Public-private contracting under limited commitment

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Abstract

A government delegates construction and operation of an essential facility to a private firm. When parties sit at the contracting table, they are uncertain about the operating cost. At the construction stage, the firm can improve its distribution by exerting some non-contractible effort. As soon as the facility is in place, the firm learns the realized cost privately. If any of the parties breaks down the relationship and the firm is replaced during the operation phase, the government bears a cost that is more important the earlier the interruption, relative to the stipulated duration. Under limited commitment, the optimal full-commitment allocation is implementable if and only if the firm has some minimum amount of own funds that can be destined to the project, it is able to borrow funds for that specific project, and the replacement cost is sufficiently high. Implementation is made by instructing the firm to invest some intermediary amount of own and borrowed funds, by conditioning the loan guarantee (provided under the aegis of a third party not suffering from commitment problems) on the outcome of the potential renegotiation process between the government and the firm, and by setting duration neither too short nor too long. Making duration contingent on the realized operating cost lessens either the moral-hazard or the commitment problem, depending upon how the compensation scheme is structured.

Keywords: Public-private contracting; limited commitment; duration; private funds; debt; guarantees; replacement cost

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1 Introduction

In contracts between governments and private firms for building and operating essential facilities, firms are often required to invest.\(^1\) Concerned investments are generally huge, and largely financed with debt.\(^2\) An important but still under-explored issue is whether it is desirable to involve private capital in large public projects and, if so, to what extent this capital is to be drawn from the own funds of the firms and/or borrowed on the credit market. Our study investigates this issue, nesting financial considerations into optimal public-contracting design.

We pursue our objective taking into account two major problems that typically plague the implementation of contracts awarded for the construction and operation of essential facilities. First, as these contracts last long (usually, some decades), there is often uncertainty about operating conditions when they are drawn up, and asymmetric information between the involved parties when they are executed. Second, especially (though not only) in developing countries, where institutions are weak, commitment is limited. Hence, contracts that fail to be self-enforcing are often reneged either by the government (non-commitment) or by the firm (limited enforcement), and possibly renegotiated.\(^3\)

To capture these two problems, we adopt a model that, in its basic elements, is similar to Laffont \([32]\) and the related studies of Guasch et alii \([21]\) and \([22]\). We nonetheless innovate on a variety of aspects, which allow us to represent the situations that we have in mind more closely, and address the issue of our interest.

Specifically, following Laffont \([32]\) and related studies, we assume that, when the government and the firm sign the contract for the construction and operation of an essential facility, they both face uncertainty about the cost that the firm will bear during the operation phase. Furthermore, once the facility is in place, the firm observes the cost realization privately.\(^4\)

\(^1\)Contracts for public projects in which firms are required to invest have first appeared in the UK under the denomination of private finance initiative (PFI).

\(^2\)In June 2008, The Economist reported that infrastructure spending (representing a large share of world GDP, with $22 trillion allocated to projected investments over a ten-year horizon only in emerging economies) was mainly funded with corporate bonds issued by the private firms running the projects before the economic crisis, and with senior debt after the crisis. That leverage is core in large infrastructure projects is further witnessed by the circumstance that, in the UK, in March 2009, the Government created the Treasury Infrastructure Finance Unit with the task of lending funds to PFI projects for which it is difficult to borrow from commercial banks (House of Lords \([25]\)). On top of that, in 2010, the Association for Consultancy and Engineering proposed the creation of a National Investment Bank, along the lines of the European Investment Bank that has lent €3-4bn. of funding on a not-for-profit basis since 2005 (ACE \([1]\)). See also Flyvbjerg et alii \([18]\) on debt financing of large public projects.

\(^3\)We use the labels "non-commitment" and "limited enforcement" as reported in Estache and Wren-Lewis \([17]\), who recall that non-commitment is explored in Chapter 9 of Laffont and Tirole (1993), and limited enforcement in Laffont \([32]\) and, more widely, in Guasch et alii \([21]\). Lack of enforcement is referred to as a cause of pervasive renegotiations in Guasch \([20]\), Engel et alii \([15]\), Estache \([16]\), Chong et alii \([7]\), Saussier et alii \([39]\).

\(^4\)Laffont \([32]\) makes this assumption to study monopoly regulation; Guasch et alii \([21]\) - \([22]\) use it with regards to concession contracts.
an illustration, one can think about the realization of a tunnel. Prior to construction, maintenance costs are not perfectly predictable. The firm has the possibility of learning them as soon as it starts managing the facility. By contrast, the government, which does not perform the activity, does not observe them directly. In line with the same literature, we capture the limited-commitment problem by allowing the contract to be reneged during the operation phase.

First of all, our approach departs from previous work in terms of contract duration. The time length of the second stage of the project (operation) is not exogenously given. As we will illustrate at a later stage, the duration choice is of core relevance for the parties’ achievements, especially under limited commitment.

Secondly, our approach departs from previous work in terms of information structure. In our model, at the construction stage, the firm decides whether to exert some non-contractible effort that can affect the operating conditions. This is in line with the studies about public-private partnerships (Bennett and Iossa [5], Hart [23], Iossa and Martimort [28] - [27], Martimort and Pouyet [35], among others), which evidence the presence of synergies between the phases of the project. Specifically, in our setting, effort provision raises the probability of a low operating cost.

Thirdly, rather than focusing on either firm-led or government-led renegotiation, we look at commitment problems more broadly, and allow for any of the contractual parties to renege on the contract. This helps us capture the far-from-abstract possibility of institutional weaknesses making contract enforcement difficult for either party. Moreover, the benefits/penalties accruing to parties after the contract is reneged are strictly related to its time length, which is endogenous in our study, as we said. If the relationship is stopped and the firm replaced with a new operator, then the government bears a cost that is more important the earlier the interruption, relative to the deadline of the contract. This modelling device is meant to reflect the acknowledged circumstance that, by being unable to keep the relationship in place, governments lose reputation *vis-à-vis* current and prospective partners, customers, and voters, and that the loss is lower when the end of the contract is closer.\(^5\) The presence of this cost is of crucial importance as, even in the event of renegotiation, parties’ payoffs do depend upon that cost, hence, indirectly, upon the time period in which reneging occurs.

Financial aspects, which are the very interest of this study, matter in our model because the realization of the project can be financed with a combination of public and private funds. The former can be provided through an up-front transfer from the government to the firm. The latter can be drawn by the firm from two sources, namely its own resource endowment

\(^5\)Irwin [29] stresses that, in real world, government-firm games that involve infrastructure investments are repeated games in a double sense. Not only the government is concerned with its reputation *vis-à-vis* the firm involved in the concerned project. It also cares about the information that its behaviour and achievements convey to third parties with whom it can potentially interact in the future.
and lenders (such as banks). This representation is coherent with the real-world evidence that, during construction, expenses are generally financed with own funds of the concerned firms and bank loans, sometimes complemented with governmental subsidies. Among the studies aforementioned, financial considerations appear in Guasch et alii [21] in a framework of firm-led renegotiation. However, the focus is there on outside financing only. That is, the firm does not inject its own resources up-front to fund the investment, which is covered with a bank loan and, possibly, governmental funds.

Time aspects emerge as a companion interest of this study as the design of an appropriate financial structure cannot spare reference to the time length of the contract. The reason for this is twofold. First, the profit that the firm obtains at each instant of the operation phase depends upon how much it invested and how long the contract lasts. Second, the total payment that the government makes at each instant of the operation phase depends upon how much the firm invested, as just said, and how much the firm needs to reimburse to the lender at each instant, which is in turn determined according to the size of the loan and the contract duration. As the outcome of the renegotiation process depends upon the instantaneous returns that parties attain under the contract and, in turn, those returns depend upon the amount of private resources injected into the project, one cannot study the optimal financial structure making abstraction from duration concerns.

We begin by considering the benchmark situation in which parties fully commit to the contract. As usual with ex-ante contracting, full commitment and no liability concerns, the original agreement stipulates the efficient allocation and remains in place for the whole duration agreed upon. However, a restriction in terms of duration is imposed by the moral hazard problem that arises at the construction stage. That is, in line with the findings of the literature on public-private partnerships (see Iossa and Martimort [28]), the contract must be sufficiently long to allow the firm to benefit from effort provision. The mix of sources used to fund the investment is irrelevant, instead.

We then move to study the situation of our interest, that in which commitment is limited and some party may have incentives to renege during the operation phase. Clearly, preventing this behaviour casts additional constraints on contract design. Taking the original contract to be the one that stipulates the optimal full-commitment allocation, we investigate whether such constraints translate into restrictions in terms of financial structure and duration and, if so, how the resources and the termination date should be chosen and mixed for the contract to be still effected under limited commitment. In so doing, we address a number of specific questions. Should the firm be instructed to use its own funds and, if so, in large or small amount? Should it be induced to borrow money on the credit market, and should the loan be guaranteed somehow?

\[6\] Engel et alii [12] stress that this is a major financial characteristics of private-public partnerships.
(say, by involving a third party, such as an international institution, that does not suffer from commitment problems)? How much private funds should be injected, overall, into the project? How should duration be picked to ease contract enforcement?

We obtain a number of results that we illustrate hereafter.

To begin with, three conditions are necessary for the contract that stipulates the optimal full-commitment allocation to be implementable under limited commitment.

First, replacing the firm during the operation phase must be sufficiently costly to the government. As long as replacement does not yield an important penalty, the government finds it convenient to interrupt the relationship before the termination date originally stipulated, and thus expropriate the investment made by the firm and the lender at the outset of the project.

Additionally, it is necessary that both own funds of the firm and funds borrowed on the credit market be invested in the project. To see why injection of own funds is essential, consider that, to induce information release at no cost, the government offers an incentive scheme under which the firm is rewarded in the event that a low cost is realized, and punished otherwise. Clearly, the firm may no longer be willing to operate if it finds out that the true cost is high. To prevent the firm from reneging on the contract in the latter case, it must be required to contribute a sufficiently important amount of own funds up-front, and be entitled to recover those funds during the operation phase by making its instantaneous return sufficiently large. This theoretical prediction has a non-negligible practical implication. That is, projects of the kind that we refer to should be delegated only to firms that do have some endowment to fund investments up-front.

In turn, the loan is essential in that it allows the government to harden the firm’s budget constraint, which is another way to lessen the firm’s incentives to abandon the project. This is made by offering the lender a guarantee conditional on the relationship between the government and the firm going on till the termination date. It means that the guarantee should be provided in the event that the contract is either executed or renegotiated, and denied otherwise. Of course, in a limited-commitment framework, guarantees are only feasible under the aegis of a third party (say, an international institution) that does not suffer from commitment problems. By conditioning the guarantee on the prosecution of the relationship, contract break-down followed by firm replacement becomes less costly to the government than renegotiation. The incentive of the firm to renege on the contract in the hope for a profitable renegotiation is, thus, removed.

Once it is ensured that these requirements are all met, the subsequent step is to set the amount of resources and the duration such that the contract stipulating the optimal allocation is effected, indeed. The next contribution of our study consists in predicting how this should be done.
Let us first consider duration. As under full commitment, the contract should not last too little due to the moral-hazard problem. At the same time, under limited commitment, the contract should not last very long either, unless the replacement cost is sufficiently high. Actually, with a low replacement cost and a long-lasting contract, the government is unable to use the firm’s own funds as an instrument to remove both its own and its partner’s incentives to renege. Indeed, refraining the firm from reneging in the bad state requires it investing a very large amount of own funds. By raising the instantaneous operating profit, this helps the government keep the firm in the contract. However, the more the firm invests, the more the government is tempted to interrupt the relationship and expropriate the investment in the good state, when the operating profit is large. Clearly, under these circumstances, a long-lasting contract does not survive. With a high replacement cost, the difficulty is circumvented by involving some "intermediate" amount of firm’s funds, neither too little to induce the firm to renege in the bad state, nor too high to induce the government to renege in the good state.

We now move to the optimal financial structure of the project. Not only the firm ought to invest an intermediate amount of own funds, as just explained. It should also borrow some intermediate amount of funds on the financial market. The reason why a loan is to be taken to run the project has been already illustrated. We still need to explain why the loan should not be too large, in turn. Again, in that case, the government would be tempted to renege on the contract so as to appropriate the facility without reimbursing the lender (through the firm). This points to the conclusion that, in environments where contractual parties suffer from commitment problems, reliance on private capital in public projects should not be massive, even when private partners have deep pockets and/or unconstrained access to financial markets.

As a final step of the work, we extend the model to investigate how results are affected if, rather than offering a fixed-term contract, in which the termination date is the same no matter the realized state of nature, the government designs a flexible-term contract, in which it chooses a different duration for each possible state. Our study predicts that making duration state-dependent can be beneficial in two ways, depending upon how the compensation scheme is structured. First of all, it can help the government lessen commitment problems. More precisely, the contract that stipulates the optimal allocation becomes implementable in the limited-commitment framework even when the replacement cost is not very large. In that case, the government is tempted to renege if a low operating cost is realized, the firm if a high operating cost is realized. Either such incentive is removed by differentiating the contract length between states. Specifically, in the good state, duration should be long enough to render replacement sufficiently costly, hence unattractive, for the government. In the bad state, duration should be short enough to make the operating profit sufficiently high to persuade the firm to remain in the activity. On top of that, making duration state-dependent can help the
government address the moral-hazard problem, which arises also under full commitment. In particular, the firm can be induced to exert effort even when the associated disutility is very large. In that case, the firm has no incentive to provide effort unless its return is sufficiently uncertain, which is again made by differentiating the contract length between states.

1.1 Mainly related literature

Our work is related to the literature about reliance on private resources for the realization of public projects. Engel et alii \[11\] argue that requiring the private firm to fund the initial investment entirely and recover costs directly from user-fees, rather than receiving public transfers, is a desirable option in situations where the budgetary authority that monitors the governmental agency in charge of shifting funds from the public budget to the firm faces agency problems. Indeed, private financing helps remove incentive issues between different tiers of the governmental hierarchy, which can plague the performance of the project. Our results suggest a different motivation for involving private funds in public projects. In environments with limited commitment, private resources represent a useful device to induce the firm to remain in the contract when an unfavorable state of nature is realized.

