examines some of the empirical implications of multitask theory. Prendergast (1999) provides a convenient summary of the theoretical and empirical work on agent response to incentives.

state-space approach to principal–agent problems was intractable (Hart and Holmström 1987). As a consequence, much of the theoretical discussion of moral hazard has been cast in the "parametrized distribution formulation" (Hart and Holmström 1987). However, Quiggin and Chambers (1998) and Chambers and Quiggin (2000) show that this apparent intractability emerged from an implausible specification of state-contingent technologies in early principal–agent treatments, and that a more plausible specification leads to a tractable and easily manipulable moral-hazard model. This paper extends that observation to the study of multitasking.

When viewed from the perspective of a plausible state-contingent production technology, the issue of multitasking takes a different flavor. The characteristics of the stochastic technology assume center stage. The effect of multitasking on contract design in a state-contingent framework hinges on the role that noncontractible activities play in allowing the agent to control the production risk associated with the contractible outputs. The emphasis, therefore, switches from agent "tasks" to the outcomes from these tasks. The crop-runoff example illustrates: the tasks might include fertilization of the crop, weeding, and activities aimed at controlling runoff. The outputs would include the realized crop and the actual runoff pollution. Presumably, a regulator would be more interested in the runoff than in the activities undertaken to control that runoff. Our focus is on those cases where it is more reasonable to presume that the principal is primarily concerned with outcomes from the tasks rather than with the tasks themselves.

In what follows, we first specify our model and develop some preliminary results. Then we examine when fixed payments are optimal in the presence of multitask considerations. Holmström and Milgrom (1991) identify conditions under which fixed payments can be optimal in the presence of moral hazard. They show that, when agent effort is perfectly substitutable between tasks, fixed wages can dominate incentive schemes. The intuition is relatively simple. If tasks are perfect substitutes, introducing incentives for a subset of the tasks leads the agent to direct all his attention to the rewarded tasks. If the principal values some of the nonrewarded tasks, he may be better off not using performance-indexed incentive schemes. Sinclair-Desgagne (1999), however, shows that by an appropriate design of monitoring schemes, tasks that are viewed as substitutes by the agent can be rendered more complementary. Under appropriate conditions on the agent's prudence, this can lead to the restoration of higher-powered incentive schemes.

Our focus is somewhat different. For the sake of concreteness, we presume that the noncontractible outputs are socially damaging. Examples include pollution, teacher collusion on student cheating, damage to the firm's capital stock, and consumer ill-will. Our result is that, provided an incentive scheme is implementable, a fixed payment is only optimal for strictly monotonic agent cost structures when the noncontractible output plays a "risk-complementary" role in the production of the contractible output. By risk-complementary, we mean that higher levels of the noncontractible output are associated with riskier outcomes with respect to the contractible output. (This is made precise below.) For weakly monotonic cost structures, fixed payments are the norm if the production of the contractible outputs is technically efficient. If incentive schemes are to be used with weakly monotonic cost structures, they must also encompass technical inefficiency.

We then take up the issue of higher-powered and lower-powered incentive schemes. Here our method is to examine how the introduction of multitask concerns affects the contracts that would have been chosen in the absence of multitasking. We show that higher-powered incentives tend to emerge when the noncontractible outputs play a risk-substituting role in the production of the contractible outputs. Lower-powered incentives emerge when the noncontractible outputs play a risk-complementary role. After a general discussion of higher-powered and lower-powered incentive schemes, we present two illustrative examples: one based on a strongly risk complementary technology and one based on a strongly risk-substituting technology. We derive, for these examples, specific conditions under which the introduction of multitasking concerns can lead the principal to switch from an incentive scheme to a production standard and when their introduction leads the principal to switch from a standard to an incentive scheme. The latter is particularly interesting because it identifies a case when the introduction of multitask concerns leads the principal to introduce a higher powered incentive scheme instead of a lower-powered incentive scheme.

2. The Model

There are two individuals: a principal and an agent. The agent produces two outputs, a socially bad output, p, say pollution, and a valuable output, z, under conditions of production uncertainty. Only the agent engages in productive activity. Uncertainty is modeled by "Nature" making a choice from a state space, $\Omega = \{1, \ldots, S\}$. Production relations are governed by a state-contingent input correspondence (Chambers and Quiggin 2000), $X : \Re^S_+ \times \Re_+ \rightarrow \Re^n_+$, defined by

$$X(\mathbf{z}, p) = \left\{\mathbf{x} \in \Re^N_+ : \mathbf{x} \text{ can produce } (\mathbf{z}, p)\right\}, \quad \mathbf{z} \in \Re^S_+, p \in \Re_+.$$

Here $\mathbf{x}$ represents an input vector that is committed prior to the resolution of uncertainty, that is, before Nature makes her choice from Ω, and $\mathbf{z}$ is a vector of state-contingent output also chosen before Nature makes its choice.[3] In

[3]The set of feasible outputs is assumed to be an interval. This assumption contrasts with that adopted in the literature following Grossman and Hart (1983) in which the outcome set is assumed to be discrete, and frequently to consist of only two elements. This restriction was imposed for reasons of tractability. The model presented here allows, as a special case, the possibility that state-contingent outputs z_s are constrained to lie in a discrete set. In this case, however, cost functions will not be continuous, and the statement of results is more complex.

line with much of the recent literature on multitasking, one can alternatively interpret $\mathbf{x}$ as measuring effort allocated to the various "tasks" undertaken by the agent. If Nature picks state s, then the *ex post* or realized value of the output is z_s. Pollution, denoted by p, is a nonstochastic output,[4] which is socially undesirable, associated with the production of the state-contingent output vector and the input commitment.[5]

We impose the following properties on X:

X.1 $X(0) = \Re^N_+$, and $O_N \notin X(\mathbf{z}, p)$ for $\mathbf{z} \geq \mathbf{0}$ and $\mathbf{z} \neq \mathbf{0}$.

X.2 $\mathbf{z}' \geq \mathbf{z} \Rightarrow X(\mathbf{z}, p) \subseteq X(\mathbf{z}', p)$.

X.3 $\lambda X(\mathbf{z}^0, p^0) + (1-\lambda)X(\mathbf{z}^1, p^1) \subseteq X(\lambda \mathbf{z}^0 + (1-\lambda)\mathbf{z}^1, \lambda p^0 + (1-\lambda)p^1)$ $0 \leq \lambda \leq 1$.

X.4 $X(\mu \mathbf{z}, \boldsymbol{\mu} p) = \boldsymbol{\mu} X(\mathbf{z}, p), \quad \boldsymbol{\mu} > 0$.

Property X.1 allows for the technical possibility of input and output inaction and requires that a positive state-contingent output can only be produced with a positive commitment of inputs. Property X.2 (free disposability of state-contingent outputs) will eventually (see below) ensure that the marginal cost of state-contingent outputs is nonnegative. Properties X.3 and X.4 require respectively that the technology set associated with the input correspondence be convex and that it exhibit constant returns to scale.

The principal is risk-neutral and does not directly value the agent's effort, but he does value the outputs that emerge from that effort.[6] The damage associated with p is given by a twice differentiable, strictly increasing, and strictly convex function $m(p)$. The agent's preferences depend upon his payment from the principal, which we denote as y, and the vector of inputs, $\mathbf{x}$, that he commits. His *ex post* utility function is given by

$$w(y, \mathbf{x}) = u(y) - g(\mathbf{x}),$$

where $u : \Re_{++} \rightarrow \Re$ is a strictly increasing, strictly concave, and twice differentiable function, and $g : \Re^{n+1}_+ \rightarrow \Re$ is continuous, positively linearly

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[4] p is taken as nonstochastic to preserve notational simplicity. This specification, however, is consistent with the treatment in other multitasking studies where "tasks" are typically viewed as nonstochastic activities undertaken by the agent. More generally, however, our approach allows p to be stochastic at the cost of increased notational complexity.

