Holding an Auction for the Wrong Project∗

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Abstract

How does the probability of being involved in a renegotiation during the execution of a procurement contract affect the behavior of the agents? What are its implications for the optimal contractual choice made by the principal? This paper investigates these issues in a context characterized by uncertainty about the adequateness of the project initially specified by the buyer. The main result of this paper establishes that, in several circumstances, the buyer may find it profitable to hold an auction for the project design which ex-ante does not have the higher probability of being appropriate.

Keywords: Procurement, Asymmetric Auctions, Renegotiation, Bargaining under Asymmetric Information.

1 Introduction

Contracts concerning the provision of customized goods or services are often granted through auctions. It is claimed that an auction guarantees transparency and fosters competition, allowing the buyer to obtain the desired good at the most favorable economic conditions. Nonetheless, once awarded, a number of procurement contracts require modifications which may significantly change the design itself of the good. Moreover, in a lot of cases the events which trigger the revision of the original agreement could have been anticipated at the time at which the initial contract was drawn up1.

If a renegotiation significantly alters the scale and the scope of the original contract, one may question the optimality of the auction outcome. A firm suitable to provide the original service may no longer be the most appropriate operator when the contract is reviewed. Nevertheless, contract clauses may prevent (or just may make it unprofitable) the buyer from turning

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1In this regard, Guasch (2004) provides extensive empirical evidence of strategic renegotiation of concession contracts granted in Latin America and the Caribbean in the period 1985-2000.
to another firm for the provision of the service.

A compelling example involves the construction of the new railway station in Mons (Belgium). At the time of the call for tenders in 2006, the stated objective of the local authorities was to preserve the original station which dated back to the 1950s. The renowned Spanish architect Santiago Calatrava was granted the contract since his design was the only one which met that requirement. Because of technical problems, the initial winning design could not be undertaken and Calatrava’s architectural firm worked out a new project which would lead to the replacement of the old station. Presumed similarities existing with other project designs rejected at the bidding stage have sparked a lot of criticism: indeed, rival and lesser-known architectural firms have claimed that they could deliver the same project at a lower price.

This example raises the question of whether a buyer would find it advantageous to choose a design which has a high ex-ante probability of being infeasible. Similarly, we may wonder whether the procurer would select a contractor who is not the most appropriate for the project design ultimately implemented. In this paper, we provide an answer to these questions showing the existence of a non-trivial trade-off between the cost of an ill-specification of the project design and the benefits of intensifying competition ex-ante.

We show that a procurer may decide to hold an auction for a project specification which is highly likely to be inappropriate (that is, a wrong project) to stiffen competition at the bidding stage when the potential contractors have different design specialization.

Bidders know that with some positive probability the initial design will be flawed and rationally discount the rents they expect to earn at the renegotiation stage when they submit their bids. If renegotiation is always successful and the firms have the same bargaining power, the expected rents enter the firms’ bidding functions in the same way and, as a result, the initial choice of the design is neutral as it affects neither the efficiency of the contract allocation nor the expected buyer’s payoff. On the other hand, the renegotiation rents may not be high enough to compensate some contractors for the higher cost of production that an alternative project design entails. If so, renegotiation fails with some positive probability and, then, there is scope for a strategic choice of the initial design. In this case, the expectation that the starting design of the project may default causes an asymmetric shift in the bidding functions of the firms. The buyer can thus take advantage of the firms’ heterogeneous reactions to countervail the existing cost asymmetry among bidders and receive lower and more aggressive bids. The buyer may optimally choose the wrong specification to intensify competition at the bidding stage even at the expenses of going through a costly renegotiation with a higher probability. Nevertheless, we stress that this choice can have negative effects in terms of ex-ante allocative efficiency, for the contract could be granted to the firm whose expected cost of production is the highest. Additional effects come into play when the buyer holds all the bargaining power. Some firms

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2LeVif.be - 20th April 2010.
3Firms may incur different costs to deliver alternative designs when they are heterogeneous for reasons related, for instance, to their size or to the skills of their human resources.
4In the model, we draw extensively on the insights laid by Maskin and Riley (2000a) on the bidding behavior of asymmetric bidders.
5Clearly, it must be the case that the buyer is imperfectly informed about the firms’ cost of production, for otherwise he would not need to stiffen competition in the auction.
incur a lower cost of production when the design change takes place and the buyer will rationally attempt to enjoy a fraction of the cost saving at the renegotiation stage. In this case, the procurer may intentionally choose the wrong project also to increase the probability of entering a renegotiation and this effect adds up to the competition effect pointed out above.

In this paper, we assume that a buyer wishes to procure a project and holds an auction to select the contractor. There are two alternative specifications of the good, $A$ and $B$, and the prior probability that the design $A$ turns out to be flawed, if implemented, is $\beta \in (0, 1)$, which is common knowledge to all the players of the game. We assume that there are two bidders who are specialized in delivering one specification of the project each and, as a consequence, its alternative designs entail different production costs. In particular, we assume that the relative cost advantage enjoyed by one bidder in undertaking project $A$ is reversed when it is the alternative project specification to be carried out.

As it is, our model fits very well the Design-Bid-Build (DBB) project delivery system which is the traditional and, to date, the most widely used method to award procurement contracts in the US\(^6\). Its distinguishing feature is that the design and the building tasks are carried out by two different entities. First, the buyer engages architects and engineers to prepare the desired specification of the project which is then put out for tender to interested general contractors. The insights of our model can then be applied to all those procurement environments where there are alternative designs available to produce the same good, but ex-ante there is uncertainty about which specification is the most appropriate.

### 2 The Model

Consider a risk-neutral government who wishes to procure a good from the outside and works out on his own its design, say $A$, to which he attaches a positive value, $v > 0$. If the initial design $A$ turns out to be flawed, then the project yields the buyer utility $v - h \geq 0$, if it is not modified. Whereas, if a change in the building phase occurs and the alternative design $B$ is adopted, the buyer again attains utility $v$. However, the new project requires different capabilities from the engaged contractor and it may thus entail either a higher or a lower cost of production. Henceforth, we assume that once the tender process has taken place, the buyer is stuck to the selected contractor and cannot hire the other firm (in practice, one may think of a large cost of breaching the initial contract which makes it unprofitable for the buyer to back out).

Bidders are risk neutral and, as mentioned earlier, have different project design specialization. In particular, we assume that when $A$ is carried out, bidder $a$ incurs a low cost of production, $\tilde{c}_a$, while bidder $b$ bears a high cost, $\tilde{c}_b$, whereas the dominance is reversed if the project specification happens to be $B$. Table 1 summarizes the relationship between the cost borne by the firms and

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\(^{6}\)A recent study by RSMeans Reed Construction Data Market Intelligence for The Design-Build Institute of America (DBIA) shows that in 2010 more than 50% of non-residential constructions in the US were procured through the DBB system.

\(^{7}\)The results will not change if we assume that he contracts out the delivery of the alternative project specification $B$, though.