Engel et alii \[11\] further suggest that, when the project has an uncertain outcome, undesirable involvement of public funds can be avoided by offering a flexible-term contract. This contract is such that, once uncertainty is resolved, the duration and, indirectly, the profits of the firm are adjusted to the realized state of nature. Hence, it is not necessary to employ public transfers to make the project financially viable. However, the main motivation for relying on flexible-term contracts seems to come from the authors’ previous works (Engel et alii \[13\] and \[14\]), in which such contracts are put forward as a useful tool to tackle limited-enforcement problems. The authors argue that, by adjusting duration to let the firm obtain exactly the same payoff whatever the state realization, the government removes the temptation of the firm to renege when an unfavorable state is realized. In our model, the incentive of the firm to renege on the contract is not removed by simply eliminating uncertainty from the payoff scheme. Whether the return is uncertain or not, the firm might be able to extract some extra-benefit through renegotiation because replacing it is costly to the government. Moreover, asymmetric information requires that payoffs be, indeed, differentiated between states. Yet, in our framework, state-dependent duration is still a useful incentive tool, and in a broader sense than emerged in previous studies. With both asymmetric-information and limited-commitment concerns, making duration contingent on the state enables the government to adjust the payoff distribution by choosing whether to make it more or less spread, depending upon the relative importance of the incentive problems.\(^7\)

\(^7\)Flexible-term contracts have not only advantages but also drawbacks. Danau \[8\] shows that, when uncer-
According to de Bettignies and Ross [10], private investment is beneficial because private firms credibly commit to early termination of socially inefficient projects, when the latter generate low cash flows. By contrast, a public authority would not do so for political reasons. Indeed, the termination of any project (whether it generates high or low cash flow) provides a bad signal to society about the activity of the government. While de Bettignies and Ross [10] focus on projects for which early termination is socially desirable, we explore situations in which this is not the case. From this standpoint, our analysis is related to that of Dewatripont and Maskin [9]. They show that, under decentralized financing, borrowing money from a small financier provides the firm with good incentives to avoid default. In our model, in which the firm runs a public (rather than a private) project, incentives are provided by the government (rather than by the lender) also by instructing the firm on how much to borrow.

Our study is further related to the literature about capital structure in agency problems. Spiegel [41] and Spiegel and Spulber [42] - [43] investigate the effects of the capital structure chosen by the agent/firm on the contractual relationship with the principal/regulator. They assume that the regulated firm exercises discretion in its choice of a capital structure as this accords with what they observe to occur, in practice, for the U.S. regulated utilities. By contrast, we are interested in identifying the mix of financing sources, including debt, that allows the government to decentralize the optimal full-commitment allocation through the contract offered to the firm.\textsuperscript{8} From this standpoint, our approach is closer to that of Lewis and Sappington [33]. However, in the latter’s framework, renegotiation issues are ruled out as parties are taken to fully commit to the initial agreement.

Lastly, our paper is related to the literature about contract renegotiation after an investment cost is sunk. Hart and Moore [24] consider a credit contract for a project, the outcome of which is observable by the parties but not verifiable. Based on the observed cash-flow, the firm and the creditor either renegotiate or break down the agreement. In the event of break-down, the firm does not share the cash-flow with the creditor. The latter liquidates the project and obtains some benefit out of this. In our model, the cash-flow of the firm is endogenous and thus verifiable, at least in part. Indeed, during the operation phase, the firm receives transfers from the government. However, the firm does not commit to return money to the lender. Moreover, the creditor is not in a position to liquidate assets, which belong to the government and have no other potential use than the public project for which they were created. Under these circumstances, a credit contract can be drawn up not because the creditor can exercise

\textsuperscript{8}This seems to be in line with the attitude, displayed by U.S. regulators before the Eighties, to control utility company debt, as detected in Taggart [44].
residual control rights on the assets, as in Hart and Moore [24]. Rather, it can be signed because the government pledges a guarantee (under the aegis of a third party) in favour of the private firm for the latter to be able to raise funds from external sources.

1.2 Outline

The remainder of the paper is organized as follows. In section 2, we describe the model. In section 3, we present the benchmark situation in which parties fully commit and characterize the optimal contract accordingly. In section 4, we introduce limited commitment into the picture, explaining how it is approached formally. In section 5, we present the renegotiation game. In section 6, we describe the whole set of conditions under which the contract that stipulates the optimal full-commitment allocation is implemented in the limited-commitment framework. Section 7 concludes. Some of the mathematical details are relegated to an appendix.

2 The model

We consider a contract between a government (denoted G) and a private firm (denoted F) for the provision of a service of general interest. The project unfolds over two stages. The first stage, which takes place at time $\tau = 0$, represents the construction phase, during which the facility that is needed to provide the service is financed and built. The second stage, which begins as soon as the facility is in place and lasts till time $T > 0$, represents the operation phase, during which the service is provided. As frequent in recent decades, the private party F is delegated both stages of the project. At time $T$, when the contract ends, the infrastructure is transferred to G, which manages the activity thereafter.

Technology, production, consumer surplus, demand

At time 0, F sinks a cost $I$ to build the facility and exerts an unobservable and non-contractible effort $e \in \{0, 1\}$ that yields a disutility $\psi(e)$, with $\psi(0) = 0$ and $\psi(1) = \psi > 0$. At each instant $\tau \in (0, T)$, F bears a cost $\theta q + K$ to provide the service, with $\theta > 0$ the marginal cost, $q \geq 0$ the production quantity, and $K > 0$ the fixed cost. As a return from production, it receives a transfer $t$ from G and obtains market revenues $p(q)q$. Observe that allowing the private firm to receive a combination of subsidies and fees warrants that a variety of real-world situations be encompassed, ranging from conventional infrastructure provision, in which the government pays for the activity and the firm earns no money from consumers, to traditional concession, in which the firm only relies upon market revenues.$^9$

$^9$As an example, in E.U. it is required that BOT (Build-Operate-Transfer) concession holders rely upon revenues from market sales only, in order to ensure that they bear operation and demand risks entirely (as an
Consumption of $q$ units of the service yields the instantaneous gross surplus $S(q)$, with $S(0) = 0$, $S'(q) > 0$ and $S''(q) < 0$. Furthermore, $S'(0) = +\infty$ and $\lim_{q \to +\infty} S'(q) = 0$, involving that the quantity solution is interior. Consumers cannot store the service and transfer consumption to future periods. Hence, the output produced at some given $\tau$ is entirely consumed at that same time and sold on the market at price $p(q) \equiv S'(q)$. This defines the inverse demand function.

Once the investment is made, both technology and demand parameters remain constant for the whole duration of the project, including the period in which the activity is run by $G$ (say, through a public firm).

**Information structure** The contract between $G$ and $F$ is signed, the investment $I$ made and the disutility of effort $\psi(e)$ borne *ex ante* i.e., when the unit variable cost $\theta$ is unknown to either contractual party. The assumption of *ex-ante* contracting, which is in line with Laffont [32] and Guasch *et alii* [21] and [22], mirrors situations in which the contours of the activity are designed before the firm receives any piece of information on its productivity. At the contracting stage, it is commonly known that $\theta$ will be either low ($\theta_l > 0$) or high ($\theta_h > \theta_l$) with probabilities $\nu_1$ and $1 - \nu_1$, if $F$ exerts effort, and $\nu_0$ and $1 - \nu_0$ if it does not. Let $\Delta \nu = \nu_1 - \nu_0 > 0$, meaning that exerting effort at the construction stage propitiates a lower operating cost stochastically.\(^{10}\)

After the investment is made and the effort exerted (if any), and before production takes place, $F$ observes privately the realized state of nature *i.e.*, whether the marginal cost is $\theta_l$ or $\theta_h$. Hence, at that point, $F$ enjoys an information advantage on the realized cost *vis-à-vis* the contractual partner, as it is typically the case in delegation settings. We denote $i \in \{h, l\}$ the state of nature and $\Delta \theta = \theta_h - \theta_l$ the degree of uncertainty about production cost.

**Project financing** Three financing sources are used to fund the investment. First, $F$ injects an amount $M \in [0, E]$ out of its resource endowment $E > 0$. Second, $F$ borrows an amount of funds $C \geq 0$ on the competitive credit market. Third, $G$ makes an up-front payment $t_0$ to $F$. We assume that a positive effort is desirable for $G$, which thus provides incentives for $F$ to choose $e = 1$. Then, the three financing sources must cover the investment that is made initially, including both the monetary cost and the disutility of effort:

$$t_0 + M + C = I + \psi.$$  \(^{(1)}\)

\(^{10}\)Synergies between project phases are pervasively represented in models on public-private partnerships that take them to provide a rationale for bundling. As an illustration, in Iossa and Martimort [28] effort provision at the construction phase reduces costs at the operation stage.
The transfer $t_0$ is positive when the project is partially financed with public funds. It is negative when the project is financed only with private funds and F makes a payment $[I + \psi - (M + C)]$ to G to be awarded the contract. Given (1) and because $C$ is non-negative, F ultimately borrows

$$C = \max \{0; I + \psi - (M + t_0)\}.$$  

The private funds injected into the project, together with the operation expenses, are covered by the overall stream of revenues (obtained from market sales) and payments (made by G) during the operation phase.\(^{11}\)

### 3 Payoffs and allocation under full commitment

Suppose that F fully commits to the contract signed with the lender and that both F and G fully commit to the contract that they stipulate.

First, the credit contract establishes the amount of money $C$ that the lender must provide to F. Additionally, it fixes the instantaneous repayment, denoted $d_i \geq 0$, that F should make to the lender in state $i \in \{h, l\}$ during the operation phase. The reason why the loan, unlike the repayment, is not differentiated between states of nature is that money is transferred to F before the unit production cost is realized.\(^{12}\) Further denoting $r$ the discount factor, the value of the debt of F in state $i$ at instant $\tau$ is given by

$$D_{i,\tau} = \int_\tau^T d_i e^{-r(x-\tau)} d\tau, \forall i \in \{l, h\}, \forall \tau \in (0, T).$$  

Assuming, for simplicity, that there is a large number of lenders on the market, each facing a zero outside opportunity, $d_i$ is set to yield neither a surplus nor a loss \(i.e., E_i[D_{i,0}] = C\) or, equivalently,

$$E_i[d_i] = \frac{rC}{1 - e^{-rT}},$$  

where the expectation is taken over $\theta_i$, $i \in \{l, h\}$, using the probabilities associated with a positive effort.\(^{13}\)

The contract that G designs for F is an incentive scheme that induces F to participate in the activity, to make an effort at the construction phase and, additionally, to release information

\(^{11}\)This is typical of public-private partnerships, in which the stream of payments that private partners receive during the operation phase of the project covers both the up-front capital expense (capex) and the subsequent operation and maintenance expenses (opex). See Engel et alii [12], for instance.

\(^{12}\)Kartasheva [31] takes the delegated firm to observe the project profitability (its type) privately at the first stage of the relationship. In that context, unlike in our setting, the debt contract is used to screen projects.

\(^{13}\)This meaning of the operator $E_i$ is maintained all throughout the paper, provided $e = 1$ is induced at equilibrium.
about the operating cost once this is realized and observed privately. The scheme includes three components. First, it specifies how much private and public resources (the triplet \((M, C, t_0)\)) should be devoted to fund the investment during the construction phase. Second, as the Revelation Principle applies and attention can be restricted to direct revelation mechanisms under which F not only exerts the desirable level of effort but also reports the true cost value, the scheme includes the allocation \((q_i, t_i)_{i=1,h}\), with \(q_i\) the quantity to be produced and \(t_i\) the transfer to be made at each instant \(\tau \in (0, T)\), in the event that the realized cost is \(\theta_i\). Third, the contract stipulates for how long F should run the project \(i.e.,\) the overall duration \(T\).

### 3.1 The payoff of F

The operating profit of F in state \(i\) at instant \(\tau \in (0, T)\) is given by

\[
\pi_i = t_i + p_iq_i - (\theta_iq_i + K) - d_i, \tag{5}
\]

with \(p_i \equiv p(q_i)\). The present value, at instant \(\tau\), of the whole stream of profits from \(\tau\) to \(T\), is written as

\[
\Pi_{i,\tau} = \int_\tau^T \pi_ie^{-r(x-\tau)}dx. \tag{6}
\]

The payoff of F in state \(i\) includes \(\Pi_{i,0}\) together with the resources initially used to fund the investment, namely \(t_0, M\) and \(C\), net of the total investment cost \((I + \psi)\) and of the amount injected by the firm itself into the project \(i.e.,\) \(\Pi_i = \Pi_{i,0} + (t_0 + M + C - I - \psi) - M\). Relying upon the financing condition \((1)\), the payoff of F is further expressed as

\[
\Pi_i = \Pi_{i,0} - M, \tag{7}
\]

and is thus equal to the difference between the discounted operating profits and the own resources used in the project.

### 3.2 The payoff of G

G is a benevolent government that aims at maximizing the discounted consumer surplus, net of the market expenditure and of the social cost of transferring resources from taxpayers to producers, over the whole time horizon. This includes not only the surplus generated under the contract, while F runs the activity, but also the surplus generated after the end of the contract, under public management.

Whatever the regime, to finance the transfers G needs to raise distortionary taxes. Each

\[^{14}\text{See Section D for the case in which duration is made contingent on the realized state of nature.}\]
transferred euro requires that $1 + \lambda$ euros be collected from taxpayers, with $\lambda > 0$.\textsuperscript{15} The imperfections of the taxation system are taken not to vary over time so that $\lambda$ remains constant.