[5] p is taken to be socially undesirable for the sake of clarity and because it reflects the role that p plays in our theoretical example. As pointed out below, our results can be extended in an obvious fashion to the case where p is socially desirable.

[6] This may seem to represent a departure from much of the recent literature on multitask agency problems, where the principal is assumed to have direct preferences over some of the agent's tasks. However, the difference is essentially semantic and corresponds closely to the semantic difference between inputs, outputs, and netputs in the axiomatic literature on production.

homogeneous, and convex in $\mathbf{x}$.[7] Both u and g satisfy the von Neumann–Morgenstern postulates.

The principal and the agent share the same probabilities of a state s occurring and that probability is denoted by $\pi_s \in \Re_{++}$ with $\sum_s \pi_s = 1$. For simplicity, we will assume that all states are equally probable so that $\pi_s = 1/S, \forall s$. As is noted by Blackorby et al. (1977), this assumption does not involve any real loss of generality since states can always be subdivided so as to make the probabilities equal.[8]

Ex post, the principal cannot observe either the state of nature or level of p. Hence, the informational asymmetry between the principal and the agent encompasses both hidden action (input commitment, p emitted) and hidden knowledge (*ex post* about the state of nature that occurs) on the part of the agent. What is observable, and by assumption contractible to both parties, is the *ex post* output.

Two indirect representations of the state-contingent input correspondence prove useful. The first is the agent's *effort–cost function* for a given vector of state-contingent output and a pollution level. It is defined by

$$c(\mathbf{z}, p) = \min_{\mathbf{x}}\{g(\mathbf{x}) : \mathbf{x} \in X(\mathbf{z}, p)\},$$

if there exists an $\mathbf{x} \in X(\mathbf{z}, p)$, and ∞ otherwise. Hence, for fixed $(y, \mathbf{z}, p)$, the agent's *ex post* welfare is

$$u(y) - c(\mathbf{z}, p).$$

The *effort–cost function* satisfies, among others, the following properties (Chambers and Quiggin 2000).

C.1 $\mathbf{z}' \geq \mathbf{z} \Rightarrow c(\mathbf{z}', p) \geq c(\mathbf{z}, p)$.

C.2 $c(\lambda\mathbf{z}^0 + (1-\lambda)\mathbf{z}^1, \lambda p^0 + (1-\lambda)p^1) \leq \lambda c(\mathbf{z}^0, p^0) + (1-\lambda)c(\mathbf{z}^1, p^1)$ $0 \leq \lambda \leq 1$.

C.3 $c(\mu\mathbf{z}, \mu p) = \mu c(\mathbf{z}, p), \quad \mu > 0$.

It is always assumed that the agent's production of $(\mathbf{z}, p)$ results in jointness in $c(\mathbf{z}, p)$. More precisely, $c(\mathbf{z}, p)$ is not additively separable in $\mathbf{z}$ and p. Given the structure of the agent's evaluation of effort, this jointness can

[7] More generally, g could be taken to be a function of p as well to allow the agent to have direct preferences over the noncontractible and socially undesirable output. For example, if g is taken to be increasing in p, the agent need not be assumed to have pathological preferences over the socially undesirable output. If p were socially desirable, the parallel assumption would be that g is decreasing in p. However, it is easy to see that these assumptions could be easily relaxed if one's interest is in modeling purely "anti-social" behavior.

[8] In the context of state-contingent production, this argument requires that the production technology remains well-behaved when the state space is subdivided. This assumption is not problematic in the present context.

arise from several sources. It can be embedded in the production technology itself.[9] Alternatively, the input correspondence could be input nonjoint, but the nonlinearity of g could mean that $c(\mathbf{z}, p)$ is not additively separable. Even in the presence of linear pricing of the inputs, $\mathbf{x}$, one generally does not expect $c(\mathbf{z}, p)$ to be additive.

The second cost function that we consider is the one that is relevant when the agent privately chooses the level of p associated with a given vector of state-contingent outputs. Denote the agent's *private cost function* by

$$C(\mathbf{z}) = \operatorname*{Min}_{p}\{c(\mathbf{z}, p)\}.$$

Denote

$$p(\mathbf{z}) \in \arg\min_{p}\{c(\mathbf{z}, p)\}.$$

C is nondecreasing, positively linearly homogenous, and convex in $\mathbf{z}$. Convexity implies continuity over any open set and differentiability at all but a countable number of points so long as $C(\mathbf{z})$ is finite (Rockafellar 1970). Hence, little generality is lost in taking C to be smoothly differentiable almost everywhere, and for convenience we do so. And, except where explicitly noted, we assume that C is strictly increasing in each element of $\mathbf{z}$. We also assume that $p(\mathbf{z})$ is unique and smoothly differentiable to avoid increased technical complications.

If $c(\mathbf{z}, p)$ does not exhibit jointness, the agent's choice of $\mathbf{z}$ and p is independent and

$$p(\mathbf{z}) = p(\mathbf{z}'),$$

for all $\mathbf{z}$, $\mathbf{z}'$. The principal, therefore, cannot influence the choice of p through an incentive scheme based on observed output z. In effect, the principal must delegate the authority for the optimal choice of p to the agent.[10]

Following Chambers and Quiggin (1997, 2000), we denote by $\preceq_\pi$ a partial ordering of $\Re_+^S$ that ranks vectors of state-contingent output vectors with the same mean. The notation

$$\mathbf{z} \preceq_\pi \mathbf{z}'$$

means that $\frac{1}{S}\Sigma_{s\in\Omega} z_s = \frac{1}{S}\Sigma_{s\in\Omega} z'_s$, and that $\mathbf{z}$ is less risky in the Rothschild–Stiglitz sense than $\mathbf{z}'$.[11] $p(\mathbf{z})$ is *risk-complementary* at $\mathbf{z}$ if

$$\mathbf{z} \preceq_\pi \mathbf{z}' \Rightarrow p(\mathbf{z}) \leq p(\mathbf{z}')$$

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[9]Formally, in this case, the production technology would not be input nonjoint (Chambers 1988).

[10]In fact, if one were to use our model to study the optimal delegation of authority for certain tasks by the principal to the agent, the optimality of various degrees of delegation would depend critically upon the degree of jointness of $c(\mathbf{z}, p)$.

[11]Given the assumption of equal probabilities, this is equivalent to saying that $\mathbf{z}$ is majorized by $\mathbf{z}'$ (Marshall and Olkin 1979).

and *risk-substituting* at $\mathbf{z}$ if

$$\mathbf{z} \preceq_{\pi} \mathbf{z}' \Rightarrow p(\mathbf{z}) \geq p(\mathbf{z}').$$

Intuitively, our notion of risk substitutability is that the producer matches risky output vectors with relatively low levels of $p(\mathbf{z})$ (Chambers and Quiggin 2000, Chapter 4). Notice that, for general probabilities, $p(\mathbf{z})$ is risk-complementary at $\mathbf{z}$ if and only if

$$\left(\frac{p_s(\mathbf{z})}{\pi_s} - \frac{p_t(\mathbf{z})}{\pi_t}\right)(z_s - z_t) \geq 0, \quad \forall s, t \in S,$$

where subscripts on functions denote partial derivatives (Chambers and Quiggin 2000, Chapter 4). $p(\mathbf{z})$ is risk-substituting if and only if the above inequality is reversed.[12] Under the assumption that $\pi_s = 1/S, \forall s$, this condition simplifies to

$$(p_s(\mathbf{z}) - p_t(\mathbf{z}))(z_s - z_t) \geq 0, \quad \forall s, t \in S.$$

