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Construction of Consistent Panels of Purchasing Power Parities (PPPs) for Comparisons of Real Incomes across Countries: A State-Space Approach

Abstract

Purchasing power parities (PPPs) are necessary for the conversion of nominal incomes into real incomes for purposes of comparison across countries. The International comparison Program (ICP) is the main source of PPPs but PPPs are compiled only for a few benchmark years with coverage limited to participating countries. Consequently, the Penn World Tables (PWT) has become the main source of PPPs and real income data for growth studies. Despite the popularity of the PWT over the last two decades, the methodology underlying PWT construction has received little attention. The main objective of the paper is to propose a method for the compilation of a consistent panel of PPPs, and real incomes, using an econometric framework that integrates various steps involved in the compilation of the PWT and also uses all the PPP benchmark data from various phases of the ICP. The new approach proposed allows us to compute PPP predictors with standard errors for both ICP- participating and non-participating countries and non-benchmark years that are consistent with observed trends in national prices. The econometric approach suggested utilises a state-space formulation of a model with errors spatially correlated across countries. The new method is illustrated using an OECD data set for the period 1970 to 2000.

JEL Classification: C53, C33

Keywords: purchasing power parities, ICP, state-space

Construction of Consistent Panels of Purchasing Power Parities (PPPs) for Comparisons of Real Incomes across Countries: A State-Space Approach

1. Introduction

In a globalised world there is an ever increasing demand for internationally comparable data on major economic aggregates such as gross domestic product (GDP), private and government consumption, and gross fixed capital formation. Over the last four decades, there has been a consensus that market exchange rates are not suitable for converting economic aggregate data from different countries expressed in respective national currency units¹. Instead, purchasing power parities (PPPs) of currencies which measure price level differences across countries are widely used for purposes of converting nominal aggregates into real terms.² PPP-converted real per capita incomes are used in influential publications like the World Development Indicators of the World Bank (World Bank, 2006 and other years) and the Human Development Report (UNDP 2006) which publishes values of the Human Development Index (HDI) for all countries in the world. The PPPs are also used in a variety of areas including: the study of global and regional inequality (Milanovic, 2002); measurement of regional and global poverty using international poverty lines like \$1/day and \$2/day (regularly published in WDI, World Bank, 2006); the study of convergence and issues surrounding carbon emissions and climate change (Castles and Henderson, 2003; McKibbin and Stegman, 2005); and in the study of catch-up and convergence in real incomes (Durlauf et al, 2005; Sal-i-Martin, 2002; Barro and Sal-i-Martin, 2004).

What are the main sources of PPP data? The PPP data are compiled under the International Comparison Program which began as a major research project by Kravis and his associates at the University of Pennsylvania in 1968 and more recently conducted under the auspices of the UN Statistical Commission. Due to the complex

¹ For a detailed discussion of the issues relating to the use of exchange rates, the reader is referred to Kravis et al (1982) as well as the ICP Handbook available on the World Bank website. In addition the most recent publication from the Asian Development Bank on the 2005 comparisons in the Asia Pacific (<http://adb.org/Documents/Reports/ICP-Purchasing-Power-Expenditures/default.asp>) also provides an in-depth discussion on the use of exchange rates and purchasing power parities.

² Nominal values refer to aggregates expressed in national currency units, and, in contrast, real aggregates are obtained by converting nominal values using PPPs. These are termed “real” since the use of PPPs eliminates price level differences.

nature of the project and the underlying resource requirements, the project has been conducted roughly every five years beginning in 1970. The latest round of the ICP for the 2005 benchmark year has just been completed. In the more recent years, beginning from early 1990's, the OECD and EUROSTAT have been compiling PPPs roughly every three years. The country coverage of the ICP in the past marks has been limited with 64 countries participating in the 1993 benchmark comparisons. However this coverage has increased dramatically to 147 for the 2005 benchmark year. Details of the history of the ICP and its coverage are well documented in the recent report of the Asian Development Bank (<http://adb.org/Documents/Reports/ICP-Purchasing-Power-Expenditures/default.asp>).

Generally the coverage of countries in various ICP benchmarks has been limited³. However, international organizations such as the World Bank and the United Nations, as well as economists and researchers, seek PPP data for countries not covered by the ICP and also for the non-benchmark years. For most analytical and policy purposes there is a need for PPPs covering all the countries and a three to four-decade period⁴. The Penn World Tables has been the main source of such data. Summers and Heston are pioneers in this field. Summers and Heston (1991) provides a clear description of the construction of the earlier versions of the Penn World Tables. The most recent version, PWT 6.2, is available on URL: <http://pwt.econ.upenn.edu>, covers 170 countries and a period in excess of five decades starting from 1950. In addition to the PWT, there are real gross domestic product (GDP) series constructed by Angus Maddison (Maddison, 1995; 2007). The Maddison series are available on Groningen Growth and Development Centre website: www.ggdcc.net/dseries/totecon.html) and the series constructed by the World Bank. The Maddison series make use of a single benchmark and national growth rates to construct panel data of real GDP and no estimates are available for non-benchmark countries. The World Bank series are based on the methodology described in Ahmad (1996) and the series make use of a single benchmark year for which extrapolations to non-benchmark countries are

³ A notable exception is the current 2005 Round of the ICP which has an impressive coverage of 146 countries. It covers the People's Republic of China for the first time and India participated in 2005 only for the second time in history (its last participation was in 1985).

⁴ For example, the Human Development Index is computed and published on an annual basis. Similarly, the World Development Indicators publication provides PPP converted real per capita incomes for all the countries in the world for every year.

derived using a regression-based approach. The benchmark and non-benchmark comparisons are extrapolated using national growth rates⁵.