In state $i \in \{l, h\}$, the discounted benefit of $G$ over the period $(\tau, T)$ is given by

$$V_{i,\tau} \equiv \int_{\tau}^{T} [S(q_i) - p_i q_i - (1 + \lambda) t_i] e^{-r(x-\tau)} dx.$$ 

Using (3), (5) and (6), and defining

$$w(q) \equiv S(q) + \lambda p(q) q - (1 + \lambda) (\theta q + K),$$

we rewrite

$$V_{i,\tau} = \int_{\tau}^{T} w(q_i) e^{-r(x-\tau)} dx - (1 + \lambda) (\Pi_{i,\tau} + D_{i,\tau}). \quad (8)$$

The benefit of $G$, net of the social cost of the up-front payment $t_0$, is expressed as

$$U_i = V_{i,0} - (1 + \lambda) t_0$$

$$= \int_{0}^{T} w(q_i) e^{-rx} dx - (1 + \lambda) (\Pi_{i,0} + I + \psi - M).$$

Assuming that no further investment is necessary to continue the activity after the end of the contract, the optimized return of $G$ under the public regime at time $T$ is equal to $\int_{T}^{\infty} w_i^* e^{-r(y-T)} dy$, where $w_i^* \equiv w(q_i^*)$ and $q_i^*$ is the quantity that maximizes $w(q_i)$. This is characterized by the Ramsey-Boiteaux condition

$$\frac{p(q_i^*) - \theta_i}{p(q_i^*)} = \frac{\lambda}{1 + \lambda \varepsilon(q_i^*)}, \quad (9)$$

with $\varepsilon(q) \equiv -(dp(q)/dq) q/p(q)$ the absolute value of the price elasticity of market demand.

Overall, given that the project lasts forever, although the contract with $F$ can be terminated at some finite $T$, the payoff of $G$ in state $i$ is given by

$$W_i = U_i + \int_{T}^{\infty} w_i^* e^{-ry} dy$$

$$= \int_{0}^{T} w(q_i) e^{-rx} dx - (1 + \lambda) (\Pi_{i,0} + I + \psi - M) + \int_{T}^{\infty} w_i^* e^{-ry} dy. \quad (10)$$

\textsuperscript{15}According to Snow and Warren \cite{40}, the shadow cost of public funds is around 0.3 in developed economies. The World Bank \cite{45} provides a figure of 0.9 with regards to developing countries.
3.3 Full commitment

Choice variables of G are the amount of own funds $M$, the pair of quantities $\{q_l, q_h\}$, the pair of discounted cumulated profits $\{\Pi_{i,0}, \Pi_{h,0}\}$ and the contract duration $T$. $G$ sets these variables so as to maximize its payoff

$$E_i [W_i] = \int_0^T E_i [w(q_i)] e^{-rx} dx - (1 + \lambda) (E_i [\Pi_{i,0}] + I + \psi - M) + \int_T^\infty E_i [w_i] e^{-ry} dy$$

subject to the incentive-compatibility constraints

$$\Pi_{l,0} \geq \Pi_{h,0} + \int_0^T \Delta \theta q_h e^{-rx} dx$$

(11a)

$$\Pi_{h,0} \geq \Pi_{l,0} - \int_0^T \Delta \theta q_l e^{-rx} dx,$$

(11b)

the moral-hazard constraint

$$\Pi_{l,0} - \Pi_{h,0} \geq \frac{\psi}{\Delta \nu},$$

(12)

the ex-ante participation constraint

$$E_i [\Pi_{i,0}] \geq M,$$

(13)

and the financing conditions (1) to (4).

At optimum, production is fixed at the level $q_i^*$ for all $i \in \{l, h\}$. Moreover, the expected profit $E_i [\Pi_{i,0}]$ is such that (13) is binding, namely $E_i [\Pi_{i,0}^*] = M$, meaning that no information rent is given up to F. Neglecting (12) for a moment, the pair of optimal profits satisfying (11a) and (11b) is any pair

$$\Pi_{l,0}^*(z) = M + (1 - \nu) \int_0^T \Delta \theta ze^{-rx} dx$$

(14a)

$$\Pi_{h,0}^*(z) = M - \nu \int_0^T \Delta \theta ze^{-rx} dx$$

(14b)

that is determined, for any given duration $T$, by picking the "sharing rule" $z$ within $Z \equiv [q_h^*, q_l^*]$ (see Appendix A for details). For (14a) and (14b) to satisfy (12) as well, it is necessary that

$$\frac{\Delta \theta z}{r} > \frac{\psi}{\Delta \nu}$$

(15)

$$T \geq T(z) = \frac{1}{r \ln \frac{\Delta \nu \Delta \theta z}{\Delta \nu \Delta \theta z - r \psi}}.$$

(16)

$^{16}$Actually, the pair of discounted cumulated profits $\{\Pi_{l,0}, \Pi_{h,0}\}$ replaces the pair of instantaneous transfers $\{t_l, t_h\}$, with a standard change of variables.
Moral hazard imposes two restrictions. First, the sharing rule ought to be such that the discounted return that F would get in state $h$ if the contract were to last forever ($\Delta \theta z/r$) is larger than that it would get if the contract were to last the minimum number of periods $T(z)$ corresponding to that sharing rule ($\psi/\Delta \nu$). Of course, if (15) is satisfied for $z = q_i^*$, then it is for all $z \in Z$ and G has full flexibility at choosing $z$. More generally, as long as some sharing rule can be found, among those that are incentive-compatible, under which (15) holds, there exists a range of contract durations $[T(z), \infty)$ for which the loss accruing to F in state $h$ (hence, the risk transferred to the firm) is sufficiently large to induce effort provision at the construction stage. Condition (16) precisely requires that $T$ be drawn from that range so that the contract lasts long enough to ensure that the firm is repaid for that effort. Observe that (16) is most relaxed when $z = q_i^*$ as $T(z)$ is smaller the larger $z$. This is easily explained. By raising $z$, G introduces more risk in the rent distribution and, thus, reinforces the incentives to effort provision. The duration requirement that ensues from the moral-hazard problem is thereby softened.

Consider now the payoff that G obtains from the project. At optimum, with $E_i [\Pi_{i,0}^*] = M$, this is written as

$$E_i [W_i^*] = \int_0^\infty E_i [w_i^* e^{-\nu x} dx] - (1 + \lambda) (I + \psi).$$  (17)

This evidences that, in expectation, G reaps the same net benefit that it would obtain if F were not to observe the realization of $\theta$ privately when it starts operating. Importantly, this result is attained no matter the way in which $M$ and $C$ are mixed to fund the project.

**Proposition 1 (Benchmark)** Under full commitment, the payoff $E_i [W_i^*]$ is achievable if and only if $\exists z \in Z$ such that (15) holds. This result is attained by setting $T$ according to (16). The mix of financing sources used in the project is irrelevant.

For later use, we let

$$\Pi_{i,\tau}^*(z) = \left( \frac{Mr}{1 - e^{-rT}} + (1 - \nu) \Delta \theta z \right) \frac{1 - e^{-r(T-\tau)}}{r}$$  (18a)

$$\Pi_{h,\tau}^*(z) = \left( \frac{Mr}{1 - e^{-rT}} - \nu \Delta \theta z \right) \frac{1 - e^{-r(T-\tau)}}{r}$$  (18b)

denote the discounted streams of optimized profits of F from $\tau$ to $T$, respectively in the good and bad state, given the sharing rule $z$. Correspondingly, evaluated at the optimal quantity $q_i^*$ and profit $\Pi_{i,\tau}^*(z)$, the expression in (8), which represents the discounted payoff of G from $\tau$ to $T$ in state $i = l, h$, becomes

$$V_{i,\tau}^* (z) = w_i^* \frac{1 - e^{-r(T-\tau)}}{r} - (1 + \lambda) \left( \Pi_{i,\tau}^*(z) + D_{i,\tau} \right).$$  (19)
In what follows, we consider frameworks where commitment is limited and new constraints appear in the programme of G. We investigate whether there exist values of \( M, C \) and \( T \) satisfying those constraints such that the contract that stipulates the full-commitment allocation can still be effected.

4 Limited commitment

Under limited commitment, the contract between G and F may end earlier than originally agreed upon. Two scenarios are possible. First, F induces G to come back to the contracting table, despite that this is not the latter’s will. According to the terminology adopted in previous works, this is the case of limited enforcement. Second, G breaks the initial agreement during the contract execution, despite that this may disadvantage F. This is the case of non-commitment. Both situations lead either to the revision of the contract between G and F, if renegotiation is successful, or to the replacement of F with another operator, if renegotiation fails. Moreover, the execution of the contract between F and the lender is affected as well, under both scenarios. Before presenting the renegotiation game that unfolds between G and F, we describe these issues from a practical viewpoint and motivate the way in which they are approached formally thereafter.

4.1 Limited enforcement

Once it is informed about the realized value of \( \theta_i \), F may credibly threaten G to stop operating and quit the activity unless the contract is revised. F takes this initiative in two cases. First, conditional on parties both knowing the realized state (which will be shown to occur at equilibrium), F would like to renege on the initial agreement if, under the latter, it obtains a low return \textit{ex post}. For instance, this occurs when \( M \) is small and F has a high operating cost, in which case the operating profit under the full-commitment compensation scheme is negative. Second, F may threaten to abandon the project as a deliberate strategy to retain more surplus in the relationship with G, when it is aware that replacement with another firm would be costly to G. Examples of limited enforcement leading to firm-led renegotiation are pervasive in public contracting. In institutionally weak contexts (developing countries, in general), strong rules of law seldom exist and renegotiation is likely to take place. For instance, Estache and Wren-Lewis [17] recall that, in Ghana, the incumbent monopoly for fixed telephony entered the mobile business despite the explicit interdiction. In Tanzania, the regulator failed to enforce regional mobile license and the dominant operator began to expand at the national level. Guasch [20] and Guasch et alii [21] - [22] provide further examples in Latin America and in the Caribbean regions. Although less often, firms renege on contracts also in frameworks where institutions
are solid (typically, developed countries) and contracts should be, in principle, more easily enforced, say, by fining firms that are reluctant to produce. For instance, Gagnepain et alii [19] detect a progressive increase in the subsidies paid to French urban transport concessionaires all over the contract execution, suggesting that governments are weak and/or not prone to engage in costly and time-consuming litigations to enforce contracts.

4.1.1 Consequences for the credit contract

In a limited-enforcement framework, not only the execution of the contract between G and F is problematic. It is also that of the credit contract, provided F cannot be compelled to return money to the lender. In turn, this involves that F may be unable to borrow on the credit market in the first place.\footnote{F would credibly commit to repay the lender if it had reputation concerns. However, a reasonable conjecture is that reputation losses are smaller for private firms than for governments, especially when firms have the possibility of diversifying activities and locations and/or disguising themselves behind subsidiaries with different denominations. A very simple way to formalize this circumstance is to assume that the government does bear a loss, whereas this is not the case of the firm. This is how we proceed in our model. More precisely, and consistently with the issues most seriously plaguing public-private contracts in real world, below we assume that, when the project is not entirely executed, all involved players incur a cost but of a different nature: a reputation/credibility loss for the government, an expropriation cost for the firm and the lender that are not repaid for their investments.} Anticipating this, G can induce financiers’ participation by stipulating that, as long as the relationship with F is not broken down (meaning that either the original contract or a renegotiated contract is on-going), it will pay some guaranteed amount directly to the creditor by abating the instantaneous transfer to F of that same amount. By contrast, G is not responsible for the residual debt in the event that F quits the project and is replaced.\footnote{Guasch et alii [21] assume that the assets of the firm can be used to pledge debt collateral. By contrast, we do not consider this possibility, for the following reason. In the private sector, when the debt is not repaid, the creditor undertakes the activity and either liquidates or reorganizes it. By contrast, in the situations that we represent, the government undertakes the activity, which goes on even when it is no longer run by the initial firm. Hence, in our model, if the relationship between G and F breaks down and F stops repaying the debt, the assets that F sunk in the project cannot be liquidated to reimburse the creditor. The latter is paid or not according to the guarantee that the government provides indirectly, through its contract with F. Yet, even if there were assets that could be liquidated without compromising the project execution under the new management, relevant in our model would only be the residual debt \textit{i.e.}, the part of the debt not protected by the collateral. Therefore, allowing for the firm to use its assets to pledge some debt collateral would bring no qualitative change in our analysis and results.}

Reliance on \textit{conditional} guarantees of this sort (\textit{i.e.}, on guarantees that remain in force as long as the firm-government relationship is in place) is coherent, in particular, with the project finance technique. The latter requires making the project legally and economically self-contained, an outcome that is attained in two ways. First, a stand-alone firm, the so-called \textit{Special Purpose Vehicle} (SPV), is created to undertake no other business than building and operating the concerned project, and endowed with the sole assets pertaining to it, which are
kept separated from the assets of the parent firm.\(^{19}\) Second, lenders are provided no guarantees beyond the right to be paid out of the resources generated within the project (namely, user fees and governmental transfers, if provided for, as represented in our model), which means that any repayment guarantee is foregone in the event that the firm abandons the activity.\(^{20}\)

In practice, however, it is often the case that, while firms remain responsible for their debts as long as they earn profits from the project, governments bail out the activity as difficulties arise, and debt responsibilities are passed onto taxpayers. This happened, for instance, with the 2002-03 London Underground maintaining-and-upgrading project. The public sector was uncertain over whether Metronet, the consortium in charge of the project, could borrow enough funds on the credit market. To boost the banks’ appetite, during the bidding stage, Transport for London guaranteed 95% of Metronet’s debt obligations. Eventually, Metronet failed and the Department for Transport had to make a £1.7 billion payment to help Transport for London meet the guarantee (House of Lords [25] - [26]). According to the National Audit Office [36], taxpayers incurred a direct loss of between £170 million and £410 million. Therefore, two events, both negative for taxpayers, took place: the contract between public administration and private investors broke down (involving a cost of replacement) and the risk of the debt was transferred to taxpayers. This epilogue points to the conclusion that it is not desirable that public guarantees for debt repayment be provided in the event that the relationship is interrupted. To stimulate investors’ participation, governments should rather offer a contract under which the relationship is preserved at equilibrium. Resting on this, in our model, we take G to guarantee the debt only as long as F does remain in the project, whether the original contract is maintained or revised. Formally, we let \(g_{i,\tau}^{rn}\) denote the instantaneous amount guaranteed to the creditor, from date \(\tau\) on, in the event that the contract is renegotiated in state \(i = l, h\) at date \(\tau\). By contrast, if the contract is reneged and the relationship between G and F ends at \(\tau\), starting from that moment, no amount is guaranteed to the creditor.