Our production technology is general enough that, when comparing two states s and t, either state s or state t in the definition above could be the "good" state of Nature in the sense that a risk-neutral individual facing this technology would choose to produce a higher output in that state of Nature. To order the states of Nature, therefore, we assume that higher-number states are better than lower-number states in the sense that rearranging state-contingent outputs in an increasing fashion reduces costs. More precisely, consider any $\mathbf{z}$ such that for some $t > r$, $z_t < z_r$, and define $\mathbf{z}'$ so that

$$z'_s = \begin{cases} z_t & s = r \\ z_r & s = t \\ z_s & s \neq r, t. \end{cases}$$

We assume

ASSUMPTION 1: *Let* $\mathbf{z}, \mathbf{z}'$ *be as above. Then*

$$C(\mathbf{z}') < C(\mathbf{z}). \tag{1}$$

Therefore, a risk-neutral individual facing this technology, and seeking to maximize $\sum_s \pi_s z_s - C(\mathbf{z})$ would always choose $z_1 \leq z_2 \cdots \leq z_S$. Following, Quiggin and Chambers (1998), a state-contingent output vector is defined as *monotonic* if $z_1 \leq z_2 \cdots \leq z_S$.

Suppose

$$(p_s(\mathbf{z}) - p_t(\mathbf{z}))(s - t) \leq 0, \quad \forall \mathbf{z} \in \Re^S_+, s, t \in S.$$

Then p is risk-complementary for monotonic $\mathbf{z}$. The converse also holds.

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[12] To derive these results, take a differentiably small multiplicative spread of $\mathbf{z}$ under the assumption that p is risk complementary or risk substituting (Chambers and Quiggin 2000, Chapter 4).

In the state-contingent framework, it is natural to think of different state-contingent outputs as having different relative input intensities. For example, returning to the chemical-runoff problem, if the set of the states of nature includes states favorable to insect infestations, output in those states will be relatively pesticide intensive. Whether pesticides are risk complements or risk substitutes depends on whether the relevant states of nature have relatively high or relatively low output. For any given technology, this, in turn, will depend on whether the output vector under consideration is monotonic or nonmonotonic.

With the ability to monitor both p and the *ex post* state of Nature s, the principal chooses p and $\mathbf{z}$ to maximize

$$\sum_s \pi_s z_s - c(\mathbf{z}, p) - m(p).$$

Optimality would require making a nonstochastic payment y to the agent such that

$$u(y) - c(\mathbf{z}, p) = \bar{u},$$

where $\bar{u}$ represents the agent's reservation utility, while choosing $\mathbf{z}$ and p to maximize expected surplus. The solution to this problem will be referred to as the *first-best*.

If pollution is a risk substitute for monotonic $\mathbf{z}$, then, since the socially optimal p will be lower than the privately optimal p, the *first-best* optimal $\mathbf{z}$ will be riskier than the private optimum, and hence will also be monotonic. Hence, a sufficient condition for the *first-best* $\mathbf{z}$ to be monotonic is that pollution should be a risk substitute for monotonic $\mathbf{z}$.

Because the principal cannot observe the pollution level p, the state of Nature s, or the agent's input use $\mathbf{x}$; the principal's *second-best* problem is to design a contract that rewards the agent on the basis of observable, *ex post* output z_s, while ensuring the agent the reservation utility and still coming as close as possible to maximal expected social surplus. Let $\mathbf{Y}$ be the class of all functions $y : \Re_+ \to \Re$ that the principal can choose from, in designing an agent reward scheme. The reward scheme works as follows: If the agent realizes an output of z, his income is set at $y(z)$ by the principal. In picking such a reward scheme, the principal knows that, if she wants to implement a particular state-contingent output vector $\mathbf{z}$, that state-contingent output vector must be both technically feasible and consistent with the agent's private optimization in the sense that

$$\begin{aligned}\mathbf{z} &\in \arg\max_{p,\mathbf{z}} \left\{ \frac{1}{S} \sum_s u(y(z_s)) - \operatorname*{Min}_p \{c(\mathbf{z}, p)\} \right\} \\ &= \arg\max_{\mathbf{z}} \left\{ \frac{1}{S} \sum_s u(y(z_s)) - C(\mathbf{z}) \right\}.\end{aligned}$$

Notice, in particular, that here we use the agent's private-cost function because pollution is not observable or contractible.

The principal's problem, therefore, is to choose the payment scheme in order to

$$\max \left\{ \begin{array}{l} \frac{1}{S}\sum_s \pi_s(z_s - y(z_s)) - m(p(\mathbf{z})): \\ \mathbf{z} \in \arg\max \left\{ \frac{1}{S}\sum_s u(y(z_s)) - C(\mathbf{z}) \right\}, \\ \frac{1}{S}\sum_s u(y(z_s)) - C(\mathbf{z}) \geq \bar{u} \end{array} \right\},$$

for technically feasible **z**.

Without loss of generality, suppose that in the optimum that $z_2 > z_1$, then the second condition requires in particular that

$$\frac{1}{S}u(y(z_2)) - C(\mathbf{z}) \geq \frac{1}{S}u(y(z_1)) - C(z_1, z_1, \mathbf{z}_{-12}),$$

where $\mathbf{z}_{-12}$ represents the vector with $S - 2$ elements obtained by dropping (z_1, z_2) from **z.** Hence,

$$\frac{1}{S}[u(y(z_2)) - u(y(z_1))] \geq C(\mathbf{z}) - C(z_1, z_1, \mathbf{z}_{-12}) > 0$$

by the monotonicity of C. The payment structure must be monotonic in output. We state this as a lemma for future reference.

LEMMA 1: *Any solution must satisfy*

$$(y(z_s) - y(z_t))(z_s - z_t) \geq 0, \quad \forall s, t \in S$$

with equality only when both terms on the left-hand side are zero.

Lemma 1 implies that the principal only chooses a contract with a fixed payment if the contract also has a fixed production level. An example of a mechanism which would implement such a contract is a bonus-type scheme in which the agent receives no payment for production below a certain threshold, and a constant payment once that threshold is reached, that is,

$$y(z) = \begin{cases} -\infty & \text{if} \quad z < \hat{z} \\ \hat{y} & \text{if} \quad z \geq \hat{z}. \end{cases} \tag{2}$$

Provided that $\hat{y}$ allows the agent to reach his reservation utility level, the agent will accept this contract and devote all his effort to ensuring that he realizes $\hat{z}$ in every state of nature. He will not devote any effort to producing output greater than $\hat{z}$ in any state.

Following Grossman and Hart (1983), Weymark (1986), and Quiggin and Chambers (1998), the principal's problem can be rewritten as

$$\max_{\mathbf{z}} \left\{ \frac{1}{S} \sum_s z_s - m(p(\mathbf{z})) - Y(\mathbf{z}, \bar{u}) \right\},$$

where

$$Y(\mathbf{z}, \bar{u}) = \min_{\mathbf{y}} \left\{ \begin{array}{l} \frac{1}{S} \sum_s y_s : \mathbf{z} \in \arg\max \left\{ \frac{1}{S} \sum_s u(y(z_s)) - C(\mathbf{z}) \right\}, \\ \frac{1}{S} \sum_s u(y_s) - C(\mathbf{z}) \geq \bar{u} \end{array} \right\}.$$

$Y(\mathbf{z}, \bar{u})$ represents what Quiggin and Chambers (1998) term as the *agency-cost function* and is defined as the least costly way, in an expected-value sense, for the principal to get the agent to adopt $\mathbf{z}$ in an incentive compatible manner. $Y(\mathbf{z}, \bar{u})$, thus, represents the agent's expected payment for producing $\mathbf{z}$.