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Table 1: Firms’ costs of production per project design

<table>
<thead>
<tr>
<th>Firm/Project Design</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(\tilde{c}_l)</td>
<td>(\tilde{c}_h)</td>
</tr>
<tr>
<td>b</td>
<td>(\tilde{c}_h)</td>
<td>(\tilde{c}_l)</td>
</tr>
</tbody>
</table>

the project design undertaken.

In practice, one may think of a large and a small firm. The former has higher fixed costs but can take advantage of economies of scale, whereas the latter has higher variable costs, but negligible fixed costs. If \(A\) and \(B\) differ with respect to the size of the project, it is conceivable that the large firm will bear a lower total cost of production than the small firm when the bigger design is undertaken and vice versa.

Firms’ production costs \(\tilde{c}_l\) and \(\tilde{c}_h\) are distributed independently over the intervals \([c_l, \overline{c_l}]\) and \([c_h, \overline{c_h}]\), respectively, with \(c_l < \overline{c_l} \leq c_h < \overline{c_h} < v\).

In light of these cost parameters, if the buyer holds an auction for project \(A\) and this subsequently turns out to be flawed, then a type-a bidder will be reluctant to shift to project \(B\) as it involves a higher cost of production while a type-b bidder will eagerly accept the design change as it will allow him to cut the production cost. Such a feature clearly alters the renegotiation claims of the two bidders and under some circumstances can even prevent a design change from occurring. In turn, these considerations undoubtedly affect the bidding strategies pursued by the two firms.

The timing of the game works as follows:

- At time 0, \(\beta\) is observed by the potential contractors and by the buyer. The latter decides which project to auction off, between \(A\) and \(B\), and chooses the auction format.
- At time 1, the auction takes place and the contract is granted to either firm \(a\) or \(b\).
- At time 2, the uncertainty is realized. If the project design chosen at time 0 exhibits imperfections, a renegotiation between the buyer and the selected contractor occurs and, if successful, the design changes.
- At time 3, the project is delivered and the payoffs are realized.

Throughout, we focus on fixed-price contracts where the awardee does not receive any reimbursement for the costs he incurs. We make this choice for several reasons. First, in our setting agents are risk-neutral and, consequently, the parties do not have to write an incentive contract where a fraction of the costs of production is borne by the buyer to strike the optimal risk-sharing agreement\(^8\). Furthermore, fixed price contracts are most often awarded through auctions: in particular, the Federal Acquisition Regulations (FAR) in the United States recommend to use auctions of fixed price-contracts for public sector purchases (see Bajari et al., 2008). Last but not least, Bajari and Tadelis (2001) have emphasized the merits of a cost-plus

\(^8\)Following McAfee and McMillan (1986), the optimal linear contract takes on a fixed-price format if agents are risk-neutral. Here we ignore what they define as the bidding-competition effect.
contract when it comes to renegotiating an agreement, and their argument is mainly based on
their greater flexibility to adapt to ex-post adjustments than fixed-price contracts. Here, we
wonder whether the award of a fixed-price contract may prove a wise decision by the buyer in
presence of a known positive probability that the initial agreement will warrant some changes.

We consider what project design would be selected in presence of imperfect information
about the production costs incurred by the firms. In absence of information asymmetries, a
competition effect could not arise and, as a result, there would not be scope for a strategic
choice of the initial design.

3 Renegotiation Under Asymmetric Information

Assume that the cost parameters of the awardee are unobservable. Then, to determine
the utility function of the buyer and the profit functions of the two bidders, we need to make
an assumption on the way renegotiation takes place. If the initial specification happens to be
flawed and the buyer wishes to change the design not to incur the net loss of welfare, \( h \), the cost
parameters of the contractors still remain private information and do not enter directly into the
renegotiation requests of the parties. This assumption is consistent with the idea spelled out
by Bajari and Tadelis that only the total production costs may be verifiable while modification
costs may not. Since the extent to which the design change has hurt the cost efficiency of the
contractor cannot be verified, any agent’s reimbursement claim for the increased cost of delivery
the good cannot be trusted. However, the contractor can reject any revision of the original
agreement.

Henceforth, we rest on the assumption that the procurer is able to identify the type of the
firm he deals with at the renegotiation stage. This is in line with the idea that the two firms
differ with respect to exogenous characteristics which can be observed by the buyer, such as
their size.

In light of the above, we need to work out a bargaining model in presence of asymmetric
information. One comfortable option is to focus on two polar cases, as Bajari and Tadelis do in
their 2001 paper, namely, when either party to the contract makes a take-it-or-leave-it offer on
which he can wholly commit. This is also consistent with the results of the literature on bar-
gaining under asymmetric information (Samuelson, 1984) which predicts that the parties may
sometimes fail to reach an agreement and that the first best is attainable provided that either
party may stand by the first-and-final offer he makes.

Therefore, we will take into account two settings: one in which it is the awardee who can
make the take-it-or-leave-it offer at the renegotiation stage and one where it is the buyer who is
entitled to make the first-and-final offer.

4 The Contractor Makes the First-and-Final Offer

Clearly, if the firm who has been granted the contract happens to be involved in a renego-
tiation with the procurer, he will make the most of his perfect information about the buyer’s
utility, asking for $h$, which is publicly observable. However, in some occurrences the contractor may be still unwilling to accept the design change: this event occurs when the awardee has to incur a higher cost of production to deliver the new project design and the hold-up rent, $h$, is not large enough to reimburse him for the increased cost. Therefore, we need to take into account the probability that the renegotiation breaks down when an auction for $A$ ($B$) has been held, $a$ ($b$) has been awarded the contract, and the procurer requests a design change.

It is thus necessary to determine under which conditions a profitable renegotiation occurs when the proposed design change entails a higher cost of production for the awardee. To do so, we denote by $\tilde{c}$ the random variable $\tilde{c}_h - \tilde{c}_l$, whose distribution and density are $F(\tilde{c})$ and $f(\tilde{c})$, respectively. Consequently, when the buyer auctions off project design $A$ and, then, he is willing to adopt the alternative design, there is a probability equal to $1 - F(h)$ that the higher cost of production firm $a$ has to incur prevents the renegotiation process. In what follows, we distinguish between two cases: when the value of the hold-up rent is so high that renegotiating the contract always succeeds (i.e., $h \geq \tilde{c}_h - \tilde{c}_l$) and when a requested renegotiation may not lead to a design change.

First note that irrespective of whether there exists some positive probability that renegotiation breaks down, the buyer’s expected utility when he has no bargaining power is given by:

$$EU(A) = v - t_w - \beta h,$$

when he initially avails himself with project design $A$, and $t$ is the transfer from the procurer to the firm, and where the subscript $w$ in the above equation can be equal to either $a$ or $b$ and denotes the contract winner.

Whereas, when $B$ is the starting project design, his expected utility is:

$$EU(B) = v - t_w - (1 - \beta)h.$$

### 4.1 Neutrality of the Initial Design

Suppose first that the parties always succeed in renegotiating the contract, regardless of the identity of the winning bidder (i.e., $F(h) = 1$).