### 4.2 Non-commitment

As we previously mentioned, non-commitment means that G can break the initial agreement during its execution, despite that this may be detrimental to F. Specifically, once the investment cost has been sunk and G has received a report from F, it may wish to modify the allocation designed \textit{ex ante}, in case it proves inefficient \textit{ex post}. For instance, this happens when F (correctly) declares \(\theta_i\), in which case the firm should be rewarded. In developing countries, government failure to honor contractual terms is even bigger a concern than limited enforcement,

\(^{19}\)Actually, in project finance initiatives, it is sometimes the case that a plurality of firms forming a consortium (rather than a single firm) concur to create the SPV.

\(^{20}\)See Engel \textit{et alii} [12], who further refer to Yescombe (2002) and (2007), for details about financial arrangements in public-private partnerships.
because large-scale investments, which are there desperately needed, especially in utilities, may not take place if governments cannot warrant investors’ remuneration. That this may occur is suggested by the result, which Banarjee et alii [4] draw from a cross-country analysis (see also Estache and Wren-Lewis [17]), that governments’ opportunistic behaviour does not propitiate private investment. It would also be in line with the observation that political risk has challenged public contracting in Central and Eastern Europe in various occasions over the last decades.21

4.2.1 Consequences for the credit contract

As under limited enforcement, execution of the credit contract is problematic also in a framework where the government may not comply with the contractual obligation to make transfers. The guaranteed debt may remain unpaid. To avoid this, G can use "external" means to commit itself to this payment. For instance, one can think of G as depositing resources with a third party, which should then be released to the creditor, in the event that it would not receive money directly from G. In practice, strategies of the kind just described are adopted when governments mandate an Investment Insurance Agency (IIA) to act as an intermediary, providing insurance and/or direct cover in the event of any default in payment by a borrower (or its guarantor) under some loan agreement. Originally created as government entities to promote, facilitate and support the exports of goods and services, starting from the Nineties, IIAs have begun to operate in project financing as well, and are now widely spread across countries.22

Moreover, in developing countries, the World Bank and other multilateral development banks (such as the Inter-American Development Bank) provide guarantees that are less subject to project and country limits, as compared to insurance, and are intended to cover debt up to 100% of principal and interest. According to Irwin et alii [30], if appropriately managed, these guarantees are essential at reinforcing governments’ resolve to abide by their commitments.

21Brench et alii [6] evidence that repeated changes in political attitude towards partnerships with private firms have slowed down the development of transportation projects in Hungary.

22Most European governments have set up IIAs for the purposes described in the text. All countries that have official IIAs, alternatively labelled Export Credit Agencies, are now party to the "Arrangement on Guidelines for Officially Supported Export Credits," which provides specific rules for project finance, derogating from the usual Consensus Rules to allow, among other things, for a longer repayment term (of up to 14 years). Examples of European IIAs are Compagnie Française d’Assurance pour le Commerce Extérieur (Coface), Euler Hermes Kreditversicherungs (Hermes), Istituto per i Servizi Assicurativi del Credito all’Esportazione (SACE), Office National du Ducroire (ONDD), to mention only a few. See Sader [37] on the core role that both bilateral and multilateral Export Credit Agencies play in developing countries at providing political risk insurance by pledging guarantees on major parts of the debt package in the realization of BOT-type infrastructure projects (debt covering the three-quarters of the costs of a typical such project).
5 The renegotiation game

We now come back to the formal analysis and consider the possibility that, at some instant $\tau \in (0, T)$, either F or G wishes to renege on the initial agreement. The former might threat to abandon the project and the latter to stop making payments during the operation phase, unless the agreement is modified. In either scenario, execution of the initial contract is suspended and parties come back to the contracting table. If renegotiation fails, F is relieved of the activity and a new firm, denoted $F'$, steps in.

Break-down of the relationship and resort to a new firm yields a "replacement cost" to G. Under limited enforcement, this cost reflects the reputation loss that a government bears for not being sufficiently authoritative to have a contract executed by the partner with which it was signed, despite that the partner invested (own and borrowed) funds in the concerned project (see also Guasch et alii [21]). Under non-commitment, the cost mirrors the loss of credibility that a government incurs by not keeping its promises vis-à-vis, in primis, the private financiers of the project (here, F and the lender) and, additionally, other potential investors and customers (see Irwin [29]; see also Martimort [34] on the information value that contractual deviations by the government have vis-à-vis third parties, and on the negative consequences for its credibility). Naturally enough, the magnitude of the replacement cost depends upon how much time is left till the end of the contract when the latter is reneged. We thus denote it $R_\delta$, with $\delta = T - \tau$. Specifically, the cost is larger the earlier the replacement, and diminishes as the time at which replacement occurs comes closer to the termination date. Formally, taking $R_\delta$ to be continuously differentiable on $(0, T)$, $R'_\delta \equiv (dR/d\delta) > 0 \forall \delta \in (0, T)$. The cost is nonetheless positive even in the event that the contract is broken down just before the date originally stipulated i.e., $R_\delta > 0 \forall \delta \in (0, T)$, with $\lim_{\delta \to 0} R_\delta = \varepsilon > 0$. It only vanishes when $\tau = T$ so that $R_0 = 0$.

We begin by considering the situation in which one party proposes the other to renegotiate, without specifying which party takes the initiative. This occurs in some state $i \in \{l, h\}$, which is commonly known at this stage of the relationship.²³

5.1 Replacement payoffs

When renegotiation fails and F is replaced, it no longer receives any compensation. That is, starting from the instant $\tau$ at which replacement occurs, its instantaneous profit becomes $\pi^r_p = 0$, the superscript $rp$ being appended to denote the replacement scenario. Thus, its

²³At a later stage, we will show that, in fact, the realized state of nature is revealed before any party might renege on the initial contract (compare (24a) and (24b) below).
payoff at instant $\tau$ is simply given by

$$\Pi_{i,\tau}^{rp} = \int_{\tau}^{T} \pi_i^{rp} e^{-r(x-\tau)} dx = 0.$$ 

Let us next come to $G$. We take the production technology to be related to the inner characteristics of the facility. Under this circumstance, once the facility is in place, the operating cost remains the same whatever the firm. $G$ is thus aware that $F'$ operates at marginal cost $\theta_i$ and offers to $F'$ the quantity-transfer pair $(q_i^{rp}, t_i^{rp})$ that maximizes its own payoff. Specifically, $G$ chooses the transfer that makes the operating profit of $F'$ equal to zero at all instants in $(\tau, T)$ i.e., $t_i^{rp} = \theta_i q_i^{rp} + K - p(q_i^{rp}) q_i^{rp}$, together with the output $q_i^{rp} = q_i^*$.\(^{24}\) As a result, $G$ obtains

$$V_{i,\tau}^{rp} = \int_{\tau}^{T} w_i^* e^{-r(x-\tau)} dx - R_\delta. \quad (20)$$

### 5.2 Renegotiation payoffs

When renegotiation succeeds, the relationship between $F$ and $G$ proceeds under the revised contract. We now describe the renegotiation process and present parties’ payoffs following to renegotiation.

In principle, parties could renegotiate all variables concerning the operation phase agreed upon in the original contract, namely quantity, transfer (or profit), duration. However, in this context, neither party has something to gain from a change in the termination date (see Appendix B). The reason is that any benefit from renegotiation is reaped through a variation in the transfer. Because of this, we hereafter concentrate on quantity-transfer proposals.

First consider $G$ making a take-it-or-leave-it offer to $F$ at date $\tau$. This occurs with probability $\alpha \in [0, 1]$ . The offer requires that, at each instant between $\tau$ and $T$, $F$ produce the quantity $q_i^G$ and receive the transfer $t_i^G$, which includes the amount $g_i^{rn}$ destined to the lender.\(^ {25}\) The offer of $G$ is such that it leaves $F$ just indifferent between renegotiating and abandoning the project i.e., $\Pi_i^G = \Pi_i^{rp}$. This requires setting $t_i^G = \theta_i q_i^G + K - p(q_i^G) q_i^G + g_i^{rn}$. Furthermore, $G$ offers the quantity $q_i^G = q_i^*$ that maximizes its own payoff at $\tau$, which is then written as

$$V_{i,\tau}^{G} = \int_{\tau}^{T} w_i^* e^{-r(x-\tau)} dx - (1 + \lambda) D_i^{rn},$$

\(^{24}\)One can easily deduce that neither $F'$ nor $G$ would have an interest in reneging on the contract that they sign when $F$ is replaced. On the one hand, under that contract, the instantaneous profit accruing to $F'$ is just equal to its best outside opportunity (which is zero). Moreover, as $F'$ does not invest in the project, $G$ would bear no reputation loss if it were to replace $F'$, hence $F'$ cannot extract any benefit from $G$ in view of a renegotiation. On the other hand, $G$ already retains the whole surplus from $F'$ and could not do better.

\(^{25}\)Instead of assuming that the transfer $t_i^G$ includes the guaranteed amount $g_i^{rn}$, meaning that $F$ receives $t_i^G$ and then transfers $g_i^{rn}$ to the lender, one could alternatively think of $G$ as making a payment $t_i^G - g_i^{rn}$ to $F$ and a payment $g_i^{rn}$ directly to the lender. The two alternatives are formally equivalent.
where
\[ D^{rn}_{i,\tau} = \int_{\tau}^{T} g^{rn}_{i,\tau} e^{-r(x-\tau)} dx \]

is the value of the guaranteed debt at instant \( \tau \) for a contract that is renegotiated precisely at \( \tau \).

With probability \((1 - \alpha)\), F makes a take-it-or-leave-it-offer to G. The offer consists in producing the quantity \( q^F_i \) and receiving the transfer \( t^F_i \), which again includes the amount \( g^{rn}_{i,\tau} \) destined to the creditor. The transfer \( t^F_i \) is set to ensure that G gets the payoff \( V^G_{\tau} = V^{rp}_{\tau} \), i.e.,
\[ t^F_i = \frac{1}{1+\lambda} \left( S(q^F_i) - p(q^F_i) q^F_i - w^*_i + \frac{\tau R_\delta}{1 - e^{-r(T^F-\tau)}} \right). \]

Observe that this payment is deflated by one plus the shadow cost of public funds. That is, all else equal, the larger the cost of collecting resources from taxpayers and/or distorting production away from the efficient level, the smaller the surplus that F can extract from G at the renegotiation stage. F further chooses the output \( q^F_i = q^*_i \) that maximizes its own payoff at \( \tau \), which is then written as
\[ \Pi^F_{i,\tau} = \frac{R_\delta}{1+\lambda} - D^{rn}_{i,\tau}. \]

Overall, the expected payoffs of F and G from renegotiation at \( \tau \) are respectively given by
\[ \Pi^{rn}_{i,\tau} = (1 - \alpha) \left( \frac{R_\delta}{1+\lambda} - D^{rn}_{i,\tau} \right) \quad (21a) \]
\[ V^{rn}_{i,\tau} = \int_{\tau}^{T} w^*_i e^{-r(x-\tau)} dx - (1 - \alpha) R_\delta - \alpha (1 + \lambda) D^{rn}_{i,\tau} \quad (21b) \]

for all \( i \in \{l, h\} \). Noticeably, while a larger replacement cost \( R_\delta \) benefits F, it penalizes G in the renegotiation process. By contrast, the larger the guaranteed debt \( D^{rn}_{i,\tau} \), the lower the expected payoff for either party, the weaker the incentives to renegotiate the contract.

### 6 Implementation of the full-commitment allocation under limited commitment

We now suppose that, at time 0, G and F sign a contract that precisely stipulates the quantity-and-transfer allocation that would be optimal in the full-commitment framework. We denote this contract \( \Psi \), for the sake of shortness. We look for the conditions under which \( \Psi \) is implemented in the limited-commitment environment.
6.1 Additional constraints

We begin by identifying the additional constraints to be met under limited commitment. First of all, given the realized state \( i \in \{ l, h \} \), and conditional on having renegotiated \( \Psi \) at some instant \( \tau \in (0, T) \), neither F nor G should be willing to renegotiate again at some date \( \tau' \in (\tau, T) \). This is the case if and only if, for all \( \tau' \in (\tau, T) \):

\[
\Pi_{i,\tau}^{rn} \geq e^{-r(\tau'-\tau)}\Pi_{i,\tau'}^{rn} \quad (22a)
\]
\[
V_{i,\tau}^{rn} \geq e^{-r(\tau'-\tau)}V_{i,\tau'}^{rn} \quad (22b)
\]

Second, conditional on F truthtelling about the observed state \( i \in \{ l, h \} \), no party should have any incentive to either renegotiate or stop the relationship, once the execution of \( \Psi \) has begun. In formal terms, this requires that, for all \( \tau \in (0, T) \):

\[
\Pi_{i,\tau}^* (z) \geq \max \{ 0; \Pi_{i,\tau}^{rn} \} \quad (23a)
\]
\[
V_{i,\tau}^* (z) \geq \max \{ V_{i,\tau}^{rp}, V_{i,\tau}^{rn} \}. \quad (23b)
\]

Third, F should not be tempted to lie about its marginal cost \( i \in \{ l, h \} \) at time 0, even in the event that some party would renegade at date \( \tau \in (0, T) \). Denote \( \Pi_{i,\tau}^{rn} \) the stream of profits that F obtains in state \( i \), discounted at time \( \tau \), in the event that the initial contract is reneged at \( \tau \). Then, truthtelling in either state requires that, for all \( \tau \in (0, T) \):

\[
\Pi_{i,0}^* (z) \geq \int_0^\tau (\pi_{i,x}^* + \Delta \theta q_h^*) e^{-rx} dx + \max \{ 0; \Pi_{i,\tau}^{rn} \} \quad (24a)
\]
\[
\Pi_{h,0}^* (z) \geq \int_0^\tau (\pi_{i,x}^* - \Delta \theta q_h^*) e^{-rx} dx + \max \{ 0; \Pi_{h,\tau}^{rn} \}. \quad (24b)
\]

It turns out that (24a) and (24b) are satisfied as long as (23a) holds true jointly with (11a) and (11b), respectively (see Appendix C.2). We can thus neglect (24a) and (24b) and concentrate on the remaining constraints.