Some brief comments about the solution to the agency cost problem are appropriate before proceeding. Under (1), the optimal state-contingent production structure must be weakly monotonic, since for any nonmonotonic $\mathbf{z}$, there exists an increasing rearrangement of $\mathbf{z}'$ such that

$$\frac{1}{S} \sum_s u(y(z_s)) = \frac{1}{S} \sum_s u(y(z'))$$

$$C(\mathbf{z}') < C(\mathbf{z}),$$

and hence $\mathbf{z} \notin \arg\max\{\frac{1}{S} \sum_s u(y(z_s)) - C(\mathbf{z})\}$. In other words, the principal will order the states of Nature exactly as would a risk-neutral individual facing the technology. Because the production structure is weakly monotonic in this sense, Lemma 1 then implies that the state-contingent payment structure must be as well.

3. Optimality of Fixed-Payment Schemes

By Lemma 1, fixed payments only emerge in association with fixed production. Our notion of a *fixed-payment production standard* is the lowest-powered incentive scheme possible, where the optimal contract can be implemented by a mechanism of the form (2). Both the state-contingent production and the payment are fixed. We refer to the case where the contract does not involve a fixed payment as an *incentive contract*. Suppose that $z_s = z \forall s$, is optimal. By Lemma 1,

$$Y(z\mathbf{1}, \bar{u}) = h(\bar{u} + C(z\mathbf{1})),$$

with $h(\bar{u} + C(z\mathbf{1}))$ representing the fixed payment to the agent. The optimal value of the principal's objective function is then

$$z - h(\bar{u} + C(z\mathbf{1})) - m(p(z\mathbf{1})).$$

The only implementable alternatives to the fixed-payment production standard are monotonic contracts. Consider the spread of the output vector defined by increasing z_S by the small positive amount δz_S and decreasing z_1 by $-\delta z_1$. Observe that the resulting contract is monotonic. The induced change in the principal's welfare in the neighborhood of $z\mathbf{1}$ has the same sign as

$$-h'(\bar{u} + C(z\mathbf{1}))(C_S(z\mathbf{1}) - C_1(z\mathbf{1})) - m'(p(z\mathbf{1}))(p_S(z\mathbf{1}) - p_1(z\mathbf{1})), \quad (3)$$

where $'$ denotes a derivative. If $p(z\mathbf{1})$ is a risk substitute, moving to a riskier $\mathbf{z}$ decreases p, whence $(p_S(z\mathbf{1}) - p_1(z\mathbf{1})) < 0$. Thus, by (1), such an increase in the riskiness of $\mathbf{z}$ unambiguously increases the principal's welfare in this case. Hence, a fixed-payment production standard cannot be optimal if $p(z\mathbf{1})$ is a risk substitute, and the spread is implementable. For a fixed-payment production standard to be optimal, such an increase in the riskiness of $\mathbf{z}$ must make the principal's welfare fall. By (1), this can only happen if $(p_S(z\mathbf{1}) - p_1(z\mathbf{1})) \geq 0$. But this requires that $p(\mathbf{z})$ rises as a result of the move to the riskier $\mathbf{z}$.

PROPOSITION 1: *If $p(\mathbf{z})$ is a risk substitute, and the principal can implement an incentive scheme, a fixed-payment production standard cannot be optimal.*

Proposition 1 may be given some heuristic content by considering the case where $\mathbf{z}$ is the measured quality of a fixed amount of output over the states of Nature. For concreteness sake, take the case where that output is a perishable agricultural commodity, such as citrus fruit, which receives chemical-pesticide treatments to control cosmetic damage. $p(\mathbf{z})$ would then be interpreted as toxic damage to consumers of the commodity associated with ingestion of the pesticide and to the environment. In such circumstances, pesticide is usually perceived as a "damage-control" input, which serves not to increase the maximal quality but to control deviations from the maximal quality. Now consider a severe quality standard and a fixed payment, which offers no reward for quality below that standard. Such a scheme could encourage socially excessive use of the pesticide as producers farmed to the standard recognizing that any cosmetic damage would render the fruit unsaleable. In this instance, if the principal were concerned with social welfare, he or she might enhance social welfare by departing from a strict quality standard.

Proposition 1 remains true even if (1) holds as an equality. In that instance, the risk-increasing spread of $\mathbf{z}$ leaves the agent's cost, and hence the principal's agency cost, unchanged. But if $p(\mathbf{z})$ is a risk substitute, then the riskier $\mathbf{z}$ still carries with it a lower p, which is welfare improving. For the general probability case, Chambers and Quiggin (1997, 2000) have defined a class of technologies for which (1) always holds as an equality—the *generalized Schur convex technologies*. C is generalized Schur convex if

$$\mathbf{z} \preceq_\pi \mathbf{z}' \Rightarrow C(\mathbf{z}) \leq C(\mathbf{z}'), \quad \mathbf{z} \in \Re_+^S.$$

When all probabilities are equal, generalized Schur convexity is equivalent to Schur convexity. Intuitively, it is the class of stochastic technologies which leads a risk-neutral individual to always pick a nonstochastic production vector.[13]

Quiggin and Chambers (1998, Corollary 8.1) show that a principal facing single task moral hazard with an agent using a Schur convex technology can completely eliminate that moral hazard. If costs are Schur convex, there is no tension between risk allocation and production efficiency. Production efficiency then requires producing a nonstochastic output which, in turn, is implementable by a fixed payment.

In the multitask case, the principal's ability to resolve the agency problem must be traded off against her ability to control $p(\mathbf{z})$ indirectly through the contract. Thus, even when the technology is Schur convex, the principal prefers an incentive contract if p is a risk substitute.

COROLLARY 1: *If C is Schur convex, and $p(\mathbf{z})$ is a risk substitute at $\mathbf{z}$ optimal for the single task moral-hazard problem, there exists a risk-increasing spread $\mathbf{z}'$ of $\mathbf{z}$, such that the principal prefers $\mathbf{z}'$ to $\mathbf{z}$.*

Corollary 1 and Lemma 1 imply that the principal always gains, as compared to the single task optimal contract, by introducing a riskier production structure. Hence, when costs are Schur convex, the optimal regulatory scheme will involve higher-powered incentives for production than in the single task case if $p(\mathbf{z})$ is a risk substitute.

Corollary 1 establishes a seemingly paradoxical result. Even when it is privately cheapest to implement a nonstochastic $\mathbf{z}$ and doing so completely removes agency costs, the principal's concerns about p will lead him to insist on the agent deploying a stochastic production vector. Multitasking causes the principal to insist on an incentive scheme where he would otherwise implement a standard.

If there is to be a case made for standards when incentive schemes are feasible, it must rest on the presumption that the noncontractible activities are risk-complementary. In fact, it is an easy consequence of preceding arguments to state a sufficient condition for standards to be optimal.

PROPOSITION 2: *A fixed-payment production standard is always optimal if C is Schur convex and $p(\mathbf{z})$ is risk-complementary.*

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[13]As Chambers and Quiggin (1997) demonstrate, the nonstochastic technology is a degenerate special case of the class of generalized Schur convex technologies. However, generalized Schur convex technologies are not trivially stochastic, as also demonstrated by Chambers and Quiggin (1997).

When the cost structure is Schur convex, the incentive problems associated with the presence of moral hazard can be efficiently surmounted by implementing a production standard. If $p(\mathbf{z})$ is risk-complementary as well, the principal's evaluation of the damage caused by the unobservable task also pushes him toward a standard.