The current method for the construction of time series of PPPs, PWT, for a large number of countries is a two-step method: (i) extrapolation of PPPs to non-benchmark countries in an ICP benchmark year using ICP benchmark data (normally from the most recent available exercise) and national level data through the use of cross-sectional regressions; and (ii) extrapolation to non-benchmark years. The second step combines the information from step (i) with GDP deflators from national accounts data, to produce the tables. Details of the PWT methodology can be found in Summers and Heston (1991) and Heston, Summers and Aten (2002).⁶

There are several important issues associated with the PWT methodology. First and foremost is the problem of time-space consistency of the data produced from different benchmarks. It is quite clear that a set of time-space comparisons can be derived using PPPs from just one benchmark and that such comparisons are not invariant to the choice of the benchmark data used. For example, use of 1990 benchmark data may result in one set of tables and the use of 1996 or 1999 may result in a very different set of tables of PPPs, real incomes and other aggregates. In solving this problem, the PPP data from the most recent benchmark comparison from the ICP is taken as the preferred starting point and the extrapolations across space and over time are derived using country-specific growth rates. This choice of a single benchmark to construct PWT means that a large body of data from other benchmarks are not utilised. Even when attempts are made to make use of information from several benchmarks, no clear methodology for combining information from different benchmarks is currently available.

A related problem associated with the use of PWT and other available series is the absence of any measures of reliability such as standard errors. Most researchers using PWT data consider them to be similar to data from national accounts or other national

⁵ We define “national growth rates” in the next section.

⁶ A description of the earlier attempts to construct panels of PPPs can be found in Summers and Heston (1988).

or international sources. There is no general recognition that the data presented in the Penn World Tables are indeed based on predictions from regression models and that they are also projections over time. Thus the PWT data should be treated as predictions with appropriate standard errors. Though the PWT data provide an indication of the quality of data for different countries, there are no quantitative indicators of reliability in terms of confidence intervals for predictions.

The main objective of this paper is to propose a methodology that will allow the joint use of *all* benchmark PPP data with data on national price deflators available for purposes of extrapolations and projections. The methodology makes *efficient* use of all the information available and obtains optimal predictors of PPPs for all the countries and time periods, as well as making possible the derivation of standard errors associated with the PPPs thereby providing measures of errors in predictions for various macroeconomic aggregates.

The paper proposes an econometric model and uses a state-space formulation of the model that can generate predictions for non-participating countries in different benchmark years and at the same time provides projections of PPPs over time that are consistent with country-specific temporal movements in prices. As an illustration, we develop a fairly general econometric model that allows for cross-sectional correlations through an appropriately specified spatially correlated error structure. The feasibility and performance of the method is demonstrated using data from 23 OECD countries.

The structure of the paper is as follows. Section 2 describes the basic framework underlying the econometric model proposed in the paper. This section provides a brief description of the economic theoretic foundations for the extrapolation of PPPs to non-benchmark countries in any ICP benchmark year. Section 3 provides an econometric formulation of the problem of construction of a consistent panel of purchasing power parities. The basic econometric model and its state-space formulation are also presented. Section 4 discusses the estimation strategy adopted in the paper. An empirical application based on data for 23 OECD countries is presented in Section 5. The paper concludes with a few remarks in Section 6.

2. Combining Economic Theory with Available Data

The econometric methodology proposed is designed to make optimal use of all the information available for the purpose of constructing a panel of PPPs. There are four principal sources of data available from national and international sources. First and foremost are the PPPs for the currencies of all the countries participating in various benchmarks of the ICP since its inception, i.e., from the first benchmark comparison in 1970 till to date. Due to differing degrees of participation of countries in different benchmarks and due to the fact that the benchmark comparisons are conducted roughly once in five years, we have an incomplete panel of PPPs. By definition, PPPs are determined only when currency of a country is chosen as the *base* or *reference* currency.⁷ Therefore, by definition the PPP of the reference currency is always equal to *unity* in all periods. So if country k is chosen as the base currency, then PPP_k is equal to 1 in all periods. This is the second source of information available. The third source of data is in the form of implicit GDP deflators which provide a measure of movements in prices in different countries over time. These deflators provide critical information on country-specific temporal movements in prices. The main source of data on deflators are the national accounts published by countries, generally on an annual basis. The fourth type of data is in the form of information on various socio-economic variables that are used in modeling national price levels or deviations of PPPs from the market exchange rates.

The variable of interest will be denoted by $p_{it} = \ln(PPP_{it})$ for country $i = 1, \dots, N$ and time $t = 1, \dots, T$ where PPP_{it} represents the purchasing power parity of the currency of country i with respect to a reference country currency. Although it is directly unobservable, we can identify four noisy sources of information that can be combined to obtain an optimal prediction⁸, p_{it}^* . They are: theory of national price levels, derived growth rates, ICP benchmark exercises, and reference country identification. We discuss each source in turn and formally develop an econometric model in the next section.

⁷ PPP of currency of country j with respect to the currency of country k is defined as the number of currency units of country j required to purchase the amount of goods and services purchased with one unit of currency of country k . Therefore, PPPs are always defined relative to the currency of a reference country.

⁸ We return to the optimality of the prediction in Section 3.3

2.1 A model derived using the theory of national price levels

There is considerable literature focusing on the problem of explaining the *national price levels*. If ER_{it} denotes the exchange rate of currency of country i at time t , then the national price level for country i (also referred to as the *exchange rate deviation index*) is defined by the ratio:

$$R_{it} = \frac{PPP_{it}}{ER_{it}} \quad (1)$$

For example, if the PPP and ER for Japan, with respect to one US dollar, are 155 and 80 yen respectively, then the price level in Japan is 1.94 indicating that prices in Japan are roughly double to that in the United States.

Most of the explanations of price levels are based on productivity differences in traded and non-traded goods across developed and developing countries. A value of this ratio greater than one implies national price levels in excess of international levels and *vice versa*. Much of the early literature explaining national price levels (Kravis and Lipsey, 1983, 1986) has relied on the structural characteristics of countries such as the level of economic development, resource endowments, foreign trade ratios, education levels. More recent literature has focused on measures like openness of the economy, size of the service sector reflecting the size of the non-tradeable sector and on the nature and extent of any barriers to free trade (Clague, 1988; Bergstrand, 1991, 1996; Ahmad, 1996).