When $A$ is the initial design, bidders’ profit functions take on the following forms:

$$E\pi_a(A) = \begin{cases} \{t_a - (1 - \beta)c_a - \beta(c_h - h)\}Pr(t_a < t_b) \\ t_b - (1 - \beta)c_h - \beta(c_h - h)Pr(t_b < t_a) \end{cases}$$

It is important to characterize the bidders’ types which consist of two components: the expected cost of production (which we label $c_{iA}$, with $i = \{a, b\}$) and the expected rent which merely shifts to the left the expected total cost of delivering the good.

$$\theta_{aA} = (1 - \beta)\tilde{c}_l + \beta \tilde{c}_h - \beta h$$

$$\theta_{bA} = (1 - \beta)\tilde{c}_h + \beta \tilde{c}_l - \beta h$$

$\theta_{iA} = (1 - \beta)\tilde{c}_h + \beta \tilde{c}_l - \beta h$

$9f(\tilde{c})$ is the convolution of the density functions of $\tilde{c}_h$ and of $\tilde{c}_l$. 


It is also helpful to determine the interval on which the bidders’ types are defined:

\[
\begin{cases}
\theta_{aA} \sim \Phi_a[(1 - \beta)c_l + \beta c_h - \beta h; (1 - \beta)c_l + \beta c_h - \beta h] = \Phi_a[\theta_{aA}; \theta_{aA}]
\end{cases}
\]

\[
\begin{cases}
\theta_{bA} \sim \Phi_b[(1 - \beta)c_h + \beta c_l - \beta h; (1 - \beta)c_h + \beta c_l - \beta h] = \Phi_b[\theta_{bA}; \theta_{bA}]
\end{cases}
\]

As a matter of fact, what characterizes the bidders is only the cost function while the rent they expect to earn is the same. The profit functions we have set out above are correct as long as \( h \) is greater than the actual cost of modifying the design of the project, \( c \), so that the renegotiation between the buyer and the contractor always proves successful. In particular, the parties to the contract succeed in renegotiating the contract even if the awardee for project designs \( A \) and \( B \) are firms \( a \) and \( b \), respectively.

Before presenting our first result, we introduce this following lemma concerning the bidding behavior of the agents:

**Lemma 1.** If (i) bidders’ cost parameters are drawn independently and if (ii) their expected profit functions are both monotonically decreasing in the firms’ types and (iii) are weakly supermodular, then they bid according to a weakly monotonic bidding function. That is, if \( t_i = \gamma_i(\theta_i) \), then \( \gamma_i'(\theta_i) \geq 0 \) for any \( \theta_i \in \Phi_i, \) for \( i = a, b \).

The proof can be found in the appendix. Here we only note that the profit functions written above are both monotonically decreasing and weak supermodular (the latter condition is always satisfied when bidders are risk neutral). We can now proceed with our first result:

**Proposition 1.** If (i) the contractor is entitled to make, and can commit to, a take-it-or-leave it offer to the buyer at the renegotiation stage; (ii) the conditions of lemma 1 are fulfilled ; (iii) it holds that \( h \geq c_h - c_l \), then:

1. **Competitive bidding mechanisms such as a First Price Auction and a Second Price Auction achieve ex-ante allocative efficiency.**

2. **The rent \( h \) is not a concern for the buyer.**

The first point of the proposition tells us that the bidder whose expected cost of production of the project is the lowest will always win the auction. Since the rent the bidders expect to enjoy is the same, the only distinguishing feature between them is their cost parameters. Furthermore, if there is little uncertainty about which project design will eventually be undertaken, that is, if \( \beta \) is either sufficiently low or sufficiently high, a First Price Auction (FPA) proves weakly superior to a Second Price Auction (SPA): the lack of uncertainty about the project design strengthens the asymmetry between the two bidders and can give rise to a distribution shift which, as Maskin and Riley (2000, proposition 4.3) show, induces the buyer to favor a FPA mechanism.

The second part of the proposition is due to the observation that the renegotiation rent is entirely discounted at the bidding stage. The reason is straightforward: both bidders are aware of the probability with which they will earn a rent, if they are granted the project, and they
know the magnitude of the rent itself. Therefore, a competitive bidding process will work in a Bertrand fashion, allowing the buyer to extract all the winning bidder’s willingness to pay to be granted the right to potentially earn the hold-up rent, that is, $\beta h$ for design $A$ and $(1-\beta)h$ for design $B^{10}$.

From proposition 1 we derive the following corollary:

**Corollary 1. Neutrality of the project design at the auction stage.** When renegotiation is always successful, it does not matter which project design is auctioned off as the contract is always awarded to the firm who is most efficient ex-ante and the renegotiation rent is entirely discounted at the bidding stage.

### 4.2 Auction of the Wrong Project

Now, we turn to the more interesting case where the hold-up rent may be lower than the increased cost of production that a design change requires.

When project $A$ is auctioned off, only the expected profit function of bidder $a$ may change with respect to the previous setting. This happens when $h < c_{ha} - c_{la}$, in which occurrence $a$’s expected profit function becomes:

$$E\pi_a(A) = (t_a - c_{la})Pr(t_a < t_b)$$

Whereas, if $h \geq c_{ha} - c_{la}$, the expected profit function of firm $a$ is the same as the one we wrote in the previous subsection:

$$E\pi_a(A) = [t_a - (1-\beta)c_{la} - \beta(c_{ha} - h)]Pr(t_a < t_b)$$

Adopting the buyer’s perspective, we focus on how the bidders’ pseudo-types change if he decides to contract out project specification $A$ or $B$. In the former case, the buyer will face the following bidders’ pseudo-types:

$$\begin{align*}
\theta_a^A &= [1 - \beta F(h)][\tilde{c}_l + \beta F(h)][\tilde{c}_h - h] \\
\theta_b^A &= (1-\beta)[\tilde{c}_h + \beta[\tilde{c}_l - h]
\end{align*}$$

In the latter case, the buyer will be confronted with different bidders’ pseudo-types:

$$\begin{align*}
\theta_a^B &= \beta \tilde{c}_h + (1-\beta)[\tilde{c}_l - h] \\
\theta_b^B &= [1 - F(h) + \beta F(h)][\tilde{c}_l + F(h)(1-\beta)][\tilde{c}_h - h]
\end{align*}$$

Adapting the Maskin and Riley’s framework to a procurement auction, we can define the weak bidder $(w)$ as the one whose pseudo-type’s distribution first order stochastically dominates that of the strong bidder $(s)^{11}$:

$$\Phi_w(\theta) < \Phi_s(\theta), \forall \theta \in (\theta_w, \theta_s) \quad (2)$$

10Furthermore, consider that the buyer should set a ceiling to the bids he receives. Specifically, when the design auctioned off is $A$, the buyer should turn down all the bids above $v - \beta h$, as they yield him an expected negative utility. In addition, if bidders bid according to a monotonic bidding function the minimum observable bid is given by $\theta_{aA}$ and, as a result, they always subtract $\beta h$ from their true cost parameter, $c_{iA}$.