There are, however, instances where incentive problems are so severe that even in the presence of risk-substitutability, they will prevent the principal from implementing anything other than a production standard. One such limiting case is given by the case of complete aversion to risk[14] where the agent's attitudes toward state-contingent incomes are given by

$$W(\mathbf{y}) = \min\{y_1, y_2, \ldots, y_S\}. \tag{4}$$

In the Appendix, we show

LEMMA 2: *If the agent's preferences are given by (4),* $\mathbf{z}$ *is implementable if and only if* $\mathbf{z} = z\mathbf{1}$.

Because the agent is so averse to risk, she only cares about the lowest payment she may receive. Thus, offering her a reward for realizing a higher output in one of the states of Nature does not elicit greater effort. *Ex ante,* it carries no marginal gain in welfare. All her effort is concentrated on ensuring that she realizes the lowest acceptable output. The principal's problem, therefore, becomes one of picking the optimal standard. A natural conjecture, therefore, is that as the agent's risk aversion increases, the principal becomes less likely to rely upon a high-powered contract because the cost of implementing such contracts grows with the agent's risk aversion.

By Lemma 2, the agency cost function is now $C(z\mathbf{1}) + \bar{u}$. The principal's problem is to

$$\max_z\{z - C(z\mathbf{1}) - m(p(z\mathbf{1}))\} - \bar{u}.$$

The solution is straightforward, and therefore we do not discuss it in detail. Even here, however, a multiplicative spread about the optimal $\mathbf{z}$ improves the principal's welfare if $p(\mathbf{z})$ is a risk substitute. But as Lemma 2 establishes, this riskier $\mathbf{z}$ is never implementable.

COROLLARY 2: *If the agent's preferences are given by (4), ignoring incentive effects, the principal always prefers an incentive scheme to a standard if* $p(\mathbf{z})$ *is a risk substitute.*

Our notion of a fixed-payment production standard involves the principal specifying a contract where the agent produces a nonstochastic output in return for a nonstochastic payment. Holmström and Milgrom (1991),

[14]This case departs from the expected-utility hypothesis that underlies the rest of the paper. However, we can arrive at this characterization by assuming constant absolute risk aversion and allowing the risk parameter to go to infinity.

however, have identified conditions under which a principal finds it optimal to offer the agent a nonstochastic reward in return for a stochastic output. By Lemma 1, this cannot happen in our model. The reason lies in our assumption of strict monotonicity of the agent's private cost structure. If that assumption is relaxed, then incentive compatibility only requires that the payment structure be weakly monotonic. The possibility of a fixed payment for a stochastic output then emerges. However, as we show immediately below, fixed payments typically will be optimal for cost structures that are not strictly monotonic, regardless of the presence of multitask concerns, unless the production of **z** is technically inefficient. Original specifications of agency problems in terms of a stochastic production function, such as those of Ross (1973; Harris and Raviv, 1979) presumed that production was always technically efficient. Therefore, in addition to identifying how our results change as a consequence of relaxing the strict monotonicity assumption, the discussion in the next section also helps explain some of the difficulty early moral hazard studies encountered in identifying monotonic contracts.

3.1. Weak Monotonicity

Thus far, we have assumed that the cost structure is strictly monotonic. Chambers and Quiggin (2000, Chapter 4) have shown that specifying the state-contingent technology by a stochastic production function yields a cost structure that is only weakly monotonic. More precisely, cost is nondecreasing in **z**, but is strictly increasing only along a one-dimensional expansion path determined by the stochastic production function. Quiggin and Chambers (1998) and Chambers and Quiggin (2000, Chapter 9) pinpoint this weak monotonicity as the stumbling block to establishing contract monotonicity in the original state-space formulations of the principal–agent problem. The following cost structure

$$C(\mathbf{z}) = \max\left\{c^1(z_1),\, c^2(z_2) \ldots c^S(z_s)\right\},$$

corresponds to a stochastic production function satisfying free disposability of state-contingent output (Chambers and Quiggin 2000, Chapter 4), and will be referred to as weakly monotonic. Here $c^s(z_s)$ is the minimal *ex post* cost associated with producing z_s. Imposing positive linear homogeneity on this specification requires

$$c^s(z_s) = c_s z_s$$

with $c_s > 0$. To ensure consistency with (1) in this case where $C(\mathbf{z})$ is not smoothly differentiable, assume that the states of Nature are ordered so that $c_1 > c_2 \cdots > c_S$. Regardless of their risk attitudes or their subjective probabilities, residual claimants facing this technology always choose $z_1 < z_2 \cdots < z_S$.

An optimal solution to the single task problem for this cost structure is for the principal to offer a fixed payment, subject to the production of some minimum level of output z_1. For any monotonic $\mathbf{z}$ with $c_1 z_1 = \max_s\{c_s z_s\}$ consider the payment structure

$$y(z) = \begin{cases} h(\bar{u} + c_1 z_1) & z \geq z_1 \\ -\infty & \text{otherwise} \end{cases}.$$

Because $C(\mathbf{z}) = \max_s\{c_s z_s\} = c_1 z_1$, this contract offers the agent exactly his reservation utility. It is also (weakly) incentive compatible in the sense that the agent cannot do better by producing any $\mathbf{z}' \neq \mathbf{z}$. This is because, for any $\mathbf{z}'$, with $C(\mathbf{z}') < C(\mathbf{z}) = c_1 z_1$, we must have $z_1' < z_1$, so that the agent receives $-\infty$ in state 1.[15] On the other hand, for $\mathbf{z}'$ with $C(\mathbf{z}') \geq C(\mathbf{z})$, the agent receives $y(z_1) - C(\mathbf{z}') \leq y(z_1) - C(\mathbf{z})$. If $c_1 z_1 < \max_s\{c_s z\}$, the principal can achieve a higher level of z_1 with no change in cost to the agent.

Output vectors $\mathbf{z}$ with $c_1 z_1 \leq c_2 z_2 \leq \cdots \leq c_S z_S$ can be implemented with a payment structure y satisfying

$$\frac{1}{S}\sum_s u(y(z_s)) - c_S z_S = \bar{u}$$

$$\frac{1}{S}(u(y(z_s)) - u(y(z_{s-1}))) = c_s(z_s - z_{s-1}).$$

However, for any such $\mathbf{z}$, there exists $\mathbf{z}'$ with $\mathbf{z}' \preceq_\pi \mathbf{z}$, satisfying the condition $c_1 z_1' = c_2 z_2' \cdots = c_S z_S'$, which can be implemented with payment structure:

$$y(z) = \begin{cases} h(\bar{u} + C(\mathbf{z}')) & z \geq z_1' \\ -\infty & \text{otherwise} \end{cases}.$$

Note that, under the stated conditions

$$C(\mathbf{z}') = c_1 z_1' = c_2 z_2'$$
$$z_S' < z_S$$
$$\frac{1}{S}\sum_s z_s' = \frac{1}{S}\sum_s z_s.$$

Since

$$C(\mathbf{z}) = c_S z_S > c_S z_S' = C(\mathbf{z}'),$$

we have, by the convexity of h that

$$\frac{1}{S}\sum_s y(z_s) > h(\bar{u} + c_S z_S)$$
$$> h(\bar{u} + C(\mathbf{z}')).$$

[15]The argument does not depend on the ability to impose infinitely large penalties. It is sufficient that the minimum payment be less than $h(\bar{u})$, the amount required for reservation utility with zero effort.

The contract associated with $\mathbf{z}'$ thus offers the principal the same expected return but at a lower cost. Hence, he will always prefer the fixed-payment contract, eliciting output $\mathbf{z}'$, to the incentive-based contract eliciting $\mathbf{z}$.