It has been found that for most developed countries the price levels are around unity and for most developing countries these ratios are usually well below unity. In general it is possible to identify a vector of regressor variables and postulate a regression relationship:

$$r_{it} = \beta_{0it} + \mathbf{x}'_{it} \boldsymbol{\beta}_{sit} + u_{it} \quad (2)$$

where,

$$r_{it} = \ln(PPP_{it} / ER_{it})$$

\mathbf{x}'_{it} a set of conditioning variables

β_{0it} intercept parameter

β_{sit} a vector of slope parameters

u_{it} a random disturbance with specific distributional characteristics.

Equation (2) is clearly not identified as it stands and identifying assumptions about the parameters will be made subsequently.

Provided estimates of β_{0it} and β_{sit} are available, model (2) can provide a prediction of the variable of interest consistent with price level theory.

$$\hat{p}_{it} = \hat{\beta}_{0it} + \mathbf{x}'_{it} \hat{\beta}_{sit} + \ln(ER_{it}) \quad (3)$$

Thus, (3) states that price level theory provides a prediction, \hat{p}_{it} of the variable of interest. We return to the estimation of β_{0it} and β_{sit} in Section 4.

2.2 The derived growth rates of PPPs

The movements in national price level, PPP_{it}/ER_{it} , can be measured through the gross domestic product deflator (or the GDP deflator) for period t relative to period $t-1$ and through exchange rate movements. This is due to the fact that PPPs from the ICP refer to the whole GDP. GDP deflators are used to measure changes in PPP and the national price level. If the US dollar is used as the reference currency to measure PPPs and exchange rates, PPP of country i in period t can be expressed as:

$$PPP_{i,t} = PPP_{i,t-1} \times \frac{GDPDef_{i,[t-1,t]}}{GDPDef_{US,[t-1,t]}} \quad (4)$$

Equation (4) defines the growth rate of PPP_{it} .⁹ GDP Deflators are computed from national accounts. The availability of resources to national statistical offices is likely to be positively related to the level of resources (technical and human) available in

⁹ Equation (4) simply updates PPPs using movements in the GDP deflator of the country concerned. Equation (4) would be a simple identity if PPPs were based on price of a single commodity. However in the case of PPPs at the GDP level, the same argument holds if GDP is treated as a composite commodity.

individual countries. Thus, we assume growth rates are measured with error. Taking logarithm of (4) and accounting for the measurement error:

$$p_{it} = p_{i,t-1} + c_{it} + \eta_{it} \quad (5)$$

where,

$$c_{it} = \ln \left(\frac{GDPDef_{i,[t,t-1]}}{GDPDef_{US,[t,t-1]}} \right)$$

η_{it} is a random error accounting for measurement error in the growth rates

2.3 PPPs computed by the ICP for each benchmark year.

Due to the complexity in the design and collection of the ICP benchmark data (see Chapters 4-6 of the ICP Handbook which can be found on the World Bank ICP website: www.worldbank.org/data/ICP), the observed PPPs are likely to be contaminated with some measurement error. As the surveys for these benchmark exercises are conducted by national statistical offices, the argument made above in relation to measurement errors applies here also. Thus, ICP benchmark observations are given by

$$\tilde{p}_{it} = p_{it} + \xi_{lit} \quad (6)$$

where,

\tilde{p}_{it} is the ICP benchmark observation for participating country i at time t

ξ_{lit} is a random error accounting for measurement error and $E(\eta_{it}\xi_{lit}) = 0$

2.4 Reference Country Definition

The definition of PPP requires a choice of reference country. The reference country is defined to have a PPP of one for all time periods.¹⁰ Thus, we know the value of the variable of interest for the reference country for all time periods. As the USA is taken as the reference country, it then follows that for all t

$$p_{US,t} = 0 \quad (7)$$

¹⁰ PPPs between currencies of two countries are invariant to the choice of the base country. In the current study, we use US dollar as the reference currency which, in turn, gives equation (7). The method proposed here is invariant to the choice of the reference currency. This invariance result is available from the authors upon request.

3. Econometric formulation of the problem

The objective is to produce a panel of predictions of p_{it} (denoted by p_{it}^*) accompanied by standard errors which optimally uses all relevant available data, and is internally consistent in a sense to be defined subsequently.

As a matter of notation, for any quantity a_{it} we define the N-vector \mathbf{a}_t as

$$\mathbf{a}_t = (a_{1t}, a_{2t}, \dots, a_{Nt})'.$$

This notation will be used throughout without further definition. Matrices will be defined in upper case and bold face.

3.1 Assumptions

- a) The errors u_{it} in the regression relationship (2) are assumed to be spatially correlated. We assume an error structure of the form

$$\mathbf{u}_t = \phi \mathbf{W} \mathbf{u}_t + \mathbf{e}_t \quad (8)$$

where $\phi < 1$ and \mathbf{W} ($N \times N$) is a spatial weights matrix.

It follows that $E(\mathbf{u}_t \mathbf{u}_t')$ is proportional to $\mathbf{\Omega}$, where $\mathbf{\Omega} = [(\mathbf{I} - \phi \mathbf{W})(\mathbf{I} - \phi \mathbf{W})']^{-1}$

- b) The measurement errors in the observation of $\ln(PPP_{it})$ during benchmark years, equation (6), are assumed spatially uncorrelated, but might be heteroskedastic. Thus, if ξ_{1it} is a measurement error associated with country i at time t , then

$$\begin{aligned} E(\xi_{1it}) &= 0 \\ E(\xi_{1it}^2) &= \sigma_\xi^2 V_{it} \\ E(\xi_{1it}^2 \xi_{1jt}^2) &= 0 \quad j \neq i \end{aligned} \quad (9)$$

where σ_ξ^2 is a constant of proportionality¹¹.

¹¹ In the empirical section we model V_{it} as inversely related to GDP_{it} . This means that reliability of an observed PPP is lower for low-income countries.