11Note that (2) implies $\theta_{aA} \leq \theta_{aA}$ and $\overline{\theta}_w \leq \overline{\theta}_w$. 8
Since the distributions of bidders’ pseudo-types vary with the value of $\beta$ and $h$, the identities of the strong and weak bidders are endogenously determined by these parameters. Even though the buyer cannot set either of them, he can decide which project design to auction off at the beginning of the game (whether $A$ or $B$), thereby affecting the probability of a renegotiation and, in turn, the distributions of the bidders’ pseudo-types. In other words, the buyer can influence the competitiveness of the bidding process with his initial decision of the project design. However, to draw some conclusions on the buyer’s optimal strategy, we need to turn to a property slightly stronger than First Order Stochastic Dominance:

**Definition 1 (Hazard Rate Dominance - HRD).** $\Phi_w(\theta)$ is said to stochastically dominate $\Phi_s(\theta)$ according to the Hazard Rate if:

$$\forall \theta \in [\theta_w, \theta_s], \quad \text{it holds that} \quad \frac{\phi_w(\theta)}{1 - \Phi_w(\theta)} < \frac{\phi_s(\theta)}{1 - \Phi_s(\theta)}$$

The following lemma sets out the direct implications of assuming Hazard Rate Dominance:

**Lemma 2.** If HRD is fulfilled, the distribution of the equilibrium bids of the weak firm will first order stochastically dominate that of the strong firm, provided that the bids are weakly monotonic in the bidders’ types. In addition, the weak bidder will bid more aggressively than the strong bidder for any bid on the interior of their common support (proof in appendix).

Before applying the other insights stemming from the analysis of Maskin and Riley (2000a) to the current setting, where the buyer’s problem, after having observed the value of $\beta$, is that of choosing which project design to auction off, we need to introduce the following definition:

**Definition 2 (Wrong project).** A wrong project is a project design whose prior probability of being flawed exceeds that of another design specification available to the buyer.

Note that in this model where only two alternative designs are available, the wrong project is the one which has the higher probability of exhibiting imperfections. We can now present our second result:

**Proposition 2.**

1. **Auction of the wrong project.** The buyer finds it profitable to auction off the project specification more likely to be flawed ex-post, that is, the wrong project, so as to stiffen competition at the bidding stage and receive more aggressive bids, if:
   
   i the contractor is entitled to make, and can commit to, a take-it-or-leave it offer to the buyer at the renegotiation stage;
   
   ii the conditions of lemma 1 are fulfilled;
   
   iii the ratio between the hazard rate of the strong and of the weak bidder is lower under the wrong project specification;
   
   iv $h \leq h^* \leq c_l - c_l$, where $h^*$ is a decreasing function of the probability of renegotiating the initial project design.

2. If the identity of the winning bidder is not affected by the project design which is auctioned off, then ex-ante allocative efficiency is preserved when the wrong project is selected.
To understand the first result (consult the appendix for the formal proof), focus on the auction of the project specification $A$. When $\beta$ is very low, there exists a strong asymmetry between the distribution of the two bidders’ pseudo-types and bidder $a$, who is the strong bidder, can win the auction by submitting a very high bid which is detrimental to the buyer. However, as $\beta$ grows large, the distribution of $b$’s pseudo-type gets closer to that of $a$ and from a certain point on the identity of the strong and the weak bidder will change. From the buyer’s standpoint what matters is that as $\beta$ increases he receives lower bids and, as a result, the expected transfer he has to pay to the awardee decreases\textsuperscript{12}. This competition effect is partially offset by the higher expected loss of utility the buyer has to incur, $(2\beta - 1)h$. Therefore, for any value that $\beta$ can take on the interval $(\frac{1}{2}; 1)$, it is possible to pinpoint a threshold value, $h^*$, above which the benefits of increased competition are outweighed by an excessive expected rent. The function $h^*$ is decreasing in $\beta$, when $A$ is the starting design, because the buyer feels the benefits of competition when the bidders’ pseudo-types’ distributions are more homogeneous\textsuperscript{13}. Then, if $\beta$ approaches unity, bidder 2 and bidder 1 already have similar expected cost of production and a too high hold-up rent may overly favor bidder 2. The implication is that if the assumptions of proposition 2 are satisfied the buyer is better off when he auctions off project design $A$ when $\beta > \frac{1}{2}$ and design $B$ when $\beta < \frac{1}{2}$.

A graphical example may help to grasp the intuition: in Figure 1 we compare the distribution of bidders’ pseudo-types when an auction for $A$ and $B$ takes place under the following assumptions for the parameters and the distributions: $\tilde{c}_l \sim U[1, 2]$, $\tilde{c}_h \sim U[2, 3]$, $h = 0.5$, and $\beta = 0.8$. The buyer clearly gains from auctioning off the wrong specification, $A$, as the firms are made far less asymmetric in so strengthening the competition at the bidding stage. Such benefit is not outdone by the higher rent he expects to pay which is relatively small.

\textsuperscript{12}This intuition is triggered by Maskin and Riley (2000a): if a weak bidder faces a strong bidder rather than another weak bidder he will react by submitting a more aggressive bid. Along the same lines, a strong bidder who faces an increasingly (as $\beta$ rises) less weak bidder will respond by bidding more aggressively.

\textsuperscript{13}Conversely, if the buyer initially avails himself with project specification $B$, $h^*$ in increasing in $\beta$ in the interval $(0, \frac{1}{2})$. 

Figure 1: Distributions of bidders’ pseudo-types for $\beta = 0.8$, $h = 0.5$
5 The Buyer Makes the First-and-Final Offer

We can now proceed to the second scenario, where it is the buyer who is entitled to make the take-it-or-leave-it offer. Here, when it comes to renegotiating the original agreement the picture becomes more cumbersome. Again, let us focus on the case in which the project specification initially chosen is $A$. On the one hand, when faced with a type-$a$ firm, the buyer is poised to give up some fraction of the renegotiation gain that accrues to himself in order to persuade the contractor to accept the design change. On the other hand, when confronted with a type-$b$ contractor, the buyer wishes to seize some fraction of the design change gain that accrues to the firm. This feature gives rise to two consequences:

a) Irrespective of whom has won the auction, there exists some positive probability that the renegotiation breaks down and the parties remain stuck to the initial agreement which requires $A$ be delivered by the winning bidder.

b) When the buyer has bargaining power, his expected utility at the renegotiation stage depends on whom has been granted the contract. Therefore, as we show below, it is not optimal for the buyer to hold a "low-price auction" to assign the project.

To start with, consider the take-it-or-leave it offer that the buyer will make to a type-$a$ firm at the renegotiation stage. The buyer will rationally make an offer which maximizes his own ex-post payoff, knowing that the firm will turn down any offer which does not make up for the higher cost of production he has to incur to deliver $B$ instead of $A$. Thus, the buyer will make the seller an offer $\omega$ which, with probability $F(\omega)$, will prove successful. The offer $\omega$ will be chosen so as to minimize the renegotiation loss:

$$F(\omega)\omega + (1 - F(\omega))h$$

that is:

$$\omega^* = h - \frac{F(\omega^*)}{f(\omega^*)} \in [0, h]$$

(3)

If $\frac{F(\omega^*)}{f(\omega^*)}$ is increasing in $\omega$, then (3) has a unique, interior solution\(^{14}\).