Now consider the general multitask case. From the above analysis $y_s > y_{s-1}$ if and only if

$$c_s z_s > c_{s-1} z_{s-1}. \tag{5}$$

Expression (5), however, requires that the state-contingent output bundle be technically inefficient in the sense that the agent could produce strictly more z_{s-1}, while maintaining the level of z_s without incurring any higher cost. Hence, for this cost structure, we conclude on the basis of these arguments that:

PROPOSITION 3: *For the weakly monotonic cost structure*

$$C(\mathbf{z}) = \max\left\{c^1(z_1), c^2(z_2) \dots c^S(z_s)\right\},$$

the principal offers fixed payments if and only if production of $\mathbf{z}$ *is technically efficient. In the single task problem, the principal always offers a fixed payment, and production of* $\mathbf{z}$ *is never technically inefficient.*

Clearly, the only situation where the principal will desire a technically inefficient production pattern is one in which $p(\mathbf{z})$ is strongly risk-substituting, so that the benefits which arise in the single task problem from the adoption of the less risky output $\mathbf{z}'$ in place of $\mathbf{z}$ are offset by an increase in $p(\mathbf{z})$.

4. Optimal Contracts and the Structure of Incentives

We now turn to examining whether multitask problems lead to higher-powered or lower-powered incentives. We use the single task moral hazard problem as a point of reference. The single task optimum yields a particularly interesting benchmark because it corresponds to the case where the principal effectively delegates the responsibility for the choice of p completely to the agent.

4.1. The Single Task Optimum

We only consider the case where a standard is not optimal in the single task case.[16] The first-order necessary conditions for an optimum are then

$$\frac{1}{S} - Y_1(\mathbf{z}, \bar{u}) \leq 0, \quad z_1 \geq 0,$$

$$\frac{1}{S} - Y_s(\mathbf{z}, \bar{u}) = 0, \quad s \neq 1,$$

[16]This implies that $z_2 > 0$.

in the notation of complementary slackness. For an interior solution,

$$Y_s(\mathbf{z}, \bar{u}) = Y_t(\mathbf{z}, \bar{u}) \quad \forall s, t. \tag{6}$$

4.2. Higher-Powered and Lower-Powered Contracts

Intuitively, for higher-powered state-contingent contracts, the reward schedule relating payments to output (which by Lemma 1 has nonnegative slope) is in some sense steeper than for lower-powered incentives. For example, in comparing two affine payment schedules with a common intercept, the one with the steeper slope has the higher powered incentives. Now consider comparing two affine payment schedules with a common slope but a different intercept. The schedule with the higher intercept now involves higher overall incentives for all *ex post* outputs. The certainty of higher income associated with it will entice some agents to accept it who would reject the other contract. If we compare two affine payments schemes with different intercepts and slopes, the intercepts can be chosen so that the one with the steeper slope offers lower returns for all relevant *ex post* outputs than the alternative. More generally, for nonlinear contract structures, the difference between a higher-powered and a lower-powered contract involves more than just the slope of the payment schedule for risk-averse agents.[17]

Alternatively, one can also observe that the contract with what are traditionally viewed as the highest-powered incentives, where the agent is made the residual claimant, exposes the agent to the entire spectrum of income and production risk. Moreover, the contract with what are usually perceived as the lowest-powered incentives, where the agent receives a fixed payment regardless of the production outcome, exposes the agent to no income risk. Therefore, we base our formal definition of higher-powered incentives upon the relative dispersion of contracts or equivalently, the riskiness of the output and payment vectors.

There are a great many different bases on which the riskiness of vectors may be compared. The simplest, analyzed by Sandmo (1971) is that of a multiplicative spread about the mean. A more general definition, widely used in the literature, is that of Rothschild and Stiglitz (1970), also analyzed by Hadar and Russell (1969) and Hanoch and Levy (1969).

The concept of monotone spreads (also known as deterministic transformations) is intermediate between these two and has proved tractable in comparative static analysis (Meyer and Ormiston 1989, Quiggin 1991). For the case $S = 2$, and monotonic vectors, all these definitions, and most others

[17]Both Holmström and Milgrom (1991) and Baker (1992) use linear payment schedules. In Holmström and Milgrom (1991), this ambiguity is removed by restricting attention to individuals with constant absolute risk aversion. In Baker (1992), individuals are risk neutral.

that have been considered in the literature coincide. Given two monotonic variables $\mathbf{x}$ and $\mathbf{x}' \in \Re^2$ such that

$$\frac{x_1 + x_2}{2} = \frac{x_1' + x_2'}{2} = \mu,$$

$\mathbf{x}'$ is derived from $\mathbf{x}$ by a mean-preserving spread in the sense of Rothschild and Stiglitz if and only

$$x_1' \leq x_1 \leq x_2 \leq x_2',$$

and this immediately implies (assuming $x_1 < \mu < x_2$)

$$x_i' - \mu = (1 + \lambda)(x_i - \mu) \quad i = 1, 2,$$

where

$$\lambda = \frac{x_1 - x_1'}{\mu - x_1} = \frac{x_2' - x_2}{x_2 - \mu} \geq 0.$$

Thus, for the case $S = 2$, we may simply say that $\mathbf{x}'$ is derived from $\mathbf{x}$ by a mean-preserving spread, without specifying any particular definition of increasing risk.

To sharpen the analysis and simplify the statement of the condition and the derivation of results, we, therefore, focus on the case $S = 2$.

DEFINITION 1: *Contract* $\{\mathbf{y}^A, \mathbf{z}^A\}$ *has higher-powered incentives than* $\{\mathbf{y}^B, \mathbf{z}^B\}$ *if,*

$$y_2^A \geq y_2^B \geq y_1^B \geq y_1^A$$
$$z_2^A \geq z_2^B \geq z_1^B \geq z_1^A.$$

That is, our notion of higher-powered contracts is that they are "spread out" versions of lower-powered contracts. The agent is asked to produce a more dispersed output distribution in return for a more dispersed income payment distribution. Mean preserving spreads are an analytically convenient case.

Observing the solution for the single task moral hazard problem allows us to state the following lemma (proof is in the Appendix):

LEMMA 3: *At the optimal solution to the single task moral hazard problem, implementing a spread of* $\mathbf{z}$ *yields a higher-powered contract.*

In the single task moral hazard problem, a small mean-preserving spread of $\mathbf{z}$ has no impact on the principal's welfare. Hence, the principal obviously has no incentive to move in this direction. Matters can change in the multitask case.

Evaluate the principal's objective function at the solution to the single task agency problem and consider a mean-preserving increase in the riskiness of $\mathbf{z}$. The resulting change in the principal's objective function is

$$-[Y_2(z_1, z_2, \bar{u}) - Y_1(z_1, z_2, \bar{u}) + m'(p(\mathbf{z}))(p_2(\mathbf{z}) - p_1(\mathbf{z}))]\delta z_2$$

with $\delta z_2 > 0$. By (6), this reduces to

$$-m'(p(\mathbf{z}))(p_2(\mathbf{z}) - p_1(\mathbf{z}))\delta z_2.$$

Lemma 3, in turn, implies that such a mean-preserving increase in the riskiness of $\mathbf{z}$ leads to a riskier $\mathbf{y}$. We conclude:

PROPOSITION 4: *At the single task agency optimum, the principal benefits from a contract structure with higher-powered incentives if $p(\mathbf{z})$ is a risk substitute. At the single task agency optimum, the principal benefits from a contract structure with lower-powered incentives if $p(\mathbf{z})$ is a risk complement.*

Proposition 4 confirms our intuition. Suppose that the principal ignores the possibility of multitasking. He then offers the agent a contract that trades off efficiency losses against enhanced provision of incentives. It is often alleged that contracts which focus solely on measurable outcomes provide incentives that are too high-powered and encourage the agent to divert attention from other tasks toward producing the contracted outcomes. Baker (1992) illustrates this effect for risk-neutral agents. Proposition 4 qualifies this argument by showing that the principal can benefit from providing even more high-powered incentives when $p(\mathbf{z})$ is a risk substitute. However, when $p(\mathbf{z})$ is a risk complement, the principal appropriately mutes the incentives associated with the contracted outcomes, as in the case examined by Baker.