- c) The measurement error in the growth rates are assumed spatially uncorrelated, but might be heteroskedastic. Thus, η_{it} in (5) is assumed

$$\begin{aligned} E(\eta_{it}) &= 0 \\ E(\eta_{it}^2) &= \sigma_{\eta}^2 V_{it} \\ E(\eta_{it}\eta_{jt}) &= 0 \quad j \neq i \end{aligned} \tag{10}$$

where σ_{η}^2 is a constant of proportionality¹².

3.2 An Econometric Model

The econometric problem is one of signal extraction. That is, we need to combine all sources of “noisy” information and extract the signal from the noise. A state-space (SS) is a highly suitable representation for this type of problem. We start by extending equation (5) to define the ‘transition equation’ of the SS:

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{c}_t + \mathbf{a}_t + \boldsymbol{\eta}_t \tag{11}$$

where,

\mathbf{c}_t is the observed growth rate of \mathbf{p}_t (see Section 2.2)

\mathbf{a}_t is an unobserved shift in the level of \mathbf{p}_t due to (possible) structural changes, and

$\boldsymbol{\eta}_t$ is an error with $E(\boldsymbol{\eta}_t)=0$ and $E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') \equiv \mathbf{Q}_t = \sigma_{\eta}^2 \mathbf{V}_t$

Equation (11) allows for the structural change in the level of the variable of interest. These changes are likely to be country specific and would reflect substantial changes in economic policy. An example of such a change would be the floating of the exchange rate.

Furthermore, we assume \mathbf{a}_t can be written in the form

$$\mathbf{a}_t = \mathbf{A}_t \boldsymbol{\gamma} \tag{12}$$

where \mathbf{A}_t is a matrix of appropriately defined intervention dummy variables and $\boldsymbol{\gamma}$ is an unknown parameter vector. Thus,

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{c}_t + \mathbf{A}_t \boldsymbol{\gamma} + \boldsymbol{\eta}_t \tag{13}$$

¹² See footnote 2.

Also, as previously discussed, noisy observations of \mathbf{p}_t are given by (3), a prediction from the regression model, and (6) a measurement by the ICP. Equations (2) and (3) relate the conditioning variables, \mathbf{X}_t , to the price level ratio. As we wish to relate the conditioning variables to the variable of interest, \mathbf{p}_t , re-writing of (2) and (3) to eliminate $\ln(ER_{it})$ is necessary.

From equation (2)

$$r_{it} = p_{it} - \ln(ER_{it}) = \boldsymbol{\beta}_{0it} + \mathbf{x}'_{it} \boldsymbol{\beta}_{sit} + u_{it}$$

and if \hat{p}_{it} denotes the prediction of p_{it} , then

$$\hat{p}_{it} = p_{it} + (\hat{\boldsymbol{\beta}}_{0it} - \boldsymbol{\beta}_{0it}) + \mathbf{x}'_{it} (\hat{\boldsymbol{\beta}}_{sit} - \boldsymbol{\beta}_{sit}) - u_{it} \quad (14)$$

Throughout the paper we will reserve the symbol θ to represent the error in a current estimate of a parameter β .

Thus,

$$\hat{\theta}_{0it} = \hat{\boldsymbol{\beta}}_{0it} - \boldsymbol{\beta}_{0it} \text{ and } \hat{\theta}_{sit} = \hat{\boldsymbol{\beta}}_{sit} - \boldsymbol{\beta}_{sit} \quad (15)$$

it is always possible to write equation (14) in the form

$$\hat{\mathbf{p}}_t = \mathbf{p}_t + \mathbf{X}_t \boldsymbol{\theta} + \mathbf{v}_t \quad (16)$$

where,

$$\boldsymbol{\theta} = [\boldsymbol{\theta}'_1, \dots, \boldsymbol{\theta}'_T]'$$

$$v_{it} = -u_{it}$$

Because the explicit form of \mathbf{X}_t depends on the particular identifying restrictions imposed on $\boldsymbol{\beta}_{0it}$ and $\boldsymbol{\beta}_{it}$, we will define it later in the context of particular cases.

Finally, in order to express these different observations as a single equation, it is convenient to define three 'selection matrices',

$$\mathbf{S}_1 = [1, \mathbf{0}'_{N-1}] \quad (\text{selects the reference country } i=1)^{13}$$

¹³ The selection matrix can be appropriately amended if a country other than country 1 is selected as the numeraire country.

$\bar{\mathbf{S}}_1 = [0, \mathbf{I}_{N-1}]$ (selects countries $i = 2, 3, \dots, N$)

\mathbf{S}_t , is a known $[N_t \times (N-1)]$ matrix which selects N_t participating countries (excluding the reference country) in the benchmark year t .

We are now able to consolidate these sources of information into a single equation on an 'observation vector' \mathbf{y}_t , viz

$$\mathbf{y}_t = \mathbf{Z}_t \mathbf{p}_t + \mathbf{B}_t \mathbf{X}_t \boldsymbol{\theta} + \boldsymbol{\xi}_t \quad (17)$$

with variables defined as follows:

i) Non-benchmark years:

$$\mathbf{y}_t = \begin{bmatrix} 0 \\ \bar{\mathbf{S}}_1 \hat{\mathbf{p}}_t \end{bmatrix}, \mathbf{Z}_t = \begin{bmatrix} \mathbf{S}_1 \\ \bar{\mathbf{S}}_1 \end{bmatrix}, \mathbf{B}_t = \begin{bmatrix} 0 \\ \bar{\mathbf{S}}_1 \end{bmatrix}, \boldsymbol{\xi}_t = \begin{bmatrix} 0 \\ \bar{\mathbf{S}}_1 \mathbf{v}_t \end{bmatrix} \quad (18)$$

$$E(\boldsymbol{\xi}_t \boldsymbol{\xi}_t') \equiv \mathbf{H}_t = \begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & \sigma_u^2 \bar{\mathbf{S}}_1 \boldsymbol{\Omega} \bar{\mathbf{S}}_1' \end{bmatrix} \quad (19)$$

with σ_u^2 a constant of proportionality, and in (18) the countries are ordered so that the reference country is the first row¹⁴