Faced with a type-$b$ operator, the buyer will submit a different take-it-or-leave-it offer, $\nu$, seeking to reap the highest possible share of the renegotiation gain which accrues to the contractor, without compromising the renegotiation itself. Here, with probability $F(\nu)$ the renegotiation breaks down. The buyer will choose the offer so as to maximize the following expression:

$$(1 - F(\nu))\nu - F(\nu)h$$

Thus, the optimal offer is\(^{15}\):

$$\nu^* = \frac{1 - F(\nu^*)}{f(\nu^*)} - h \geq 0$$

\(^{14}\)Note that this condition is satisfied for any log-concave distribution. Since $f(\tilde{c})$ is obtained as the convolution of the density functions of $\tilde{c}_h$ and of $\tilde{c}_l$ and convolution is an operation that preserves log-concavity, what is ultimately required is that the densities of $\tilde{c}_h$ and of $\tilde{c}_l$ are log-concave (such as uniform).

\(^{15}\)Note that $\frac{1 - F(\omega)}{f(\omega)}$ is decreasing in $\nu$ for any log-concave distribution.
The following lemma shows what allocation rule the buyer should optimally adopt when he holds all the bargaining power.

**Lemma 3.** Let \( \rho = F(\omega^*)\omega^* + (1 - F(\omega^*))h + (1 - F(\nu^*))\nu^* - F(\nu^*)h \).

Let \( \eta_{aA} \) and \( \eta_{aB} \) be the probabilities of awarding the contract to firm \( a \) when the initial project specification is \( A \) and \( B \), respectively.

Then, when \( A \) is auctioned off, the contract is awarded according to the following rule:

\[
\eta_{aA} = \begin{cases} 
1, & \text{if } t_a \leq t_b - \beta \rho \\
0, & \text{otherwise}
\end{cases}
\]

Whereas, if the initial specification of the project is \( B \), the allocation rule becomes:

\[
\eta_{aB} = \begin{cases} 
1, & \text{if } t_a \leq t_b + (1 - \beta)\rho \\
0, & \text{otherwise}
\end{cases}
\]

**Proof.** The proof is straightforward. To begin with, note that the utility the buyer can attain at the renegotiation stage depends on the identity of the firm who has been granted the contract. Thus, when the buyer has received the bids for project design \( A \), his expected utility is:

\[
EU(A) = v - \eta_{aA} [T_a + \beta(F(\omega^*)\omega^* + (1 - F(\omega^*))h)] - (1 - \eta_{aA}) [T_b + \beta(F(\nu^*)h - (1 - F(\nu^*))\nu^*)]
\]

Where \( T_a \) and \( T_b \) are the expected total costs of awarding the contract to bidders \( a \) and \( b \), respectively, while \( \eta_{aA} \) denotes the probability that \( a \) wins the contract.

The buyer cannot consider the price as the only relevant variable in the allocation rule, for two different reasons: first, the firm specialized in delivering the starting project would have a huge head start over his opponent, in so harming competition; second, were the buyer to receive the same offer from the two bidders, he would much rather grant the project to the less specialized firm. This second point stems from the observation that the principal always prefers to renegotiate with the party from whom he can elicit the highest utility, i.e., the most willing to shift to the alternative design specification. As a consequence, the allocation rule at the auction stage should be:

\[
\eta_{aA} = \begin{cases} 
1, & \text{if } T_{aA} \leq T_{bA} \\
0, & \text{otherwise}
\end{cases}
\]

Since

\[
T_{aA} \leq T_{bA} \iff t_a \leq t_b - \beta \left( F(\omega^*)\omega^* + (1 - F(\omega^*))h - F(\nu^*)h + (1 - F(\nu^*))\nu^* \right)
\]

we retrieve the condition stated in the lemma.

Similarly, when the buyer holds an auction for project design \( B \), his expected utility is:

\[
EU(B) = v - \eta_{aB} [t_a + (1 - \beta)(F(\nu^*)h - (1 - F(\nu^*))\nu^*)] - (1 - \eta_{aB}) [t_b + (1 - \beta)(F(\omega^*)\omega^* + (1 - F(\omega^*))h)]
\]
And the allocation rule will be:

\[ \eta_{aB} = \begin{cases} 
1, & \text{if } T_{aB} \leq T_{bB} \iff t_a \leq t_b + (1 - \beta)\rho \\
0, & \text{otherwise}
\end{cases} \]

A corollary of this lemma is the following:

**Corollary 2.** Since \( \rho \) is always positive, the firm who is specialized in the project initially specified by the buyer is always handicapped.

Thus, a handicap will be optimally adopted to reduce the competitive advantage enjoyed by the firm which is specialized in the initial project design and will lead the favored bidder to bid more aggressively as shown in the following lemma:

**Lemma 4.** A bidder bids more aggressively when a positive handicap is put in place. That is, for the auction of design A, it holds that \( g_{\text{hand}}^a(t) \geq g_a(t) \) while for design B, it holds that \( g_{\text{hand}}^b(t) \geq g_b(t) \) where \( g_i(t) \) is the inverse bid function and the superscript hand refers to the inverse bid function of firm i when the handicap is taken into account (proof in the appendix).

However, the presence of a handicap might not be enough to guarantee a fierce competition between the bidders and as a result, the buyer may find it convenient to manipulate the auction process to further stiffen competition at the bidding stage. Thus, we wonder how the buyer will behave at the first stage in face of the allocation and transfer rules we set out above.

**Proposition 3.** 1. When the buyer is entitled to make, and can commit to, a take-it-or-leave-it offer to the contractor at the renegotiation stage, he will find it profitable to hold an auction for the wrong project when the following conditions hold:

i the conditions of lemma 1 are fulfilled;

ii \( f \) is log-concave so that \( \frac{F(x)}{f(x)} \) and \( \frac{1-F(x)}{f(x)} \) are increasing and decreasing in \( x \), respectively;

iii the ratio between the hazard rate of the strong and of the weak bidder is lower under the wrong project;

iv \( h \in [0, \hat{h}] \), where \( \hat{h} \) is decreasing in the probability of going through the renegotiation stage.

2. If \( h \leq \tilde{h} < \hat{h} \), where \( \hat{h} \) is decreasing in the probability of entering a renegotiation, then an auction for the wrong project does not preserve ex-ante allocative efficiency, namely, the contract is awarded to the firm who is less efficient ex-ante.

The intuition of the proposition can be readily explained taking on again the perspective of a buyer who starts with the project specification \( A \), for which firm \( a \) has a cost advantage over firm \( b \) (consult the appendix for the proof).

On the one hand, if \( h \) is particularly large, the buyer strives to avoid failing to change the
design when the initial specification of the project is flawed and, as a result, he will ask for a tiny $\nu^*$ when dealing with a type-$b$ firm, whereas he will submit a very generous offer to a type-$a$ firm. As a consequence, renegotiating with bidder $a$ will turn out to be very costly and a large handicap will be adopted. If $\beta > \frac{1}{2}$, firm $b$ is highly likely to win the auction for project design $A$ by offering a very high bid. Hence, the buyer will be better off holding an auction for the right project. In other words, for $h$ sufficiently large, the buyer will tend to minimize the probability that the contract will be reviewed.