5. An Example

To illustrate some of the ideas developed above, we consider two specific private-cost structures in the case where $S = 2$. Quiggin and Chambers (1998) fully characterize the solution to (6) for the case $S = 2$. The interested reader can refer to that paper for details. Let h denote the inverse mapping of u. Applying results 1–6 of Quiggin and Chambers (1998), the optimal state-contingent payments for given $\mathbf{z}$ are

$$y_1 = h(\bar{u} + C(z_1, z_1)), \tag{7}$$

and

$$y_2 = h(\bar{u} + C(z_1, z_1) + 2(C(z_1, z_2) - C(z_1, z_1))). \tag{8}$$

The principal's single task and multitask problem can be written, respectively, as

$$\max_{\mathbf{z}} \left\{ \frac{z_1 + z_2}{2} - Y(z_1, z_2, \bar{u}) : z_2 \geq z_1 \right\},$$

and

$$\max_{\mathbf{z}} \left\{ \frac{z_1 + z_2}{2} - m(p(\mathbf{z})) - Y(z_1, z_2, \bar{u}) : z_2 \geq z_1 \right\},$$

where

$$2Y(z_1, z_2, \bar{u}) = h(\bar{u} + C(z_1, z_1)) \\ + h(\bar{u} + C(z_1, z_1) + 2(C(z_1, z_2) - C(z_1, z_1)). \qquad (9)$$

The specific cost structures considered are

$$c(\mathbf{z}, p) = \gamma^2 p + c_1 z_1 + \frac{z_2^2}{p}, \quad \gamma, c_1 > 0,$$

and

$$c(\mathbf{z}, p) = \phi^2 p + \frac{z_1^2}{p} + c_2 z_2, \quad \phi, c_2 > 0.$$

For the first cost structure, straightforward calculation reveals

$$p(\mathbf{z}) = \frac{z_2}{\gamma},$$

whence

$$C(\mathbf{z}) = c_1 z_1 + 2\gamma z_2.$$

Thus, this technology is strongly risk-complementary because any mean-preserving spread of a monotonic $\mathbf{z}$ leads to an increase in $p(\mathbf{z})$. For intuitive purposes here, one might keep in mind the case where the agent is a factory employee, and p is the wear and tear on a piece of capital equipment that is used by several employees. It seems plausible that p will be directly related to the rate and intensity at which the equipment is operated. Suppose that the equipment is such that if all things go well, its productivity in terms of $\mathbf{z}$ increases dramatically when it is operated at high speed but diminishes even more rapidly if things go poorly. In our terms, p would be risk-complementary.

Parallel arguments reveal that for the second cost structure

$$p(\mathbf{z}) = \frac{z_1}{\phi},$$

and

$$C(\mathbf{z}) = 2\phi z_1 + c_2 z_2.$$

Thus, the second cost structure is strongly risk substituting. Here one might think in terms of our earlier example where $\mathbf{z}$ is measured quality of a single unit of a perishable agricultural commodity and p is the pesticide residue in the product. For this choice of parameters, a mean preserving spread of a monotonic quality distribution leads to lower pesticide residues. Conversely, a mean preserving contraction of a monotonic quality distribution leads to higher pesticide residues.

For the risk-complementary (machinery, factory employee) cost structure, the single task optimum, therefore, solves

$$\max_{\mathbf{z}}\left\{\frac{z_1+z_2}{2}-\frac{1}{2}h(\bar{u}+(c_1+2\gamma)z_1)\right.$$
$$\left.-\frac{1}{2}h(\bar{u}+(c_1+2\gamma)z_1+4\gamma(z_2-z_1)) : z_2 \geq z_1\right\}.$$

Introducing a nonnegative auxiliary variable, $\alpha = z_2 - z_1$, the single task problem reduces to the simple nonlinear program subject only to nonnegativity constraints

$$\max_{z_1,\alpha}\left\{\frac{z_1+z_1+\alpha}{2}-\frac{1}{2}h(\bar{u}+(c_1+2\gamma)z_1)-\frac{1}{2}h(\bar{u}+(c_1+2\gamma)z_1+4\gamma\alpha)\right\}.$$

Associated necessary and sufficient first-order conditions are

$$1-(c_1+2\gamma)\left[\frac{1}{2}h'(u_1)+\frac{1}{2}h'(u_2)\right]\leq 0, \quad z_1 \geq 0 \tag{10}$$

$$\frac{1}{2}-h'(u_2)2\gamma \leq 0, \quad \alpha \geq 0 \tag{11}$$

in the notation of complementary slackness where $u_i = u(y_i)$. A nontrivial production standard (i.e., one not involving a complete shutdown) is optimal if and only if

$$\frac{1}{(c_1+2\gamma)} > \frac{1}{4\gamma}$$

and, there exists $z_1 > 0$ with

$$h'(\bar{u}+(c_1+2\gamma)z_1) = \frac{1}{(c_1+2\gamma)}.$$

Hence, for a production standard to be optimal, the benefit–cost ratio from raising z_2 must be less than the benefit–cost ratio associated with increasing production nonstochastically (i.e., raising z_1 and z_2 by the same amount). Suppose, in fact, that one takes the case of constant relative risk aversion with

$$h(u) = \exp(u),$$

then the above requires

$$\frac{1}{(c_1+2\gamma)} = \exp(\bar{u}+(c_1+2\gamma)z_1) > \frac{1}{4\gamma}.$$

By an appropriate choice of parameters, it is straightforward to establish that this can happen. On the other hand, by the preceding, a nontrivial standard can never be optimal in the single task problem if

$$\frac{1}{4\gamma} > \frac{1}{(c_1+2\gamma)}, \tag{12}$$

or in words when the benefit–cost ratio from raising z_2 alone is greater than the benefit–cost ratio associated with raising production nonstochastically. In such instances, there will always be a separating or nonpooling solution in the single task optimum.

Now consider the multitask problem in the risk-complementary case, under the further assumption that damage from p is linear, that is, $m(p) = mp$ with $m > 0$. The multitask optimum solves

$$\max_{z_1,\alpha}\left\{\frac{1}{2}z_1 + \frac{1}{2}(z_1+\alpha) - m\frac{z_1+\alpha}{\gamma} - \frac{1}{2}h(\bar{u} + (c_1+2\gamma)z_1) - \frac{1}{2}h(\bar{u} + (c_1+2\gamma)z_1 + 4\gamma\alpha)\right\}$$

with necessary and sufficient conditions

$$1 - \frac{m}{\gamma} - (c_1+2\gamma)\left[\frac{1}{2}h'(u_1) + \frac{1}{2}h'(u_2)\right] \leq 0, \quad z_1 \geq 0$$

$$\frac{1}{2} - \frac{m}{\gamma} - h'(u_2)2\gamma \leq 0, \quad \alpha \geq 0,$$

again in the notation of complementary slackness.