ii) Benchmark years

$$\mathbf{y}_t = \begin{bmatrix} 0 \\ \bar{\mathbf{S}}_1 \hat{\mathbf{p}}_t \\ \tilde{\mathbf{p}}_t \end{bmatrix}, \mathbf{Z}_t = \begin{bmatrix} \mathbf{S}_1 \\ \bar{\mathbf{S}}_1 \\ \mathbf{S}_t \end{bmatrix}, \mathbf{B}_t = \begin{bmatrix} 0 \\ \bar{\mathbf{S}}_1 \\ \mathbf{0} \end{bmatrix}, \boldsymbol{\xi}_t = \begin{bmatrix} 0 \\ \bar{\mathbf{S}}_1 \mathbf{v}_t \\ \boldsymbol{\xi}_{1t} \end{bmatrix} \quad (20)$$

$$E(\boldsymbol{\xi}_t \boldsymbol{\xi}_t') \equiv \mathbf{H}_t = \begin{bmatrix} 0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_u^2 \bar{\mathbf{S}}_1 \boldsymbol{\Omega} \bar{\mathbf{S}}_1' & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \sigma_\xi^2 \mathbf{S}_t \mathbf{V}_t \mathbf{S}_t' \end{bmatrix} \quad (21)$$

$\tilde{\mathbf{p}}_t$ is an $N_t \times 1$ vector of benchmark observations.

Again, σ_u^2 and σ_ξ^2 are constants of proportionality and the first row is the reference country.

¹⁴ The inclusion of the reference country constraint is a necessary condition for invariance of the results to the chosen reference country.

3.3 A State-Space Representation

Equations (13) and (17), together with the matrix definitions (18) to (21), constitute the ‘transition’ and ‘observation’ equations, respectively of a state space model for the unobservable ‘state vector’ \mathbf{p}_t .

However, it is necessary for estimation with the Kalman filter to remove the unobserved term $\mathbf{A}_t\boldsymbol{\gamma}$ from (13). To this end, we define a new state variable $\boldsymbol{\alpha}_t$ by

$$\boldsymbol{\alpha}_t = \mathbf{p}_t - \mathbf{A}_t^*\boldsymbol{\gamma}, \quad (22)$$

where

$$\mathbf{A}_t^* = \sum_{i=1}^t \mathbf{A}_i, \quad (23)$$

It then follows that

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \mathbf{c}_t + \boldsymbol{\eta}_t \quad (24)$$

and the observation equation (17) becomes

$$\begin{aligned} \mathbf{y}_t &= \mathbf{Z}_t(\boldsymbol{\alpha}_t + \mathbf{A}_t^*\boldsymbol{\gamma}) + \mathbf{B}_t\mathbf{X}_t\boldsymbol{\theta} + \boldsymbol{\xi}_t \\ &= \mathbf{Z}_t\boldsymbol{\alpha}_t + \mathbf{X}_t^*\boldsymbol{\delta} + \boldsymbol{\xi}_t \end{aligned} \quad (25)$$

where,

$$\mathbf{X}_t^* = [\mathbf{Z}_t\mathbf{A}_t^* \quad \mathbf{B}_t\mathbf{X}_t]$$

$$\boldsymbol{\delta} = [\boldsymbol{\gamma}' \quad \boldsymbol{\theta}']$$

Given the unknown parameters, $\boldsymbol{\theta}, \boldsymbol{\gamma}$ and hyperparameters $\phi, \sigma_u^2, \sigma_\eta^2, \sigma_\xi^2$ and the distribution of the initial vector, $\boldsymbol{\alpha}_0$, under Gaussian assumptions¹⁵, the Kalman filter computes the conditional (on the information available at time t) mean $\hat{\boldsymbol{\alpha}}_t$, and covariance matrix, $\boldsymbol{\Psi}_t$, of the distribution of $\boldsymbol{\alpha}_t$. Further, $\hat{\boldsymbol{\alpha}}_t$ is a minimum mean square estimator (MMSE) of the state vector $\boldsymbol{\alpha}_t$. When Gaussian assumptions are dropped, the Kalman filter is still the optimal estimator in the sense that it minimizes the mean square error within the class of all linear estimators (see Harvey 1990, 100-112, Durbin and Koopman 2001, Sections 4.2 and 4.3).

¹⁵ The disturbances and initial state vector are normally distributed.

4. Estimation

In order for the Kalman filter to deliver an estimate of the state vector and its covariance matrix, we require estimates of the unknown parameters and a distribution of the initial state vector. The estimation of the parameters of a state-space system can be handled with likelihood based methods (Harvey 1990 125-146) or Bayesian methods (see for instance Durbin and Koopman (2002), Koop and van Dijk (2000), and Harvey et al. (2005)). The results presented in this paper are obtained using likelihood based methods. The distribution of the initial state vector, α_o , is assumed centered at zero and its covariance is assumed to be a diffuse prior unless genuine information is available¹⁶. We return to this issue in Section 5.

We note that under normality of the disturbances, the conditional distribution of the observation vector \mathbf{y}_t is given directly by the Kalman filter¹⁷ (we refer the reader to Harvey 1990 for details).

The unknown parameters of the state-space model given by (24) and (25) cannot be estimated as they stand. Specifically, the vector $\boldsymbol{\theta}$ is not identified without further structure. For the purpose of estimation alternative identifying assumptions could be made. We present three such possible assumptions as follows:

Case 1: Time Varying Intercept and Slopes Invariant

$$\begin{aligned} \beta_{oit} &= \bar{\beta}_o + \beta_{ot}, \quad \text{with } \beta_{ot} = 0 \text{ when } t = 1 \\ \boldsymbol{\beta}_{sit} &= \boldsymbol{\beta}_s \end{aligned} \tag{26}$$

Then,

$$\boldsymbol{\theta} = [\bar{\theta}_0 \quad \boldsymbol{\theta}'_o \quad \boldsymbol{\theta}'_s]'$$

$$\boldsymbol{\theta}_o = [\theta_{o2} \dots \theta_{oT}]'$$

$$\mathbf{X}_t = [\mathbf{j}_N \quad \mathbf{e}'_t \otimes \mathbf{j}_N \quad (\mathbf{x}_{1t} \dots \mathbf{x}_{Nt})']$$

\mathbf{j}_N is a vector of ones

$$\mathbf{e}'_t = \mathbf{0}'_{T-1} \quad t = 1$$

$$= (0, \dots, 1, \dots, 0) \quad t > 1 \quad (1 \text{ is in the } t-1 \text{ position})$$

¹⁶ See Harvey 1990 pp 120-124.