On the other hand, when $h$ is not particularly large, the buyer will attempt to grab a fraction of the cost saving when he is faced with a type-$b$ bidder. By contrast, with a type-$a$ bidder, he may end up not renegotiating the contract, since the reimbursement will likely exceed the loss he would experience by sticking to the original project design. Thus, holding an auction for the wrong project again becomes a device to stiffen competition at the bidding stage and, if the less suitable contractor for the initial project is hired, the buyer may enjoy a significant gain should the renegotiation successfully take place.

To summarize, we attain the same counterintuitive result as in section 4 when it is the buyer to be entitled to make a first-and-final offer at the renegotiation stage, provided that the value of the hold-up rent does not dominate the differential cost of providing alternative specifications of the project.

6 Conclusion

Renegotiation of procurement contracts seems to be a widespread practice. Furthermore, in a number of cases, renegotiation is apparently unrelated to any contract incompleteness explanations, namely, it is not the emergence of ex-ante unforeseeable contingencies to trigger substantial contract modifications.

In this paper, we have shown that if the prior probability of a partial default of a project specification is known to all the parties to a contract, the buyer may act strategically when choosing the design of the project to auction off. In particular, the buyer may decide to hold an auction for the project design which has a lower probability of being appropriate, in an effort to enhance competition at the bidding stage or to seize a fraction of the renegotiation gain (i.e., the reduced cost) which accrues to some contractors.

References


Appendix

**Lemma 1.** The bidders bid accordingly to a weakly monotonic bidding function. That is, if \( t_i = \gamma_i(\theta_i) \), then \( \gamma'_i(\theta_i) \geq 0 \) for any \( \theta_i \in \Phi_i \), for \( i = a, b \).

**Proof of Lemma 1.** The assumptions required to prove this lemma are the following:

(a) Bidders’ pseudo-types are drawn independently. Formally\(^{16}\):

\[
\begin{align*}
\phi_i(\theta_i | \theta_j) &= \phi_i(\theta_i) \\
\phi_j(\theta_j | \theta_i) &= \phi_j(\theta_j)
\end{align*}
\]

(b) Bidders’ expected profits must decrease monotonically in their own types: \( \frac{\partial \pi_i}{\partial \theta_i} < 0 \) for \( i = a, b \).

(c) Firms’ expected profits must be weakly supermodular: \( \frac{\partial^2 \pi_i}{\partial \theta_i \partial \theta_j} \geq 0 \) for \( i = a, b \).

In our model, the expected profit function of bidder \( i \) takes on the following form:

\[
E_{\theta_i} \pi_i(t_i, \theta_i) = (t_i - \theta_i) Pr(t_i < \gamma_j(\theta_j))
\]

where \( \gamma_j(\theta_j) \) is firm \( j \)'s bid function which solely depends on his pseudo-type. We can define \( p_i(t_i) \) as the conditional probability of \( i \)'s winning the procurement auction with a bid equal to \( t_i \). Formally:

\[
p_i(t_i) = \int_{Pr(t_i < \gamma_j(\theta_j))} \phi_j(\theta_j) d\theta_j
\]

\(^{16}\)Bear in mind that the generic bidder’s pseudo-type, \( \theta_i \), is drawn from a distribution \( \Phi_i \), with density \( \phi_i \), on the interval \([\underline{\theta}_i, \overline{\theta}_i]\).
which is a weakly decreasing function of \( t_i \), because of assumption (c).

Now, suppose that \( \tilde{t}_i \) and \( t'_i \) are the best response of player \( i \) when his type are \( \tilde{\theta}_i \) and \( \theta'_i \), respectively. If so, for any \( \tilde{t}_i \) and \( t'_i \) it must hold that:

\[
E_{\theta_i} \pi_i(\tilde{t}_i, \tilde{\theta}_i) = (\tilde{t}_i - \tilde{\theta}_i)p_i(\tilde{t}_i) \geq (t'_i - \tilde{\theta}_i)p_i(t'_i) \tag{5}
\]

by definition of best response. Note that the right-hand side of (5) can be written as:

\[
t'_i p_i(t'_i) - \theta'_i p_i(t'_i) + \theta_i p_i(t'_i) - \tilde{\theta}_i p_i(t'_i) = (t'_i - \theta'_i) p_i(t'_i) + (\theta'_i - \tilde{\theta}_i) p_i(t'_i)
\]

Therefore, (5) can be rewritten as

\[
E_{\theta_i} \pi_i(\tilde{t}_i, \tilde{\theta}_i) \geq E_{\theta_i} \pi_i(t'_i, \theta'_i) + (\theta'_i - \tilde{\theta}_i) p_i(t'_i) \tag{6}
\]

And, if \( \theta'_i > \tilde{\theta}_i \), we attain that:

\[
p_i(\tilde{t}_i) \geq \frac{E_{\theta_i} \pi_i(\tilde{t}_i, \tilde{\theta}_i) - E_{\theta_i} \pi_i(t'_i, \theta'_i)}{\theta'_i - \tilde{\theta}_i} \geq p_i(t'_i) \tag{7}
\]
as the numerator is always positive due to assumption (b). Furthermore, if we let \( \theta'_i \to \tilde{\theta}_i \) we have that

\[
\frac{\partial E_{\theta_i} \pi_i(t_i, \theta_i)}{\partial \theta_i} = -p_i(t_i)
\]

Since the probability that \( i \) wins the auction when his type rises does not increase and the fact that the function \( p_i \) in weakly decreasing in \( t_i \) it cannot be that \( \theta'_i > \tilde{\theta}_i \) and \( t'_i < \tilde{t}_i \).

\[\Box\]

**Lemma 2.** The distribution of the equilibrium bids of the weak firm first order stochastically dominates that of the strong firm, if hazard rate dominance is fulfilled. That is, if \( \frac{\phi_w(t)}{1 - \Phi_w(t)} < \frac{\phi_s(t)}{1 - \Phi_s(t)} \forall \theta \in [\theta_w; \bar{\theta}_s] \), then \( \rho'_w(t) \frac{\phi_w(t)}{1 - \Phi_w(t)} < \rho'_s(t) \frac{\phi_s(t)}{1 - \Phi_s(t)} \) and as a result, \( p_w(t) < p_s(t) \). In addition, the weak bidder will bid more aggressively than the strong bidder for any bid on the interior of their common support, that is, \( \forall t \in (\bar{t}, \bar{\tilde{t}}), \) it holds that \( g_w(t) > g_s(t) \)

**Proof of Lemma 2.** Again, define the bidding function as \( t_i = \gamma_i(\theta_i) \). Then, the inverse bid function, \( g_i(t_i) \) is defined as follows:

\[
g_i(t_i) = \gamma_i^{-1}(t_i) = \theta_i
\]

Bidder \( i \) chooses his bid so as to maximize his expected profit:

\[
\max_i (t_i - \theta_i)[1 - \Phi_j(g_j(t))] \]