We first investigate the circumstances under which (22a) and (22b) are satisfied. Let

\[
\hat{D}_{i,\tau/\tau'}^{rn} \equiv \int_{\tau'}^T g_{i,\tau}^{rn} e^{-r(x-\tau')} dx
\]

denote the value, at instant \( \tau' \), of the debt guaranteed in state \( i \) under the contract renegotiated at \( \tau < \tau' \).

**Lemma 1** Suppose that, in state \( i \in \{ l, h \} \), \( \Psi \) is renegotiated at some time \( \tau \in (0, T) \). Then,
(22a) and (22b) are jointly satisfied if and only if
\[
D_{i,\tau}^{rn} - \tilde{D}_{i,\tau/\tau'}^{rn} \geq \frac{1}{1 + \lambda} \max \left\{ \left( R_{\delta'} - R_{\delta} \frac{1 - e^{-r_{\delta'}}}{1 - e^{-r_{\delta}}} \right); \left( R_{\delta} \frac{1 - e^{-r_{\delta}}}{1 - e^{-r_{\delta}}} - R_{\delta'} \right) \frac{1 - \alpha}{\alpha} \right\} \quad \text{(25)}
\]
where $\delta = T - \tau$ and $\delta' = T - \tau'$.

**Proof.** See Appendix C.1. ■

To prevent subsequent renegotiations, it is necessary to guarantee, in the hypothetical event of a later renegotiation, a debt that is at least as large as that guaranteed at earlier renegotiation. In particular, (25) means that the later debt guarantee must strictly exceed the earlier one whenever the operating profit that F would obtain if a new renegotiation were to take place between instant $\tau = T - \delta$ and instant $\tau + dt$ without changing the guarantee, either increases or decreases i.e., $\frac{d}{d\delta} \left( \frac{R_s}{1 - e^{-r_{\delta}}} \right) \neq 0$. Specifically, when $\frac{d}{d\delta} \left( \frac{R_s}{1 - e^{-r_{\delta}}} \right) > 0$, with constant guarantee, F would have something to gain from further renegotiation. When $\frac{d}{d\delta} \left( \frac{R_s}{1 - e^{-r_{\delta}}} \right) < 0$, it would be G to benefit from further renegotiation. In either situation, raising enough the debt guarantee at each later renegotiation discourages parties from reneging. This can be viewed from (21a) and (21b), according to which the return from renegotiation decreases with the debt guarantee for both F and G.

We next come to constraints (23a) and (23b). To investigate the circumstances under which they are satisfied, one should consider that, after some party reneges on $\Psi$, two situations are possible. First, there is no room for renegotiation and G replaces F. Second, there is room for renegotiation and a new agreement is achieved. The following lemma specifies the condition under which the former situation arises, rather than the latter.

**Lemma 2** Suppose that, in state $i \in \{l, h\}$, some party reneges on $\Psi$ at some time $\tau \in (0, T)$ and that (25) holds. Then, replacement takes place, rather than renegotiation, if and only if
\[
D_{i,\tau}^{rn} \geq \frac{R_s}{1 + \lambda}. \quad \text{(26)}
\]

Anticipating that, conditional on some party reneging on $\Psi$, renegotiation will take place if and only if (26) is violated, the incentives to renege on $\Psi$ will ultimately depend upon how much debt is guaranteed in the event of renegotiation.

One can show that preventing replacement imposes milder requirements, in terms of private funds, as compared to preventing renegotiation. This means that $\Psi$ is more easily effected if replacement, rather than renegotiation, is anticipated after any contractual renege. Because of this, in the next lemma, we provide the conditions under which reneging on $\Psi$ is prevented.
when the guarantee is large enough that replacement is anticipated. These conditions will then allow us to draw conclusions on the implementability of $\Psi$.

**Lemma 3** Suppose that, in state $i \in \{l, h\}$, at some given $\tau \in (0, T)$, (25) and (26) hold. Then, when $i = l$, (23a) is satisfied; when $i = h$, it is if and only if

$$M \geq \nu_1 \Delta \theta z \frac{1 - e^{-rT}}{r}. \quad (27)$$

Moreover, (23b) is satisfied for $i = l$ and $i = h$ if and only if, respectively,

$$D_{l, \tau} \leq \frac{R_\delta}{1 + \lambda} - \left( \frac{Mr}{1 - e^{-rT}} + (1 - \nu_1) \Delta \theta z \right) \frac{1 - e^{-r\delta}}{r}, \quad (28a)$$

$$D_{h, \tau} \leq \frac{R_\delta}{1 + \lambda} - \left( \frac{Mr}{1 - e^{-rT}} - \nu_1 \Delta \theta z \right) \frac{1 - e^{-r\delta}}{r}, \quad (28b)$$

together with

$$M \leq \left[ \frac{r}{1 - e^{-r\delta}} \frac{R_\delta}{1 + \lambda} - (1 - \nu_1) \Delta \theta z \right] \frac{1 - e^{-rT}}{r}. \quad (29)$$

**Proof.** See Appendix C.3. ■

Condition (27) means that $F$ cannot be prevented from reneging on $\Psi$ at some time $\tau$, unless it is required to invest an amount of own funds sufficiently large to ensure that the profit to be obtained at each instant by operating till the termination date $T$, given the sharing rule $z$, is non-negative even in the bad state. In other words, $M$ is to be raised enough *ex ante* to prevent losses *ex post* and thus lessen the incentive of $F$ to renegade. While this is necessary to remove the temptation of $F$ to renegade on $\Psi$, it is however not sufficient. Provided the reputation loss of $G$ is important enough, there might still be more to reap by breaking down the agreement and threatening $G$ to abandon the project. To make this strategy unattractive, $G$ is to pledge sufficiently high guarantees for the debt of $F$, in the event that $\Psi$ is renegotiated (*i.e.* (26) must hold). This result, which might look counter-intuitive, is in fact easily explained. Guaranteeing (a part of) the debt involves abating the compensation of $F$ by the amount that is to accrue to the creditor. Given the reputation loss of $G$, the more debt is guaranteed, the less the benefit for $F$ from reneging on $\Psi$. Therefore, when the guarantee is fixed larger than the replacement cost (deflated by the shadow cost of public funds), renegotiation would yield a loss in either state, hence $F$ would prefer to quit the project.

Condition (29) means that $G$ cannot be prevented from reneging on $\Psi$ at some time $\tau$, unless $F$ is required to invest an amount of own funds sufficiently small to ensure that, even in the good state, the profit that $G$ awards to $F$ at each instant till the termination date $T$, as inflated by the shadow cost of public funds, does not exceed the reputation loss that $G$ would
incur if it were to replace F. In other words, $M$ should be kept low enough ex ante to prevent comparative benefits ex post and, thus, to weaken the incentive of G to renege. However, just as for F, the requirement on $M$ is necessary but not sufficient to remove the temptation of G to renege on $\Psi$. Indeed, provided the reputation loss is not too important, G can still gain from either renegotiating or replacing F. To make these options unattractive, G should tie its hands (through the third party) on a double ground. First, it should guarantee a sufficiently large amount of debt, in the event that $\Psi$ is renegotiated (i.e., (26) should hold). Second, it should guarantee sufficiently little debt, in the event that $\Psi$ is fully executed (as from (28a) and (28b)). This result is explained as follows. On the one hand, while replacing F yields a loss of reputation, it allows G to save on the debt, which is guaranteed only as long as F remains in the project. Given the reputation loss, the more debt is guaranteed when renegotiating, the larger the benefit from replacing F, the less attractive renegotiation as compared to replacement. On the other hand, the larger the debt guaranteed in $\Psi$, the more costly $\Psi$ is to G (as from (19), $i = l, h$), hence the less convenient with respect to the alternative options. Therefore, when the guarantee in the renegotiation scenario is fixed larger than the reputation loss (deflated by the shadow cost of public funds), renegotiation would be so costly to G that its payoff would be negative whatever the production cost of the firm. G would definitely prefer to replace F. For $\Psi$ to be more attractive than replacement, the debt burden under $\Psi$ should be not too important, in turn.

6.2 Implementation of $\Psi$

In Lemma 3 we enlisted the weakest conditions under which break-down of the relationship between G and F is prevented. We can now be based on that list to investigate the requirements that $\Psi$ is to meet to be effected in the limited-commitment environment. As a first step, we identify the set of exogenous conditions under which this outcome is achieved.

**Proposition 2** $\Psi$ is implementable if and only if $\exists z \in Z, T \in [T(z), +\infty)$ such that (15) holds together with

$$R_\delta \geq (1 + \lambda) \Delta \theta z \frac{1 - e^{-r \delta}}{r}, \quad \forall \delta \in (0, T),$$

and, additionally,

$$E \geq \nu \psi \Delta \nu$$

$$C > 0.$$  

**Proof.** See Appendix C.4. ■
According to (30), $\Psi$ is beyond reach under limited commitment unless the replacement cost that $G$ would bear at time $\tau$ is at least as large as the present value of the *ex-post* cumulated return (as inflated by the shadow cost of public funds) that $F$ would be obtain in state $h$ if it were to remain in $\Psi$ for the subsequent $(T-\tau)$ periods. This is explained as follows. For any given $T$, when $R_\delta$ is low, $G$ has an interest in reneging in the state in which $F$ would be rewarded (state $l$). In so doing, $G$ would avoid to give large operating profits to $F$ and, at the same time, it would appropriate a facility that can be operated at a low cost. To remove this temptation, $F$ should be required to put fewer own funds on the table up-front, so that the good-state operating profit in $\Psi$ is reduced. However, setting $M$ small is not desirable either because, in that case, $F$ has an incentive to reneg on the bad state, in turn, so as not to incur a low operating return. Unless $R_\delta$ is high enough, the own funds of the firm cannot be used to reconcile these two purposes that they serve, and $\Psi$ is not enforceable. We have illustrated this point taking $T$ as given. In a moment, we will investigate how duration should be chosen for (30) to be satisfied, provided both the replacement cost and the firm’s operating profit depend upon this choice.

For $\Psi$ to be enforceable, (31) and (32) must hold as well. For $F$ to be able to invest as much own funds as it is necessary to effect $\Psi$, its pockets must be sufficiently deep to begin with. Specifically, according to condition (31), $F$ must be able to contribute at least as much as the expected return that it would get if duration were contained to the lowest admissible number of periods ($T = T(z)$). Lastly, condition (32) evidences that $F$ needs to have access to the credit market and be able to get a loan within the specific relationship with $G$, under the conditions stipulated in $\Psi$.

Overall, Proposition 2 conveys a strong message. For the contract that stipulates the full-commitment allocation to be effected, private capital must be available to run the project, both in the form of own funds of the firm that is delegated to perform the activity, and in the form of outside financing. The reason is that involving private funds of a diverse nature provides two different commitment devices, which are both functional to refrain $F$ from reneging. On one side, involving own funds reinforces the firm’s willingness to remain in the relationship as at least a part of them would no longer be recovered by abandoning the project in favour of the replacement payoff. On the other side, forcing $F$ to take a loan hardens its budget constraint through the guarantees offered to the creditor, as illustrated previously. Indeed, by reducing the benefit that $G$ would obtain from renegotiation, these guarantees lower the surplus that $F$ can extract from $G$ in the event of renegotiation. As a result, $F$ finds it less attractive to renge on the contract opportunistically anticipating profitable renegotiation.

Let us now discuss the impact that the need to satisfy condition (30) has on the choice of the contract duration. Depending upon the properties of the function $R_\delta$, it may impose restrictions
on $T$, in addition to (16). The next corollary identifies those restrictions and assesses when they can be met together with (16).

**Corollary 1** Take (31) and (32) to hold and define

$$
\tilde{T}(z, E) \equiv \frac{1}{r} \ln \frac{\nu_1 \Delta \theta z}{\nu_1 \Delta \theta z - r E} < \infty.
$$

First suppose that

$$
R'_\delta \geq (1 + \lambda) \Delta \theta z e^{-r \delta}, \forall \delta \in (0, T), \forall T \in [T(z), \infty).
$$

Then, there exist values of $T$ for which $\Psi$ is implemented:

$$
T \in [T(z), \tilde{T}(z, E)] \text{ when } E \in \left[ \nu_1 \frac{\psi}{\Delta \nu}, \nu_1 \frac{\Delta \theta z}{r} \right],
$$

$$
T \in [T(z), \infty) \text{ when } E \geq \nu_1 \frac{\Delta \theta z}{r}.
$$

Next suppose that

$$
R'_\delta < (1 + \lambda) \Delta \theta z e^{-r \delta}, \forall \delta \in (0, T), \forall T \in [T(z), \infty),
$$

and define

$$
\overline{T}(z) \equiv \frac{1}{r} \ln \frac{\Delta \theta z}{\Delta \theta z - r R_{\overline{T}(z)} \psi / (1 + \lambda)} < \infty.
$$

Then, there exist values of $T$ for which $\Psi$ is implemented if and only if $R_{\overline{T}(z)} \geq (1 + \lambda) \psi / \Delta \nu :

$$
T \in [T(z), \min \left\{ \tilde{T}(z, E); \overline{T}(z) \right\}] \text{ when } E \in \left[ \nu_1 \frac{\psi}{\Delta \nu}, \nu_1 \frac{\Delta \theta z}{r} \right],
$$

$$
T \in [T(z), \overline{T}(z)] \text{ when } E \geq \nu_1 \frac{\Delta \theta z}{r}.
$$

**Proof.** See Appendix C.5. ■

The corollary identifies two possible scenarios.

The first scenario arises when $R'_\delta$ increases fast enough with $\delta$ to satisfy (34) for all $\delta \in (0, T), T \in [T(z), \infty)$, given the sharing rule $z$. Then, $R'_\delta$ is large enough to meet (30) for all feasible $T$. Hence, whether some restriction is imposed on the choice of contract duration, it depends upon "how rich" the firm is. When the own funds that the private partner could devote to the project do not even cover the expected return that it would get in the bad state if $T = \infty$, the contract cannot last more than $\tilde{T}(z, E)$ periods, a threshold that is tighter the lower the endowment. To see why this is the case, recall that, the larger $M$, the higher the operating
profit that \( F \) obtains in \( \Psi \) at each instant over the execution period, the weaker the incentive of \( F \) to quit the project. When \( E \) is low, \( G \) can no longer rely on \( M \) to discourage \( F \) from reneging on the contract. The alternative option is to shorten the relationship. This hardens the budget constraint of \( F \) and its instantaneous payoff is to be raised. As a result, \( F \) finds it less attractive to renege and the contract remains in place. By contrast, for a sufficiently rich firm, this restriction does not arise and \( T \) is to be chosen in compliance with (16) only.