¹⁷ The log likelihood is written in *prediction error decomposition form*

Case 2: Time Varying Intercept and Slopes (non-stochastic)

$$\begin{aligned}\beta_{oit} &= \bar{\beta}_o + \beta_{ot}, \quad \text{with } \beta_{ot} = 0 \text{ when } t = 1 \\ \beta_{sit} &= \beta_{st}\end{aligned}\tag{27}$$

here,

$$\begin{aligned}\theta &= [\bar{\theta}_0 \quad \theta'_o \quad \theta'_s]' \\ \theta_o &= [\theta_{o2} \quad \dots \quad \theta_{oT}]' \\ \theta_s &= [\theta'_{s1} \quad \dots \quad \theta'_{sT}]' \\ \mathbf{X}_t &= [\mathbf{j}_N \quad \mathbf{e}'_t \otimes \mathbf{j}_N \quad (\mathbf{x}_{1t} \dots \mathbf{x}_{Nt})' \mathbf{E}_t] \\ \mathbf{E}_t &= [\mathbf{0}_K \quad \dots \quad \mathbf{I}_K \quad \dots \quad \mathbf{0}_K] \text{ where, } \mathbf{0}_K \text{ are } K \times K \text{ zeros and } \mathbf{I}_K \text{ is in the } t^{\text{th}} \text{ position}\end{aligned}$$

Case 3: Time Varying Intercept and Slopes (stochastic)

$$\begin{aligned}\beta_{oit} &= \bar{\beta}_o + \beta_{ot}, \quad \text{with } \beta_{ot} = 0 \text{ when } t = 1 \\ \beta_{sit} &= \beta_{st} \\ \beta_{st} &= \beta_{s,t-1} + \boldsymbol{\varepsilon}_t\end{aligned}\tag{28}$$

In this case both the intercept and slopes are assumed to be time varying, and in addition the evolution of the slopes follows a random walk. We incorporate β_{st} into the state vector

$$\mathbf{a}_t^* = \mathbf{a}_{t-1}^* + \mathbf{c}_t^* + \boldsymbol{\eta}_t^*$$

where,

$$\begin{aligned}\mathbf{a}_t^* &= [\boldsymbol{\alpha}'_t \quad \boldsymbol{\beta}'_{st}]'; \quad \mathbf{c}_t^* = [\mathbf{c}'_t \quad \mathbf{0}']'; \quad \boldsymbol{\eta}_t^* = [\boldsymbol{\eta}'_t \quad \boldsymbol{\varepsilon}'_t] \\ \mathbf{y}_t &= \mathbf{Z}_t^* \mathbf{a}_t^* + \mathbf{X}_t^* \boldsymbol{\delta} + \boldsymbol{\xi}_t \\ \mathbf{Z}_t^* &= [\mathbf{Z}_t \quad (\mathbf{x}_{1t} \dots \mathbf{x}_{Nt})'] \\ \mathbf{X}_t &= [\mathbf{j}_N \quad \mathbf{e}'_t \otimes \mathbf{j}_N] \\ \boldsymbol{\theta} &= [\bar{\theta}_0 \quad \theta'_0]'\end{aligned}$$

Algorithm

There are two types of parameters to be estimated in the SS, namely, hyperparameters, and coefficients associated with explanatory variables and the level shifts. Hyperparameters are those associated with the covariance structure. In our case these are: $\phi, \sigma_u^2, \sigma_\eta^2, \sigma_\xi^2$ ¹⁸. These parameters must be estimated by numerical maximization of the likelihood function (in a likelihood based estimation). The other parameters, θ, γ in our case, can be estimated by a generalised least squares procedure in conjunction with the numerical maximization of the likelihood function (see Harvey 1990 pp.130-133)¹⁹, or placed in the transition equation and estimated with the state vector²⁰.

Independently of which assumptions are made to identify β_{it} , the algorithm we use can be described in 5 steps.

Step 1: Obtain an initial estimate of β_{it} , $\hat{\beta}_{it}^0$, by regressing r_t on X_t and construct an initial prediction, \hat{p}_{it}^0 , using equation (3).

Step 2: Run SS through KF (or KF/GLS) to obtain estimates of the hyperparameters and θ, γ .

Step 3: Use updated estimate of β_{it} , $\hat{\beta}_{0t} = \hat{\beta}_t^0 - \hat{\theta}_{0t}$, $\hat{\beta}_{sit} = \hat{\beta}_{sit}^0 - \hat{\theta}_{sit}$, to obtain an updated \hat{p}_{it}

Step 4: Repeat 2 and 3 until $\hat{\theta}$ are sufficiently close to zero.

Step 5: Run KF and smoother one more time to obtain \hat{p}_{it}^* and standard errors.

A prediction of PPP_{it} is given by:

$$PPP_{it} = e^{\hat{p}_{it}^* / \tau} \quad (29)$$

where,

¹⁸ and σ_ϵ^2 if Case 3 is used.

¹⁹ The SS in Cases 1 and 2 are set up for the use of the KF/GLS approach

²⁰ The SS in Case 3 is set up to use this approach

$\hat{p}_{i/T}^*$ is the corresponding element of $\hat{\mathbf{p}}_{t/T}^* = \mathbf{a}_{t/T} + \mathbf{A}_t^* \hat{\boldsymbol{\gamma}}$, and

$\mathbf{a}_{t/T}$ is the smoothed estimate of the state-vector

The standard errors for the predicted PPP are computed as follows:

$$se(\hat{P}\hat{P}_{it}) = \sqrt{e^{2\hat{p}_{i/T}^*} e^{\hat{\psi}_{ii,t}} (e^{\hat{\psi}_{ii,t}} - 1)} \quad (30)$$

where,

$\hat{\psi}_{ii,t}$ is the *ith* diagonal element of the estimated covariance of the state vector, $\hat{\boldsymbol{\alpha}}_{t/T}$.