The first order condition is

\[
\frac{1}{t - \theta_i} = g'_j(t) \frac{\phi_j(g_j(t))}{1 - \Phi_j(g_j(t))}
\]
The equilibrium inverse bid functions can be found as a solution to a system of first order differential equations\textsuperscript{17}:
\[
\begin{align*}
\frac{1}{t-g_w(t)} &= g'_w(t) \frac{\phi_w(g_w(t))}{1-\Phi_w(g_w(t))} \\
\frac{1}{t-g_t(t)} &= g'_w(t) \frac{\phi_w(g_w(t))}{1-\Phi_w(g_w(t))} \\
\frac{1}{t-H_w(p_w(t))} &= p'_w(t) \frac{\phi_w(g_w(t))}{1-p_w(t)} \\
\frac{1}{t-H_s(p_s(t))} &= p'_s(t) \frac{\phi_s(g_s(t))}{1-p_s(t)}
\end{align*}
\]
Now, define \(p_i(t) = \Phi_i(g_i(t))\) and \(H_i(p_i) = \Phi_i^{-1}(p_i(t)) = g_i(t)\). The system of equations becomes:
\[
\begin{align*}
\frac{1}{t-H_w(p_w(t))} &= p'_w(t) \\
\frac{1}{t-H_s(p_s(t))} &= p'_s(t)
\end{align*}
\]
Since hazard rate dominance implies first order stochastic dominance, \(\Phi_w(\theta) < \Phi_s(\theta)\) and as a result, \(H_s(p) < H_w(p), \forall p \in (0,1)\). Moreover, for any equilibrium bid the probability that the weak bidder wins the auction is lower as \(p_w(t) = 1 - \Phi_s(\theta_s) < 1 - \Phi_w(\theta_w) = p_s(t)\).

Then
\[
\frac{p'_w(t)}{1-p_w(t)} = \frac{1}{t-H_w(p_w(t))} < \frac{1}{t-H_s(p_s(t))} = \frac{p'_s(t)}{1-p_s(t)} \forall t \in (\bar{t}, \bar{\bar{t}})
\]
as \(H\) is decreasing in \(p\). Moreover, the above condition can be rearranged so as to explain why the weak bidder bids consistently more aggressively than the strong bidder. First consider that if \(\bar{\theta}_s < \bar{\theta}_w\), then at \(t = \bar{t}\), \(g_w(\bar{t}) > g_s(\bar{t})\). Instead if \(\bar{\theta}_s = \bar{\theta}_w\), then \(g_w(\bar{t}) = g_s(\bar{t})\), and because of HRD, it must be that \(g'_w(t) > g'_s(t)\) in a neighborhood of \(t = \bar{t}\):
\[
\frac{1}{t-g_w(\bar{t})} = g'_w(\bar{t}) \frac{\phi_w(g_w(\bar{t}))}{1-\Phi_w(g_w(\bar{t}))} = g'_s(\bar{t}) \frac{\phi_s(g_s(\bar{t}))}{1-\Phi_s(g_s(\bar{t}))} = \frac{1}{t-g_w(\bar{t})}
\]
But if so, for \(t \in (\bar{t}, \bar{\bar{t}})\), it cannot be that \(g_s(t) > g_w(t)\), otherwise:
\[
\frac{1}{t-g_s(t)} = g'_w(t) \frac{\phi_w(g_w(t))}{1-\Phi_w(g_w(t))} > g'_s(t) \frac{\phi_s(g_s(t))}{1-\Phi_s(g_s(t))} = \frac{1}{t-g_w(t)}
\]
Because of HRD, that condition would imply:
\[
\frac{p'_w(t)}{1-p_w(t)} > \frac{p'_s(t)}{1-p_s(t)}
\]
which contradicts our previous finding. Therefore, \(g_w(t) > g_s(t)\), namely, the weak bidder consistently bids more aggressively than the strong bidder (or, to put it differently, the degree of bid shading of the weak bidder is lower than that of the strong bidder):
\[
\frac{1}{t-g_s(t)} = g'_w(t) \frac{\phi_w(g_w(t))}{1-\Phi_w(g_w(t))} < g'_s(t) \frac{\phi_s(g_s(t))}{1-\Phi_s(g_s(t))} = \frac{1}{t-g_w(t)} \forall t \in (\bar{t}, \bar{\bar{t}})
\]
\]
\textbf{Proof of Proposition 2}. We first prove that when the wrong specification is auctioned off, the hazard rate of the weak bidder strictly increases and that of the strong bidder weakly increase, namely, if \(\beta > \frac{1}{2}\) then \(\frac{\phi_w}{1-\Phi_w} > \frac{\phi_w}{1-\Phi_w}\) and \(\frac{\phi_s}{1-\Phi_s} \geq \frac{\phi_s}{1-\Phi_s}\).

For the dominance of the weak bidder we have two instances:

\textsuperscript{17}Notice that under the same assumptions of Lemma 1, Maskin and Riley (2000b) prove that the distribution of the winning bids in equilibrium is an interval \([\bar{t}, \bar{\bar{t}}]\). All functions \(g_j\) and \(g'_j\) must then be evaluated at \(t \in [\bar{t}, \bar{\bar{t}}]\).
• either the weak bidder is \( b \) for the auction of \( A \) and \( a \) for the auction of \( B \) in which case it is apparent that \( b \) is better off under \( A \) than \( a \) under \( B \) as the former has a higher probability of enjoying a low cost and getting the rent (\( \beta \) rather than \( 1 - \beta \));

• or the identity of the weak bidder is always the same, i.e., player \( a \). This event takes place when \( \beta h \) is sufficiently high and therefore the weak bidder under \( A \) has a higher probability of enjoying a low cost: \((1 - \beta F(h))\) rather than \((1 - \beta)\).

Also for the weakly dominance of the strong bidder we must account for two cases:

• either the identity of the strong bidder is always the same, i.e., player \( b \). As already said, this event occurs when \( \beta h \) is high, implying that \( b \) enjoys a lower cost with a higher probability when the starting project is \( A \): \( \beta > (1 + \beta F(h) - F(h)) \);

• or the strong bidder is firm \( a \) for design \( A \) and firm \( b \) for design \( B \). It will happen whenever \( h \) is very small so that the two strong types are pretty much the same as the probability of enjoying a low cost tends at 1 and as a consequence, the probability that a renegotiation if requested will succeed is very close to zero.