The second scenario arises when the rate of increase of \( R_\delta \) is sufficiently low to satisfy (35) for all \( \delta \in (0, T) \), \( T \in [T(z), \infty) \). In this case, either (30) induces further restrictions on its time length or it prevents \( G \) from implementing \( \Psi \) at all. Necessary and sufficient for enforceability of \( \Psi \) is that the cost of replacement for a number \( T(z) \) of residual periods, as deflated by the shadow cost of public funds, does not fall below the return that \( \Psi \) assigns in state \( h \) when it lasts forever. Once this is ensured, the requirements on \( T \) reflect how the cost of replacement compares with the firm’s endowment. With a poor firm, the relevant threshold is \( T(z; E) \) as long as \( R_{T(z)} \geq (1 + \lambda) E \), in which case the maximum admissible duration is still driven by the endowment effect triggered by (31). The relevant threshold becomes \( \bar{T}(z) \) when, conversely, \( R_{\bar{T}(z)} < (1 + \lambda) E \). Then, (30) is so hard to satisfy that the maximum admissible duration is decreased even further than the poorness of the firm would require. Of course, as the firm becomes wealthier, \( T(z) \) remains the relevant threshold, provided the endowment effect is then lessened.

Importantly, in the first scenario, restrictions on the choice of \( T \), apart from (16), solely reflects the magnitude of the firm’s endowment. It follows that the sharing rule that most facilitates the implementation of \( \Psi \) is the one that makes (31) most relaxed \( i.e., z = q_h^* \). In the second scenario, admissible durations are determined not only by the endowment effect but also by the behaviour of the function \( R_\delta \). Because of this, the most convenient sharing rule cannot be univocally identified, unless specific cases are considered. Example 1 below illustrates a situation in which the best sharing rule in the second scenario is \( q_l^* \).

**Example 1** Take \( R_\delta = (1 - ae^{-r_\delta}) / r \), where

\[
\begin{align*}
    a &> (1 + \lambda) \Delta \theta q_h^* \\
    a &< 1 < (1 + \lambda) \Delta \theta q_l^*.
\end{align*}
\]

As \( \lim_{\delta \to 0} \left( (1 - ae^{-r_\delta}) / r \right) = 1 - a \), this function satisfies the condition that \( \lim_{\delta \to 0} R_\delta > 0 \). For simplicity, further take \( E \geq \nu_1 \Delta \theta q_l^* / r \) so that (33) holds and we do not need to be concerned with it. Define \( \hat{\delta} \equiv \frac{1}{r} \ln \left( \frac{(1 + \lambda) \Delta \theta - a}{1 + \lambda} \right) \) the unique value of \( \delta \) for which (30) holds as an equality. Taking first \( z = q_h^* \), \( \hat{\delta} < 0 \) and so (30) is strictly satisfied for all \( \delta \in (0, T) \) and \( T \in (0, \infty) \). Taking next \( z = q_l^* \), \( \hat{\delta} > 0 \) and (30) is satisfied if and only if \( \delta \in \left( 0, \min\{\hat{\delta}, T\} \right) \), for any given
In this case, the largest feasible interval for $\delta$ is attained when $T = \hat{T}$ i.e., when the upper extreme of the interval is $T(q_i^*)$ as defined in (36). From (16), it must be $T \geq T(q_i^*)$. Then, $\Psi$ is implementable only if the interval $[T(q_i^*), T(q_i^*)]$ exists and $T$ is drawn from this interval. Plugging $R_\delta = (1 - ae^{-rT})/r$ into (36), we find that the interval $[T(q_i^*), T(q_i^*)]$ exists if and only if $a \leq \Delta \theta q_i^{\Delta \nu - r(1 + \lambda) \psi}$, which means that $R_T(q_i^*)$ is sufficiently high to satisfy the condition required in the second scenario of Corollary 1. Overall, (30) is not an issue when $z$ is chosen equal to $q_i^*$. When $z = q_i^*$, instead, (30) is satisfied only if the interval $[T(q_i^*), T(q_i^*)]$ exists. Actually, (37) and (38) mean that (34) holds for $z = q_i^*$ and (35) for $z = q_i^*$. Results are thus explained in the light of Corollary 1.

An important point emerges from the finding in Corollary 1. When the replacement cost is not large enough in all periods till the termination date, contractual parties anticipate that, at some point, it will become convenient for one of them to interrupt the relationship and will not survive. Then, the only way to make the agreement viable is to reduce the length of the contract. Under a short-term contract, the budget constraint of $F$ is hardened and $F$ receives a relatively large instantaneous operating profit. This means that $G$ can require $F$ to invest a relatively small amount of own funds without triggering the incentive of $F$ to renege. In turn, this lessens the incentives of $G$ to renege as, with a low investment of the firm’s own funds, there is little that $G$ can gain from replacement. Therefore, duration becomes the very instrument through which $G$ can persuade the partner of its intention not to renege and, at the same time, keep the partner in the initial agreement.

Once the conditions that are necessary and sufficient for $\Psi$ to be implementable are identified, we can establish how the mix of private funds must exactly be set in $\Psi$, given the duration agreed upon, for the contract to be enforced in the limited-commitment framework.

**Corollary 2** Suppose that (30), (31) and (32) hold. Then, $\Psi$ is implemented by choosing $M$ and $C$ such that, for all $\delta \in (0, T)$,

$$
\nu_1 \Delta \theta z \frac{1 - e^{-rT}}{r} \leq M \leq \frac{R_\delta}{1 + \lambda} \frac{1 - e^{-rT}}{1 - e^{-r\delta}} - (1 - \nu_1) \Delta \theta z \frac{1 - e^{-rT}}{r},
$$

(39)

together with

$$
M + C \leq \frac{R_T}{1 + \lambda}.
$$

(40)

**Proof.** See Appendix C.6. ■

This corollary channels two main lessons. First, the amount of own funds that $F$ must be required to inject up-front should be neither too small nor too large. Second, the total amount of private resources (own funds and loan) to be invested in the project cannot be too large, in turn. Let us discuss these findings in details.
To begin with, condition (39) imposes a lower bound on the amount of own funds, which reflects condition (31). As aforementioned, the temptation of F to renege on the contract is not removed unless it invests enough own money. By executing $\Psi$, F obtains an operating profit (namely, $\Pi^*_r(z)$, $i = l, h$) that is designed to increase with $M$ so that the investment made up-front is recovered. On the opposite, by reneging on $\Psi$, F switches to a payoff that is independent on the amount of own funds put on the table initially. Indeed, at that stage, the initial contribution is no longer considered; only costs and benefits deriving from future actions come to matter. Under this circumstance, the temptation of F to renege on $\Psi$ is removed as $M$ is raised enough.

Condition (39) imposes an upper bound also on the amount of own funds to be destined to the project. The same does condition (40) on the total amount of private resources. Appearance of these restrictions on $M$ and $M+C$ is related to the need of preventing G from reneging on $\Psi$. By replacing F, G would escape reimbursing private financiers, whether in total (if the contract is reneged immediately after operation has begun) or in part (if the contract is reneged at a later stage). For this option not to be attractive for G, the amount of private funds used to run the project should not be unlimited. Importantly, this implies that it might be desirable not to fund the investment entirely with private resources ($M + C < I + \psi$). Then, G ought to provide an up-front payment $t_0 > 0$ to complete the initial investment. That is, the project should be run with a mix of private and public funds. Observe however that, as private resources must be such that $M + C > \nu_1 \Delta \theta z (1 - e^{-rT}) / r$, it might even be desirable that F makes an up-front transfer to G ($t_0 < 0$), which is to be interpreted as a fee for being awarded the contract.

Two final points are worth making. The first concerns the impact of the shadow cost of public funds on results. As $\lambda$ increases, the upper bounds on private resources (in (39) and (40)), the lower bounds on $R_\delta$ (in (30)) and that on the rate of increase of $R_\delta$ (in (35)), become all more severe. Furthermore, the longest duration that is admissible under (35), namely $\mathcal{T}(z)$, reduces. This means that a raise in $\lambda$ tightens all restrictions on the implementability of $\Psi$ that ensue from the necessity to remove the incentives of G to renege. The reason is rather intuitive. The more costly it is to collect/transfer public funds (and to distort price), the less prone G is to compensate F (and, indirectly, the lender) during the operation phase, the harder it becomes to motivate G to remain in the relationship with F. By contrast, the requirements that reflect the incentives of F are not affected by the magnitude of $\lambda$, provided the payoff of F would depend upon it only in the event of renegotiation.\footnote{This is immediately checked by looking at (31), (32), (33), and the lower bound in (39).}
7 State-dependent duration

So far we have focused on a fixed-term contract \(i.e.,\) on a contract that has the same duration whatever the realized operating cost. Given the crucial role that, according to our results, the length of the contract plays in the implementation of the optimal allocation, we now investigate whether there can be any improvement in the achievements of the government when duration is allowed to vary with the state of nature. A contract with this characteristics is referred to as the flexible-term contract.

Denote \(T_l > 0\) the duration in state \(l\) and \(T_h > 0\) in state \(h\): Further let \(T_i, i \in \{l, h\}\) and \(T \in (0, T_i)\). Then, we can rewrite the profits at time \(\tau\) in the good and bad state as

\[
\Pi_{l, \tau} = \left( M + (1 - \nu_1) \int_0^{T_i} \Delta \theta \alpha_i e^{-r x} dx \right) \frac{1 - e^{-rT_l}}{1 - e^{-rT_i}}
\]

\[
\Pi_{h, \tau} = \left( M - \nu_1 \int_0^{T_i} \Delta \theta \alpha_i e^{-r x} dx \right) \frac{1 - e^{-rT_h}}{1 - e^{-rT_i}}
\]

where \(\alpha_i > 0\) indicates the sharing rule in this framework, such that (11a) and (11b) are satisfied and (13) is binding. This formulation of the profits helps us emphasize that, with state-dependent duration, the range of feasible values of the sharing rule is possibly enlarged, as compared to the fixed-term setting. Indeed, the feasible range of \(\alpha_i\) is \(A_l \equiv \left[q_h^i \frac{1 - e^{-rT_h}}{1 - e^{-rT_i}}, q_l^i\right]\) and that of \(\alpha_h\) is \(A_h \equiv \left[q_h, q_l^i \frac{1 - e^{-rT_i}}{1 - e^{-rT_h}}\right]\). Hence, \(A_l \supset Z\) for all \(i\), as long as \(T_h < T_i\). Fixing \(T_h\) below \(T_i\) has an important implication: the wedge between good- and bad-state profit is either increased or decreased with respect to the compensation scheme in \(\Psi\). This is beneficial to \(G\) in that \(G\) can raise or reduce the wedge at its best convenience, depending upon the difficulties that it faces at implementing the optimal allocation.

Prior to stating the new results, it is however useful to devote a few words to the flexible-term contract in the full-commitment framework. Not surprisingly, the highest payoff, which is still \(E_i [W_i^*]\), is achievable if and only if, for any given \(\alpha_i \in A_i\), duration is chosen such that \(T_i \geq T_\tau (\alpha_i)\). Again, the mix of financing sources used in the project does not matter. Further remark that, once the sharing rule \(\alpha_i\) is adopted, the profit distribution is independent of the contract duration associated with the state of nature \(j \neq i\) that the sharing rule does not refer to. From now on, we denote \(\Upsilon_i, i = l, h\), the flexible-term contract that stipulates the optimal (full-commitment) allocation with sharing rule \(\alpha_i\).

We begin by identifying the conditions under which \(\Upsilon_i\) is implementable, whereas \(\Psi\) is not, in the limited-commitment framework.

\[27\text{With } T_h < T_i, q_h^i \frac{1 - e^{-rT_h}}{1 - e^{-rT_i}} < q_h^i \text{ and } q_l^i \frac{1 - e^{-rT_i}}{1 - e^{-rT_h}} > q_l^i.\] See Appendix D for details about the construction of the sharing rule \(\alpha_i\).
**Proposition 3** Suppose that (30) does not hold for \( z = q_h^* \) and \( T \geq \overline{T}(q_h^*) \). Then, \( \exists \alpha_l \in \left[ q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}}, q_h^* \right] \) for which \( \Upsilon_l \) is implementable if and only if \( T_l > T_h \), together with

\[
\frac{\Delta \theta \alpha_l}{r} > \frac{\psi}{\Delta \nu} \tag{41}
\]

\[
R_{\delta(l)} \geq (1 + \lambda) \frac{\Delta \theta \alpha_l}{r} \frac{1 - e^{-r\delta(l)}}{1 - e^{-rT_l}}, \quad \forall \delta(l) \in (0, T_l), \tag{42}
\]

and with (31) and (32).

**Proof.** See Appendix D. ■

The sharing rule \( \alpha_l \) is useful to G when the replacement cost is not very large. As we know, in that case, under \( \Psi \), it is not possible to remove at once the temptation of G to renge in state \( l \) to expropriate the private investment and the temptation of F to renge in state \( h \) to escape low returns. This might be feasible under \( \Upsilon_l \), instead, as condition (42) is less stringent than (30). The compensation scheme associated with \( \alpha_l \) is such that, at each instant of the operation phase, the profit distribution is less spread than in \( \Psi \). This facilitates the composition of the two commitment problems so that the requirement on the replacement cost is relaxed. Example 2 below illustrates this finding.