The next section presents an illustration of our proposed method.

5. An Empirical Application

In this section we present the results from the empirical implementation of our state-space approach to construct a complete panel of PPPs for 23 OECD countries for the period 1971 to 2000. We have accessed data for the OECD countries through the OECD and World Bank sites. Several of the countries in the OECD were participants in the ICP project since its first benchmark year. We include 23 countries in this empirical illustration using the US as the reference country.

The sample used provides a unique opportunity to illustrate our method in that there are a reasonable number of benchmark PPPs observations of the state variable for most of these countries. In such a case it might be argued that a simple system combining the benchmark data with the derived growth rates (equations (5) and (6)) should suffice to complete a panel of PPPs without requiring the predictions from the national price level model. We construct such a panel by running the following simplified state-space system (estimates and predictions from this model are labeled as *naïve model* below):

Observation equations:

Non-benchmark years:

$$\mathbf{y}_t = \mathbf{0}, \mathbf{Z}_t = \mathbf{S}_1, \boldsymbol{\xi}_t = \mathbf{0}, E(\boldsymbol{\xi}_t \boldsymbol{\xi}_t') = \mathbf{H}_t = \mathbf{0} \quad (31)$$

Benchmark years

$$\mathbf{y}_t = \begin{bmatrix} 0 \\ \tilde{\mathbf{p}}_t \end{bmatrix}, \mathbf{Z}_t = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_t \end{bmatrix}, \xi_t = \xi_{1t}$$

$$E(\xi_t \xi_t') \equiv \mathbf{H}_t = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \sigma_\xi^2 \mathbf{S}_t \mathbf{V}_t \mathbf{S}_t' \end{bmatrix}$$

Transition equations:

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{c}_t + \boldsymbol{\eta}_t$$

We then estimate the complete system under two alternatives. First, the state-space model is estimated under the restriction that ICP benchmark data do not suffer from any measurement error (we fix $\sigma_\xi^2 = 1E-10$, see equation (9)). However, measurement error is allowed in the transition equations and spatial correlation in the model of national price levels, and the results are labeled *constrained model*. Second, the full state-space model is estimated allowing for measurement errors in the benchmark and transition equations and spatial correlation in the model of national price levels (results are labeled *unconstrained model*).

The data for the empirical example are for the period 1970 to 2000, annual, and we discuss next the *dependent*, *explanatory*, and *covariance related variables*.

Dependent Variable

Benchmark PPP information, GDP Deflators, and exchange rates were collected from the OECD website. Benchmark years were: 1980, 1985, 1990, 1993, 1996 and 1999²¹. Table 1 lists the countries included in the analysis, the currency used and the status of participation of each of the countries in various ICP Benchmark exercises of the past.

²¹ We are indebted to Ms Francette Koechlin (OECD) for providing the ICP Benchmark data.

Explanatory Variables

The following variables were included as explanatory variables in the national price level model:

Euro Dummy: Takes the value of 1 from 1993 onwards for the countries that joined the euro currency by 2000.

FDI%: Foreign direct investment, net inflows (% of GDP)

LE: Life Expectancy in years

SERV%: Services, value added (% of GDP)

OPEN%: Trade (% of GDP)

CPI_{it}/CPI_{US,t}, for $i=1, \dots, N$

Labour Productivity = (Population \times per capita GDP)/ Labour Force

The choice of conditioning variables is based on national price level theory and data availability. The data were obtained from the OECD website and from various issues of the World Development Indicators published by the World Bank on a regular basis

The statistical model and the variables included in the regression equation are adapted from those used in the literature to suit the nature and scope of the current study. In particular, since the model is only illustrative and is applied only to OECD countries, a variable like education has not been included. The effect of productivity differentials on national price levels is captured through the inclusion of a labour productivity measure. The model is a first approximation and further work and refinements are planned for the next stage of this project.

Identifying Assumptions

The illustration has been obtained using the identifying assumptions presented in Case 1 (See equation (26)). Additionally, for the purpose of this illustration γ (see equation (12)) is assumed to be zero.

Covariance related Variables

- a) *Measuring spatial correlation*

A contiguity matrix was constructed using volumes of bilateral trade in 1990. This is the matrix \mathbf{W} (see equation (8)).

b) *Capturing accuracy of benchmark data collection and National Accounts' computation of the national price level.*

We assume that the measurement error in the collection of PPP benchmark data and in the growth rate on the GDP deflator are uncorrelated. However, in both cases they are assumed to be distributed with a zero mean and heteroskedastic with a diagonal covariance which is inversely related to the real per capita (measured in constant US\$ of 1995) of the country.

Distribution of the Initial State Vector

We note that under the assumption $\gamma = \mathbf{0}$, the state vector simplifies to (see equation (22)):

$$\alpha_t = \mathbf{p}_t$$

For this specification we can derive a non-diffuse covariance for the initial state vector, $\alpha_o = \mathbf{p}_o$ by making use of equation (3). Suppose at $t = 0$ we have socio-economic data, \mathbf{X}_o . Then we can define,

$$\mathbf{p}_o = \mathbf{X}_o \boldsymbol{\beta} + \ln(\mathbf{ER}_o) + \mathbf{u}_o \quad (32)$$

where,

$$\boldsymbol{\beta} = [\boldsymbol{\beta}_{oo} \boldsymbol{\beta}_{so}]'$$

$$\mathbf{p}_o = \begin{bmatrix} \mathbf{p}_o^{(1)} \\ \mathbf{p}_o^{(2)} \end{bmatrix}$$

$$\mathbf{X}_o = \begin{bmatrix} \mathbf{X}_o^{(1)} \\ \mathbf{X}_o^{(2)} \end{bmatrix}$$

$\mathbf{X}_o^{(1)}$ and $\mathbf{p}_o^{(1)}$ are the corresponding partition containing the observations from participating countries.