We can recall and adapt the measure of relative strength of two bidders introduced by Kirkegaard (2009), which is based on the ratio of their distribution functions. Unlike him, we focus on the ratio of the hazard rate functions. We have just shown that holding an auction for the wrong project weakly strengthens the strong bidder and strictly strengthens the weak bidder\(^{18}\). For the results of proposition 2 to hold, we require the weak bidder’s distribution to be reinforced more than the strong bidder’s distribution. Mathematically:

\[
\frac{\phi_{sA}}{1 - \Phi_{sA}} \frac{1}{\phi_{wA}} < \frac{\phi_{sB}}{1 - \Phi_{sB}} \frac{1}{\phi_{wB}}
\]  

when \( \beta > \frac{1}{2} \). Note that a necessary condition for it to happen is that: \( \frac{\phi_{sA}}{\Phi_{sA}} < \frac{\phi_{sB}}{\Phi_{sB}} \). If this is the case, bidders are made less asymmetric when an auction for the wrong project is held and for any bid in the common support of the two auctions the ratio between the degrees of bid shading of the strong and the weak bidder is reduced. Denoting by \( t \) the equilibrium bid in the auction for \( A \) and by \( t' \) the equilibrium bid in the auction for \( B \), it turns out that when \( \beta > \frac{1}{2} \) and condition (10) is fulfilled:

\[
\frac{t^* - g_{sB}(t^*)}{t^* - g_{wB}(t^*)} > \frac{t^* - g_{sA}(t^*)}{t^* - g_{wA}(t^*)} > 1 \forall t^* \in (t', T)
\]

In other words, by holding an auction for the wrong project the buyer manages to stiffen competition as he makes the bidders bid consistently more aggressively. However this benefit may be outweighed by an excessive expected rent that the buyer has to pay under the wrong specification. If \( \beta > \frac{1}{2} \) and he decides to hold an auction for \( A \), the increase in the rent he expects to pay is: \((2\beta - 1)h \). On top of that, we must emphasize that when \( \beta \) approaches unity and

\(^{18}\)In investigating firms’ incentives to invest in cost reduction in a procurement auction, Arozamena and Cantillon (2004) model the investment as a reduction of the ex-ante distribution of costs (a distributional upgrade) which is similar to the effects that the auction of a wrong project design brings about in our paper.
the value of \( h \) is negligible the two bidders are quite symmetric while an excessive value of the hold-up rent would undo the competitive effect brought about by the wrong auction. These two forces explain why for any value of \( \beta \in \left( \frac{1}{2}, 1 \right) \), there exists a threshold value \( h^* \) above which it is no longer profitable for the buyer to turn to the wrong auction. The higher \( \beta \) the lower the value of \( h^* \).

In the second part of Proposition 2, it is stated that if the identity of the strong bidder, who is the bidder more likely to win the auction, depends on which project is auctioned off, then ex-ante allocative efficiency is not achieved as the buyer grants the wrong project to the wrong firm, namely the firm whose expected cost of production is higher. Again, consider the case of \( \beta > \frac{1}{2} \). If the identity of the strong bidder changes, it means that the auction for the right project (i.e., \( B \)) would more likely award the contract to firm \( b \). Instead, if the buyer holds an auction for project \( A \), and he is induced to do so whenever the conditions of proposition 2 are fulfilled, the winner is more likely to be firm \( a \). Even though the wrong auction would be beneficial to the buyer, it would be detrimental from a social point of view. The social welfare when \( A \) is auctioned off and firm \( a \) wins the contract is:

\[
SW(A) = v - \beta(1-F(h))h - [1-\beta F(h)]\tilde{c}_l - \beta F(h)\tilde{c}_h
\]

The social welfare when \( B \) is auctioned off and \( b \) wins the contract is:

\[
SW(B) = v - (1-\beta)(1-F(h))h - [1-F(h)]\tilde{c}_l - [F(h)(1-\beta)]\tilde{c}_h
\]

Note that since \( \beta > (1-\beta) \), the expected hold-up rent is higher when \( A \) is the starting project. For the same reason, there is a higher probability that the cost of production will be high under the wrong project.

\[\square\]

**Proof of Lemma 4.** It suffices to compare the two first order differential equations which describe the behavior of the bidder when he is handicapped and when he is not. For project specification \( A \) (for design \( B \) the same reasoning applies):

\[
\frac{1}{t - g^{\text{hand}}_a(t)} = g'_a(t + \beta \rho) \frac{\phi_b(g_b(t + \beta \rho))}{1 - \Phi_b(g_b(t + \beta \rho))} \geq g'_a(t) \frac{\phi_b(g_b(t))}{1 - \Phi_b(g_b(t))} = \frac{1}{t - g_a(t)}
\]

The weak inequality is satisfied since \( g'_b \) is nondecreasing and the hazard rate of bidder \( b \) is strictly increasing. Thus, the degree of bid shading of bidder \( a \) diminishes when he is positively handicapped, namely, \( t - g^{\text{hand}}_a(t) \leq t - g_a(t) \).\(^{19}\)

\[\square\]

**Proof of Proposition 3.** First and foremost, a crucial role is played by the assumption on the ratio between the hazard rate of the strong and of the weak bidder which must be lower when the wrong project is auctioned off. As we have seen in the proof of proposition 2, as long as this condition holds the bidders are made to bid consistently more aggressively if the wrong project design is initially chosen by the buyer. It is the existence of this competition effect which creates

\(^{19}\) Furthermore, notice that the degree of risk shading of the bidder positively handicapped decreases in \( \beta \) and \( \rho \).
scope for auction manipulation and the extent to which the wrong auction can benefit the buyer depends on the values of $h$ and $\beta$, as stressed in the paper.

For combinations of $\beta$ and $h$ sufficiently low, the auction for the wrong project is won by the firm who is less efficient ex-ante as the handicap is not large enough to reduce the asymmetry brought in by the initial choice of the project design\textsuperscript{20}. If this is the case, the buyer receives lower and more aggressive bids for the wrong project than for the right one and this positive effect more than compensate for the larger payment he expects to incur at the renegotiation stage (it is larger as there is a higher probability of going through a renegotiation).

Nonetheless note that, for any given $\beta$, if $h$ increases, so does the value of the handicap on the initially favored bidder as $\rho$ is affected positively by $h$. Therefore, for values of $h$ larger than a cutoff, $\hat{h}$, the most efficient contractor ex-ante will be more likely to win the auction for the wrong project. In addition, consider that if the probability of entering a renegotiation rises, so does the weight on the optimal handicap $\rho$ and, as a result, the level of the cutoff above which allocative efficiency is satisfied, $\tilde{h}$, decreases (point 2 of the proposition).

Thus, for values of $h$ sufficiently large, the most efficient bidder ex-ante will always win the contract, irrespective of which project design is auctioned off. The buyer will still find it convenient to hold an auction for the wrong project as long as the probability of entering a renegotiation is not too high. In doing so, the buyer will receive more aggressive, though typically higher, bids but he will lean towards the wrong project design as he can gain from the sharp difference existing between the utility he expects to earn at the renegotiation stage in the two different auction settings\textsuperscript{21}. Nonetheless, this difference shrinks as the probability of going through a renegotiation increases and, as a consequence, the cutoff value $\hat{h}$ is decreasing in $\beta$.

\textsuperscript{20}That is, $\beta > \frac{1}{2}$, the buyer decides to hold an auction for project design $A$, and firm $a$ is awarded the contract, although it would have been firm $b$ to be granted the contract, if an auction for the right project, $B$, had been held.

\textsuperscript{21}For instance, assume that $\beta > \frac{1}{2}$, and the most efficient firm ex-ante, firm $b$, would always be awarded the contract, regardless of the initial choice of the design. Even though by auctioning off design $A$ the buyer would expect to pay a higher upfront payment to firm $b$, he would also expect to make substantial profits at the renegotiation stage as firm $b$ would be eager to deliver specification $B$ rather than $A$ and, therefore, he could ask for a share of the design change gain which accrues to the firm.