**Example 2** Take again the case of Example 1, except that now

\[
a \prec (1 + \lambda) \Delta \theta q_h^*
\]

so that, when \( z = q_h^* \), we are in the second scenario of Corollary 1. Then, implementation of \( \Psi \) requires existence of the interval \([T(q_h^*), \overline{T}(q_h^*)]\), which is tantamount to having \( a \leq \Delta \theta q_h^* \frac{\Delta \nu - r(1 + \lambda)\psi}{\Delta \theta q_h^* \Delta \nu - r\psi} \). With state-dependent duration, if \( T_l \) and \( T_h \) are chosen such that \( T_l \geq \overline{T}(\alpha_l) \) and \( q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}} \leq \alpha_l \) for \( \alpha_l = (a/ (1 + \lambda) \Delta \theta) \), then \( \Upsilon_l \) is implemented despite that the previous condition on \( a \) is violated.

While the sharing rule \( \alpha_l \) helps G circumvent the commitment difficulties that arise when the replacement cost is low, the sharing rule \( \alpha_h \) helps G lessen the moral-hazard problem. More precisely, when inducing F to exert effort is especially hard (\( \psi \) is large), G is to exacerbate (rather than contain) the spread in the profit distribution so as to transfer more risk to the firm. The compensation scheme associated with \( \alpha_h \) enables G to reach this outcome.

**Proposition 4** Suppose that (16) does not hold for \( z = q_h^* \) and \( T \) chosen so as to satisfy (30).
Then, \( \exists \alpha_h \in \left( q_h^*, \frac{1-e^{-rT_h}}{1-e^{-rT_i}} q^*_i \right) \) for which \( Y_h \) is implementable if and only if \( T_l > T_h \), together with

\[
\frac{\Delta \theta \alpha_h}{r} > \frac{\psi}{\Delta \nu} \quad \text{and} \quad R_{\delta(l)} \geq (1 + \lambda) \Delta \theta \alpha_h \frac{1 - e^{-rT_h}}{r} \frac{1 - e^{-r\delta(l)}}{1 - e^{-rT_i}}, \quad \forall \delta(l) \in (0, T_l),
\]

and with (31) and (32).

**Proof.** See Appendix D. ■

The requirement expressed by (45), which is here the counterpart of (30) with sharing rule \( z \), and of (42) with sharing rule \( \alpha_l \), follows from the way in which the sharing rule \( \alpha_i, i \in \{l, h\}, \) is constructed. Recall that, in the fixed-term framework, (30) is necessary to make sure that neither \( G \) reneges in the good state nor \( F \) reneges in the bad state and that, whether this is the case or not, it also depends upon how \( T \) is set. Analogous role (42) plays in the state-dependent framework with sharing rule \( \alpha_i \). When the contract length is differentiated across states \( (T_l \neq T_h) \) and the sharing rule \( \alpha_h \) (rather than \( \alpha_l \)) is adopted, the loss that is inflicted to \( F \) in the bad state is contingent on \( T_h \) (rather than \( T_l \)), whereas the gain that \( G \) would obtain by reneging in the good state is contingent on the contract duration in the reneging state \( T_l \). Condition (45), which contains both \( T_l \) and \( T_h \) (rather than \( T_l \) only), reflects precisely this circumstance.

Proposition 3 and 4 clarify the conditions under which \( Y_i, i \in \{l, h\}, \) is enforceable in a limited-commitment environment. We are now left with specifying how \( M \) and \( C \) should be set to actually effect \( Y_i \). This final result is the counterpart of Corollary (2) in a context of state-dependent duration.

**Corollary 3** Suppose that \( \exists \alpha_i \in A_i, i \in \{l, h\}, \) for which \( Y_i \) is implementable. Then, implementation is made by setting \( M \) and \( C \) are such that, \( \forall \delta(i) \in (0, T_i), \)

\[
\nu_1 \Delta \theta \alpha_i \frac{1 - e^{-rT_i}}{r} \leq M \leq \min \left\{ \frac{R_{\delta(h)}}{1 + \lambda} \frac{1 - e^{-rT_h}}{1 - e^{-r(T_h - \gamma)}} + \nu_1 \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx; \right. \\
\left. \frac{R_{\delta(l)}}{1 + \lambda} \frac{1 - e^{-rT_l}}{1 - e^{-r(T_l - \gamma)}} - (1 - \nu_1) \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx \right\}
\]

\[
together\ with \quad M + C \leq \frac{E_i [R_{T_i}]}{1 + \lambda}.
\]

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8 Concluding remarks

Resting on a relatively simple model, we drew a few important predictions about public-private contracting in limited-commitment environments. While it is rather intuitive that, under limited commitment, the financing structure of the project can be used as a commitment device to implement the contract that stipulates the optimal (full-commitment) allocation, we assessed how exactly private funds, drawn from different sources, should be mixed to achieve this objective, given the exogenous conditions that come to matter in the (potential) renegotiation process. More than that, the very contribution of our study was to show that an appropriate financing structure of the project cannot be designed sparing reference to the duration of the contract, provided early break-down of the relationship with the private firm is not as costly for the public partner as late break-down is.

One important prediction of our model was that, for the optimal allocation to be implemented at equilibrium under limited commitment, it is essential that the debt guarantees be set in an appropriate manner, despite that their value is only relevant for the out-of-equilibrium payoffs so that one would not expect these guarantees to be actually used in practice. In particular, we established that the funds to be borrowed by the firm on the credit market should be guaranteed conditionally on the government-firm relationship not being interrupted in advance, whether the contract is maintained as originally signed till the termination date or renegotiated during the operation phase. Concerning this prediction, two observations are in order. First, in several practical instances, loans are guaranteed even in that event that renegotiation fails and the relationship is broken down (as an illustration, recall the Metronet case). This behaviour is based on the argument that it would be impossible to attract outside financiers otherwise. Nonetheless, the result that, at equilibrium, the relationship is preserved and the loan reimbursed, casts doubts on the validity of this argument, and rather suggests that providing conditional guarantees does not compromise the ability to raise external funds. Second, under limited commitment, it is not desirable to have lenders actively involved in renegotiation. In our model, if the lender could say a word when parties come back to the contracting table, it would accept a partial repayment. Obviously, this would be preferable to being denied any reimbursement in the event of early interruption of the relationship between the government and the firm. But then out-of-equilibrium guarantees could no longer be used as a commitment device to keep that relationship in place. Hence, for the committing role of guarantees to be preserved, lenders should remain "passive." Importantly, this is beneficial to lenders themselves insofar as credits are entirely recovered.
References


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A Proof of (14a) and (14b)

Based on (11a) and (11b), we see that $\exists \varepsilon_1 \geq 0, \varepsilon_2 \geq 0$ such that

$$\Pi_{t,0} = \Pi_{h,0} + \int_0^T \Delta \theta (q_h^* + \varepsilon_1) e^{-rx} dx$$  \hspace{1cm} (48)

$$\Pi_{h,0} = \Pi_{t,0} - \int_0^T \Delta \theta (q_t^* - \varepsilon_2) e^{-rx} dx.$$  \hspace{1cm} (49)

Using (48) and considering that (13) is binding, we can write

$$\Pi_{t,0} = M + (1 - \nu_1) \int_0^T \Delta \theta (q_h^* + \varepsilon_1) e^{-rx} dx$$

$$\Pi_{h,0} = M - \nu_1 \int_0^T \Delta \theta (q_t^* + \varepsilon_1) e^{-rx} dx.$$  \hspace{1cm} (50)

Replacing the above expression of $\Pi_{t,0}$ into (48), we further obtain

$$\Pi_{h,0} = M + (1 - \nu_1) \int_0^T \Delta \theta (q_h^* + \varepsilon_1) e^{-rx} dx - \int_0^T \Delta \theta (q_t^* - \varepsilon_2) e^{-rx} dx.$$
As the two expressions of \( \Pi_{h,0} \) must be equivalent, it must be the case that
\[
\int_0^T \Delta \theta (q_h^* + \varepsilon_1) e^{-rx} dx = \int_0^T \Delta \theta (q_0^* - \varepsilon_2) e^{-rx} dx,
\]
which is equivalent to
\[
q_h^* + \varepsilon_1 = q_0^* - \varepsilon_2.
\]
Setting \( z \equiv q_h^* + \varepsilon_1 = q_0^* - \varepsilon_2 \), we obtain the profits in the main text. Further using \( \varepsilon_1 \geq 0 \) and \( \varepsilon_2 \geq 0 \) in the above equality, the range of feasible values of \( z \) is derived as well.

Consider now (12). Using the profits previously found, we can write
\[
\Pi_{l,0}(z) - \Pi_{h,0}(z) = \int_0^T \Delta \theta z e^{-rx} dx,
\]
showing that (12) is satisfied if and only if \( \int_0^T \Delta \theta z e^{-rx} dx \geq \frac{\psi}{2\delta} \). This is equivalent to \( T \geq T(z) \), with \( T(z) \) as defined in (16).

**B No incentive to renegotiate \( T \)**

Suppose that \( G \) makes the offer and that it proposes to end the contract at some \( T^G > \tau \), \( T^G \neq T \). If \( T^G > T \), then \( G \) proposes a quantity \( q_i^* \) and a transfer \( t_i^{G,1} \) for all \( x \in [\tau, T] \), as well as a quantity \( q_i^* \) and a transfer \( t_i^{G,2} \) for all \( x \in [T, T^G] \). To make \( F \) indifferent between renegotiation and replacement, the two transfers are to be set as follows:
\[
\begin{align*}
t_i^{G,1} &= \theta_i q_i^* + K - p(q_i^*) q_i^* + g_{i,\tau}^{rin}, \quad \forall x \in [\tau, T) \\
t_i^{G,2} &= \theta_i q_i^* + K - p(q_i^*) q_i^*, \quad \forall x \in [T, T^G].
\end{align*}
\]
The payoff of \( G \) from the renegotiated contract is given by
\[
\hat{V}_{i,\tau}^G = \int_\tau^{T^G} w(q_i^*) e^{-r(x-\tau)} dx - (1 + \lambda) \int_\tau^{T^G} g_{i,\tau}^{rin} e^{-r(x-\tau)} dx.
\]
If \( T^G < T \), then the proposal of \( G \) includes, apart from \( T^G \), the quantity \( q_i^{sb} \) and the transfer \( t_i^{G,1} \) for all \( x \in [\tau, T^G] \). Hence, \( G \) obtains the payoff \( \hat{V}_{i,\tau}^G \). The end date \( T^G \) is chosen so as to maximize
\[
W_{i,\tau} = \hat{V}_{i,\tau}^G + \int_{T^G}^{+\infty} w(q_i^*) e^{-r(y-T^G)} dy
= \int_\tau^{+\infty} w(q_i^*) e^{-r(x-\tau)} dx - (1 + \lambda) \int_\tau^{T^G} g_{i,\tau}^{rin} e^{-r(x-\tau)} dx,
\]
which is independent of \( T^G \).

Next suppose that \( F \) makes the offer and that it proposes to end the contract at some instant \( T^F > \tau \), \( T^F \neq T \). The offer includes a quantity \( q_i^* \) and a transfer \( t_i^{F,1} \) for all \( x \in [\tau, T^F] \). To
make G indifferent between the payoff under the renegotiated contract and the period thereafter and the return from immediate replacement, $t_i^{F,1}$ and $T^F$ must be set such that

$$
\int_\tau^{T^F} (S(q_i^F) - p(q_i^F) q_i^* - (1 + \lambda) t_i^* \) e^{-r(x-\tau)} \, dx + \int_\tau^\infty w(q_i^*) e^{-r(y-T^F)} \, dy
$$

$$
= \int_\tau^\infty w(q_i^*) e^{-r(y-\tau)} \, dx - R_\delta.
$$

Then, the transfer is

$$
t_i^{F,1} = \frac{1}{1 + \lambda} \left( S(q_i^F) - p(q_i^F) q_i^* - w(q_i^*) + \frac{rR_\delta}{1 - e^{-r(T^F-\tau)}} \right).
$$

The payoff of $F$ at $\tau$ is given by

$$
\Pi_{i,\tau}^F = \frac{R_\delta}{1 + \lambda} - \int_\tau^{T^F} g_{i,\tau}^{rn} e^{-r(x-\tau)} \, dx,
$$

and is thus independent of $T^F$.

C Implementation of $\Psi$  

C.1 Proof of Lemma 1  

We first rewrite the incentive constraints (22a) and (22b). Suppose that the current period is $\tau' \in (0, T)$ and that the contract has been renegotiated at $\tau < \tau', \tau \in (0, T)$. The payoff of $F$ and that of $G$ over the period $[\tau', T)$ under the contract renegotiated at $\tau$ and never again, are respectively written as

$$
\widehat{\Pi}_{i,\tau'/\tau}^{rn} = (1 - \alpha) \int_{\tau'}^{T} \frac{R_\delta}{1 + \lambda} \frac{r}{1 - e^{-r \delta}} e^{-r(x-\tau')} \, dx - \widehat{D}_{i,\tau'/\tau}^{rn},
$$

$$
\widehat{V}_{i,\tau'/\tau}^{rn} = \int_{\tau'}^{T} \left( w(q_i^{sb}) - (1 - \alpha) \frac{rR_\delta}{1 - e^{-r \delta}} \right) e^{-r(x-\tau')} \, dx - \alpha (1 + \lambda) \widehat{D}_{i,\tau'/\tau}^{rn}.
$$

If the contract is renegotiated at $\tau'$, then the discounted payoff of $F$ is $\Pi_{i,\tau'/\tau}^{rn}$, and that of $G$ is $V_{i,\tau'/\tau}^{rn}$. $F$ does not wish to renegotiate at $\tau'$ if and only if $\widehat{\Pi}_{i,\tau'/\tau}^{rn} \geq \Pi_{i,\tau'/\tau}^{rn}$, which is equivalent to (22a). $G$ does not wish to renegotiate at $\tau'$ if and only if $\widehat{V}_{i,\tau'/\tau}^{rn} \geq V_{i,\tau'/\tau}^{rn}$, which is equivalent to (22b).