We now seek to determine if there are conditions under which the single task optimum requires an incentive contract but the introduction of multitasking concerns leads to the adoption of a contract. Therefore, assume for the moment that condition (12) holds. In the presence of multitasking, a nontrivial production standard can be optimal if and only if there exists a z_1 satisfying

$$\frac{1-\dfrac{m}{\gamma}}{(c_1+2\gamma)} = h'(\bar{u} + (c_1+2\gamma)z_1) > \frac{\dfrac{1}{2}-\dfrac{m}{\gamma}}{2\gamma}.$$

We need to establish that the inequality implied by this condition

$$\frac{1-\dfrac{m}{\gamma}}{(c_1+2\gamma)} > \frac{\dfrac{1}{2}-\dfrac{m}{\gamma}}{2\gamma}, \tag{13}$$

is consistent with (12). Rearranging obtains the condition

$$\frac{m}{\gamma}\left(\frac{1}{2\gamma} - \frac{1}{(c_1+2\gamma)}\right) > \frac{1}{4\gamma} - \frac{1}{(c_1+2\gamma)},$$

and since $\frac{1}{2\gamma} - \frac{1}{(c_1+2\gamma)} \geq \frac{1}{4\gamma} - \frac{1}{(c_1+2\gamma)}$, (13) and (12) are consistent with one another so long as $m > \gamma$. Thus, by an appropriate choice of parameters, the presence of multitasking concerns can lead to the introduction of standards where none would exist in the single task optimum.

Turning to the risk substituting (fruit quality, pesticide residue) cost structure, the single task optimum solves

$$\max_{z_1,\alpha}\left\{\frac{1}{2}z_1+\frac{1}{2}(z_1+\alpha)-\frac{1}{2}h(\bar{u}+(2\phi+c_2)z_1)\right.$$
$$\left.-\frac{1}{2}h(\bar{u}+(2\phi+c_2)z_1+\frac{c_2}{\frac{1}{2}}\alpha)\right\},$$

with necessary and sufficient-order conditions

$$1-(2\phi+c_2)\left[\frac{1}{2}h'(u_1)+\frac{1}{2}h'(u_2)\right]\leq 0,\quad z_1\geq 0$$
$$\frac{1}{2}-h'(u_2)c_2\leq 0,\quad \alpha\geq 0.$$

Our focus here is on demonstrating that the introduction of multitask concerns can lead a principal who would implement standards in the single task optimum to introduce incentives in the presence of multitask concerns. This is important, because it is usually emphasized that the presence of multitasking leads to a muting and not a strengthening of incentives. Therefore, our focus is on the case where a nontrivial production standard is optimal in the single task case. By the above, this can happen if and only if there exists a z_1 such that

$$\frac{1}{2\phi+c_2}=h'(\bar{u}+(2\phi+c_2)z_1)>\frac{1}{2c_2},$$

which, as explained above, requires that the single task, benefit–cost ratio from nonstochastically increasing production be greater than the single task benefit–cost ratio from increasing only good-state production.

Under the assumption of a linear damage structure, $m(p)=mp$ with $m>0$, the first-order necessary and sufficient conditions for the multitask problem are

$$1-\frac{m}{\phi}-(2\phi+c_2)\left[\frac{1}{2}h'(u_1)+\frac{1}{2}h'(u_2)\right]\leq 0,\quad z_1\geq 0$$
$$\frac{1}{2}-h'(u_2)c_2\leq 0,\quad \alpha\geq 0.$$

A nontrivial production standard is now optimal if and only if there exists a z_1 such that

$$\frac{1-\frac{m}{\phi}}{2\phi+c_2}=h'(\bar{u}+(2\phi+c_2)z_1)>\frac{1}{2c_2}\geq 0.$$

It is easy to see, for example, that this condition can never be satisfied if $m>\phi$. In this case, the multitask, the benefit–cost ratio from nonstochastically increasing production is always less than one. In both the second-best and the multitask optimum, the principal would choose to produce zero output in the bad state.

Hence, a set of sufficient conditions in the risk-complementary case for the introduction of multitasking concerns to cause the principal to switch from standards to incentives are

$$h'\left(\bar{u} + \frac{c_2}{\frac{1}{2}} z_2\right) = \frac{1}{2c_2},$$

when $\frac{1}{2\phi + c_2} > \frac{1}{2c_2}$ and $m > \phi$. In the case of constant relative risk aversion, this simply requires that there exist a z_2 such that

$$z_2 = \frac{\frac{1}{2}\left[\ln\left(\frac{1/2}{c_2}\right) - \bar{u}\right]}{c_2} > 0,$$

when $\frac{1}{2\phi + c_2} > \frac{1}{2c_2}$ and $m > \phi$.

6. Concluding Remarks

We have studied the structure of state-contingent contracts in the presence of moral hazard and multitasking. Our analysis has identified necessary and sufficient conditions for the presence of multitasking to lead to fixed payments instead of incentive schemes. We also show, however, that the introduction of multitask concerns can lead the principal to introduce incentive schemes where standards would have prevailed in the absence of multitask concerns.

Holmström and Milgrom (1991) were the first to isolate the possibility of fixed payment schemes in the presence of moral hazard due to multitasking. We rely on the state-contingent representation of the moral hazard problem developed in our earlier work (Quiggin and Chambers 1998). That earlier model, however, did not consider multitasking concerns. This paper introduces multitasking concerns into that framework. In our original model formulation, the emphasis was on the state-contingent technology, and consequently our analysis of the multitasking problem focuses on different aspects of the problem than the Holmström and Milgrom analysis. When viewed from a state-contingent perspective, the relationship between noncontractible outputs and the costliness of the production risk (for the observable and contractible outputs) determines whether multitasking leads to higher-powered or lower-powered incentive schemes. In particular, we show that when the noncontractible outputs are socially undesirable risk substitutes, standards are never optimal. The same result holds if the noncontractible outputs are socially desirable risk complements.

Appendix: Proofs and Derivations Not in Text

Proof of Lemma 2: Without loss of generality, take $S = 2$. The incentive compatibility constraints in this case are

$$\min\{y_1, y_2\} - C(z_1, z_2) \geq y_1 - C(z_1, z_1)$$

$$\min\{y_1, y_2\} - C(z_1, z_2) \geq y_2 - C(z_2, z_2)$$

$$C(z_2, z_1) - C(z_1, z_2) \geq 0.$$

Multiply the first constraint by $\lambda (0 < \lambda < 1)$ and the second constraint by $1 - \lambda$, and add the resulting equations together to obtain after rearrangement

$$\lambda C(z_1, z_1) + (1-\lambda) C(z_2, z_2) - C(z_1, z_2) \geq \lambda y_1 + (1-\lambda) y_2 - \min\{y_1, y_2\} \geq 0.$$

There are two possible ways to avoid a standard $z_2 > z_1$ and the reverse. Consider the former, then incentive compatibility requires

$$\lambda C(z_1, z_1) + (1 - \lambda) C(z_2, z_2) - C(z_1, z_2) \geq 0, \quad 0 < \lambda < 1.$$

Letting $\lambda \to 1$ yields $C(z_1, z_1) - C(z_1, z_2) \geq 0$, contradicting the strict monotonicity of $C(\mathbf{z})$. A parallel argument yields a similar conclusion in the reverse case. This establishes necessity. Sufficiency is established by setting $y_s = y_1 = \bar{u} + C(z\mathbf{1})$ for all s. ■

Proof of Lemma 3: A multiplicative spread of $\mathbf{z}$ requires

$$\delta z_1 = -\delta z_2$$

with $\delta z_2 > 0$. Hence, by (6), any such change is mean preserving for $\mathbf{y}$. The agent's payment in state 1 is

$$y_1 = h(\bar{u} + C(z_1, z_1)).$$

Observing that y_1 varies directly with z_1 then establishes the result. ■

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