Using the definitions of $\widehat{\Pi}_{i,\tau'/\tau}^{rn}$ and $\Pi_{i,\tau'/\tau}^{rn}$, the condition $\widehat{\Pi}_{i,\tau'/\tau}^{rn} \geq \Pi_{i,\tau'/\tau}^{rn}$ is also equivalent to

$$
D_{i,\tau'/\tau}^{rn} - \widehat{D}_{i,\tau'/\tau}^{rn} \geq \left( R_\delta - R_\delta \frac{1 - e^{-r \delta'}}{1 - e^{-r \delta}} \right) \frac{1}{1 + \lambda}.
$$
Using the definitions of $\tilde{V}_{i,\tau'}^{rn}$ and $V_{i,\tau'}^{rn}$, $\tilde{V}_{i,\tau'}^{rn} \geq V_{i,\tau'}^{rn}$ is also equivalent to

$$D_{i,\tau'}^{rn} - \tilde{D}_{i,\tau'}^{rn} \geq - \left( R_{\delta'} - R_{\delta} \frac{1 - e^{-\tau' \delta}}{1 - e^{-\tau \delta}} \right) \frac{1 - \alpha}{\alpha (1 + \lambda)}. \quad (51)$$

(25) follows after combining (50) with (51).

C.2 (24a) and (24b) are satisfied

Suppose first that, in state $l$, $F$ reports $h$ at time 0 and then the contract is renegotiated at some instant $\tau \in (0, T)$. Its instantaneous profit is given by

$$\tilde{\pi}_{i,\tau}^{rn} = t_{h}^{rn} + p(q_{h}^*) q_{h}^* - (\theta_{f} q_{h}^* + K) - g_{h,\tau}^{rn},$$

where $t_{h}^{rn} = \alpha t_{h}^{G} + (1 - \alpha) t_{h}^{F}$ denotes the expected transfer that results from renegotiating at $\tau$, given the report $h$. Using the formulas of $t_{h}^{G}$ and $t_{h}^{F}$ from the main text, $t_{h}^{rn}$ is rewritten as

$$t_{h}^{rn} = \alpha (\theta_{f} q_{h}^* + K + g_{h,\tau}^{rn}) + \frac{1 - \alpha}{1 + \lambda} \left( S(q_{h}^*) - w_{h}^* + \frac{r R_{\delta}}{1 - e^{-\tau \delta}} \right) - \frac{1 + \alpha \lambda}{1 + \lambda} p(q_{h}^*) q_{h}^*.$$

Replacing into the expression of $\tilde{\pi}_{i,\tau}^{rn}$, the latter is rewritten as

$$\tilde{\pi}_{i,\tau}^{rn} = (1 - \alpha) \left( \frac{R_{\delta}}{1 + \lambda} \frac{r}{1 - e^{-\tau \delta}} - g_{h,\tau}^{rn} \right) + \Delta \theta q_{h}^*.$$

In discounted terms, the profit is written as

$$\tilde{\Pi}_{i,\tau}^{rn} = \int_{\tau}^{T} \left( 1 - \alpha \right) \left( \frac{R_{\delta}}{1 + \lambda} \frac{r}{1 - e^{-\tau \delta}} - g_{h,\tau}^{rn} \right) e^{-r(x-\tau)} d\tau + \Delta \theta q_{h}^*.$$

Replacing this into (24b), the latter is rewritten as

$$\Pi_{l,0}^{*} \geq \int_{0}^{r} (\pi_{h,x}^{*} + \Delta \theta q_{h}^*) e^{-rx} dx + e^{-rt} \max \left\{ 0; \Pi_{h,\tau}^{rn} + \int_{\tau}^{T} \Delta \theta q_{h}^* e^{-r(x-\tau)} d\tau \right\},$$

which is equivalent to

$$\Pi_{l,0}^{*} \geq \Pi_{h,0}^{*} + \int_{0}^{T} \Delta \theta q_{h}^* e^{-rx} d\tau + e^{-rt} \left( \max \left\{ 0; \Pi_{h,\tau}^{rn} + \int_{\tau}^{T} \Delta \theta q_{h}^* e^{-r(x-\tau)} d\tau \right\} - \left( \Pi_{h,\tau}^{*} + \int_{\tau}^{T} \Delta \theta q_{h}^* e^{-r(x-\tau)} d\tau \right) \right).$$
This is implied by (11a) and (23a). Hence, (24a) does hold.

The proof that (24b) is satisfied proceeds analogously, leading to the conclusion that (24b) is implied by (11b) and (23a).

C.3 Proof of Lemma 3

Using the expression of $\Pi^*_{i,\tau}(z)$ and $\Pi^*_{h,\tau}(z)$ together with (21a) and $\Pi^*_{i,\tau} = 0$, (23a) is rewritten in state $l$ and $h$ as follows:

$$M \frac{1 - e^{-r\delta}}{1 - e^{-rT}} + (1 - \nu_1) \Delta \theta z \frac{1 - e^{-r\delta}}{r} \geq (1 - \alpha) \left( \frac{R_{\delta}}{1 + \lambda} - D_{i,\tau}^a \right)$$  \hspace{1cm} (52a)

$$M \frac{1 - e^{-r\delta}}{1 - e^{-rT}} - \nu_1 \Delta \theta z \frac{1 - e^{-r\delta}}{r} \geq \max \left\{ (1 - \alpha) \left( \frac{R_{\delta}}{1 + \lambda} - D_{h,\tau}^a \right) ; 0 \right\} ,$$  \hspace{1cm} (52b)

With (26) and (52b) satisfied, (52a) is satisfied and can thus be ignored. (52b) is rewritten as (27) in the lemma.

With (26) satisfied, (23b) is equivalent to $V^*_{i,\tau} \geq V^a_{i,\tau}$, $i \in \{l, h\}$. Using (19) together with (20) for all $i$, this inequality is rewritten as

$$D_{i,\tau} \leq \frac{R_{\delta}}{1 + \lambda} - \Pi^a_{i,\tau}(M, T) .$$

Replacing the expression of $\Pi^*_{i,\tau}(z)$ for $i = l, h$, this yields (28a) and (28b) in the text. (28a) and (28b) hold only if their right-hand sides are positive i.e.,

$$\frac{R_{\delta}}{1 + \lambda} \geq \left( \frac{Mr}{1 - e^{-rT}} + (1 - \nu_1) \Delta \theta z \right) \frac{1 - e^{-r\delta}}{r} ,$$  \hspace{1cm} (26a)

$$\frac{R_{\delta}}{1 + \lambda} \geq \left( \frac{Mr}{1 - e^{-rT}} - \nu_1 \Delta \theta z \right) \frac{1 - e^{-r\delta}}{r} ,$$  \hspace{1cm} (26b)

where the former imply the latter and is equivalent to (29) in the lemma.

C.4 Proof of Proposition 2

The condition $T \geq T(z)$ is necessary for (12) to be satisfied.

Recall that (23a) and (23b) are tighter when (26) is not satisfied than they are when it is. From the proof of Lemma 3 further recall that, when (26) is satisfied, (23a) and (23b) are rewritten as (27), (29), (28a) and (28b). These are all conditions on $M$ and $D_{i,\tau}$ and represent the weakest conditions on $M$ and $D_{i,\tau}$, $i = l, h$, under which $\Psi$ is implementable.

Now take (26) to hold for either type. (27) and (29) hold jointly only if (30) is satisfied. Further, from (27) and $M \leq E$, we deduce the condition

$$E \geq \nu_1 \Delta \theta z \frac{1 - e^{-rT}}{r} .$$

If $E \geq \nu_1 \Delta \theta z / r$, then this condition is satisfied for any $T > 0$. Otherwise, the condition is
rewritten as $T \leq \tilde{T}(z, E)$. Then, since $T \geq T(z)$, it is necessary that $T(z) \leq \tilde{T}(z, E)$, which is equivalent to $E \geq \nu_1 \psi/\Delta \nu$. Overall, the above condition and $T \geq T(z)$ are both satisfied if and only if (31) is satisfied.

Suppose that $C = 0$ and $\Psi$ is implemented. Then, $D_{i, r'} = \tilde{D}_{i, r'/r'} = 0$ for all $\tau, \tau' \in (0, T)$, $\tau' > \tau$. Hence, we cannot have $D_{i, r'} = \tilde{D}_{i, r'/r'} > 0$, as it is required for (25) to be met, which is a contradiction. Hence (32) must hold.

With (30) and (31) satisfied, $G$ can find values of $M$ satisfying both (27) and (29). With (32), $G$ can find out-of-equilibrium guarantees that satisfy the out-of-equilibrium conditions. Hence, the three conditions are both necessary and sufficient for implementation of $\Psi$.

C.5 Proof of Corollary 1

By assumption, $\lim_{\delta \to 0} R_\delta$ is positive and finite. Hence, (30) holds as $\delta$ tends to zero.

First suppose that, for some given $z$, (34) holds. It means that, as $\delta$ is raised (i.e., $\tau$ is decreased and/or $T$ increased), (30) becomes less stringent. Then, provided it is satisfied as $\delta$ tends to zero, it is as for all $\delta \in (0, T)$, $T \in [\bar{T}(z), \infty)$.

Next suppose that (35) holds. Then, for any given $T \geq \bar{T}(z)$, (30) is most stringent as $\tau$ tends to zero. Then, substituting $\tau = 0$ into (30), the latter is satisfied if and only if $T \leq \bar{T}(z)$. As $T$ must be at least as large as $\bar{T}(z)$, for $\Psi$ to be implementable it must be the case that the interval $[\bar{T}(z), \bar{T}(z)]$ exists and that $T$ belongs to this interval.

The remaining conditions in the corollary are given the restrictions on $T$ from the proof of Proposition 2, namely $T \geq \bar{T}(z)$ when $E \geq \nu_1 \Delta \theta z/r$ and $\bar{T}(z, E) \geq T \geq \bar{T}(z)$ when $E < \nu_1 \Delta \theta z/r$.

C.6 Proof of Corollary 2

Again recall that, when (26) is satisfied, (23b) is rewritten as (27), (29), (28a) and (28b). (27) and (29) are rewritten as (39).

Using the definition of $E_i[D_{i, r}]$ in (28a) and (28b), we obtain

$$E_i[D_{i, r}] \leq \frac{R_\delta}{1+\lambda} - M \frac{1 - e^{-r \delta}}{1 - e^{-r T}}.$$

Recalling that $E_i[D_{i, 0}] = C$, this condition together with $C > 0$ collapses onto (40).

D Proof of Proposition 3 and 4

Under the sharing rule $\alpha_i$, it must be the case that

$$\Pi_{i, 0} - \Pi_{h, 0} = \int_0^{T_l} \Delta \theta \alpha_l e^{-r x} dx = \int_0^{T_h} \Delta \theta \alpha_h e^{-r x} dx.$$  

$\alpha_i$ attains the highest feasible value for $i = l$ and $\alpha_h = q^*_l$. Replacing above, this yields $\alpha_l = \frac{1 - e^{-r T_l}}{1 - e^{-r T_h}}$. It attains the lowest feasible value for $i = l$ and $\alpha_h = q^*_h$. Replacing above, this yields $\alpha_l = \frac{1 - e^{-r T_h}}{1 - e^{-r T_l}}$. The feasible set of $\alpha_i$ is thus determined for $i \in \{h, l\}$. 

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We now check the conditions under which $T$ is implemented. (12) is rewritten as

$$\int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx \geq \frac{\psi}{\Delta \nu},$$

which requires that $T_i \geq T(\alpha_i)$. (27) is equivalent to $\Pi_{h,\tau} \geq 0$, which is written as

$$M \geq \nu_1 \Delta \theta \alpha_i \frac{1 - e^{-rT_i}}{r}. \quad (54)$$

Using $T_i \geq T(\alpha_i)$, $E \geq M$ and (54) altogether, (31) follows. (28a) and (28b) result from $V_{i,\tau}^* \geq V_{i,\tau}^{rp}$ and are written as

$$D_{h,\tau} \leq \frac{R_{\delta(h)}}{1 + \lambda} - \Pi_{h,\tau}^*,$$

$$D_{l,\tau} \leq \frac{R_{\delta(l)}}{1 + \lambda} - \Pi_{l,\tau}^*,$$

where $R_{\delta(i)}$ denotes the replacement cost in state $i$ when the residual contractual period is $(T_i - \tau)$. Using the profits above, these conditions become

$$D_{h,\tau} \leq \frac{R_{\delta(h)}}{1 + \lambda} - \left( M - \nu_1 \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx \right) \frac{1 - e^{-r(T_h - \tau)}}{1 - e^{-rT_h}} \quad (55)$$

$$D_{l,\tau} \leq \frac{R_{\delta(l)}}{1 + \lambda} - \left( M + (1 - \nu_1) \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx \right) \frac{1 - e^{-r(T_l - \tau)}}{1 - e^{-rT_l}}. \quad (56)$$

(29) was obtained by considering that the right-hand side of either of the conditions on debt must be positive. Instead of (29), we have now

$$M \leq \frac{R_{\delta(h)}}{1 + \lambda} \frac{1 - e^{-rT_h}}{1 - e^{-r(T_h - \tau)}} + \nu_1 \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx \quad (57)$$

$$M \leq \frac{R_{\delta(l)}}{1 + \lambda} \frac{1 - e^{-rT_l}}{1 - e^{-r(T_l - \tau)}} - (1 - \nu_1) \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx. \quad (58)$$

In Proposition 2, (30) was obtained from (27) and (29), which should hold jointly. When the operating cost is $\theta_h$, they hold jointly only if $R_{\delta(h)}/(1 + \lambda) \geq 0$, which is satisfied. When the operating cost is $\theta_l$, two situations can arise. If the sharing rule is $\alpha_l$, then they hold jointly only if (30) is satisfied for $z = \alpha_l$ and $T = T_l$. If the sharing rule is $\alpha_h$, then they hold jointly only if (45) is satisfied.
D.1 Proof of Corollary 3

Condition (46) is obtained from (54), (57) and (58). Using the definition of $E_i[D_{i,r}]$ in (56) and (55), we obtain

$$E_i[D_{i,r}] \leq \frac{E_i[R_{\delta(i)}]}{1 + \lambda} - ME_i \left[ \frac{1 - e^{-r_{\delta(i)}}}{1 - e^{-r_{\lambda}}} \right].$$

Then, recalling that $E_i[D_{i,0}] = C$, this condition together with $C > 0$ collapses onto (47).