Then a prediction of \mathbf{p}_o and its associated covariance are given by

$$\hat{\mathbf{p}}_o = \mathbf{X}_o \hat{\boldsymbol{\beta}} + \ln(\mathbf{ER}_o) \quad (33)$$

$$\text{cov}(\hat{\mathbf{p}}_o) = \mathbf{\Psi}_o = \sigma^2 \mathbf{X}_o (\mathbf{X}_o^{(1)'} \mathbf{X}_o^{(1)})^{-1} \mathbf{X}_o' \quad (34)$$

We use the expression in (34) to obtain an estimate of the covariance of the initial state vector for the *constrained* and *unconstrained* models. For the *naïve* model we use a diffuse prior.

5.1 Model Estimates

Table 2 presents the parameter estimates with standard errors. Data for 1970 were used to create growth rates where needed, data for 1971-2000 were used for estimation. The years 1971 and 1972 are used as burning off periods²², and therefore not reported in the predictions.

The computed likelihood ratio test for the null hypothesis that the constrained model is correct (Table 2) is statistically significant at the 1% level and therefore the restriction that benchmark data do not suffer from measurement error is rejected by the data (we still present both sets of predictions in the next section for comparison). The time varying intercept shows a slight downward trend over time. All slope parameters are statistically significant and with expected signs with the exception of *FDI*.

5.2 Predictions of PPP

The methodology proposed generates a complete matrix of PPPs for all the 23 countries and for all the years. In order to facilitate presentation and discussion, we have chosen three countries, Australia, Spain and Turkey. These countries have been chosen for illustrating the PPP predictions obtained, and for the purpose of comparing to alternative predictions from other approaches. The reason for this choice is that Australia floated its exchange rate in 1984, providing an excellent opportunity to test our approach during a volatile exchange rate period. Spain is chosen to represent the countries that joined the euro-zone and have participated in all ICP benchmark exercises to date. Finally, Turkey is chosen as it is a country exhibiting hyperinflation

²² This is particularly important when the covariance of the initial state vector is diffuse.

during the sample period providing a challenge to any modeling approach²³. Therefore, selection of Turkey is designed to assess the performance of the method when the country under consideration exhibits extreme price movements.

The results are presented in graphical (for Australia and Spain) and table format (for Turkey). Figures 1 and 2 present the predictions from the naïve model for Australia and Spain, respectively. Figures 3 and 4 present the predictions from the constrained model, and Figures 5 and 6 those from the unconstrained model for Australia and Spain, respectively. Table 3 presents the results for Turkey.

All figures for Australia include PPP predictions computed under three alternative approaches. The first type is the time series of PPPs that can be computed by extrapolating a given benchmark ICP value using the implied growth rate (see equation (4)). One such series can be produced for each country and benchmark year. We present only two of those for illustration, labeled EXT 93B and EXT 90B. They are the extrapolations obtained using the 1990 and 1993 ICP benchmark values, respectively. The second type of predictions is that produced by the Penn World Tables (PWT 6.1). These are presented so that our results can be benchmarked against what is currently commonly used. They are plotted for Australia and Spain and included in Table 3 for Turkey to offer a comparison. Finally, the predictions computed using our approach are presented for all three countries (including a $\pm 2 \times$ Standard Error interval for Australia and Spain and the standard errors for Turkey).

From the results we note that prediction intervals increase in size when the time period under consideration is away from a benchmark year, and reduce significantly at the benchmarks even for models that are unrestricted. The naïve model's prediction interval is useful in highlighting this issue. For the early years of the sample, when no observations on the state variable are available through ICP benchmarking, prediction intervals are extremely wide. The inclusion of predictions from the national price level model becomes crucial in reducing uncertainty. Comparing Figures 1 to 3 and 5, or Figures 2 to 4 and 6 this can be observed. Further, for the case of Turkey, see Table

²³ A complete tableau is available from the authors.

3, it is clear that the absence of ICP benchmarks and predictions from the national price level model for the early period results in implausible PPPs predictions from the naïve model. The estimates are much larger than the exchange rate when they would be expected to be smaller so the resulting ratio PPP/ER is less than one. This is the case for the estimates produced by the two full models. That is, when comparing predictions and standard errors of the naïve model to those of the full system for all three countries it is evident that the full models are superior to the naïve model. As stated in the introduction, most countries in the world have participated in the current round of the ICP benchmarking for the first time²⁴. Thus, the inclusion of the national price level model in the state-space formulation is crucial to both the prediction of the mean value of PPPs as well as its variance.

Concentrating now on the predictions from the full state-space, we note that the prediction interval for benchmark years is much wider in the unconstrained than the constrained case (this can be easily observed from Table 3), while the PPP point prediction for the benchmark value differs minimally from the actual benchmark PPPs for Australia and Spain. These are predictions that quantify and weigh the measurement error from all sources of information.

Figures 1, 3, 5, and 7 present results for Australia. Figure 7 superimposes the movements of the exchange rate over the sample period (ER) on the PPP series produced by the PWT and our unconstrained model. This figure is included to demonstrate that PPP predictions are a smoother series than the exchange rate as expected. After the floating of the AUD in 1984, the ER experienced some highly volatile periods. This structural change in the ER is reflected in the movement of the PPPs. The volatility of the exchange rates combined with the absence of benchmark information before 1985 result in a wider prediction intervals (Figures 1, 3 and 5), although much narrower than those of the naïve model. The extrapolated series (from 1990 and 1993 benchmarks) plotted in Figures 1, 3 and 5 demonstrate how difficult it is to produce point predictions in this situation, given the inconsistency across observation sources and time. Thus, the use of an interval estimate, similar to that

²⁴ Preliminary estimates from the current ICP round, carried out in 2005, have still not been released at the time of submission of this manuscript.

provided by our method, should provide users of the panel with a way of assessing “best” and “worse” case scenario in further applied work.

The predictions for Spain are very representative of most of the “core” OECD group of countries that participated in the ICP from the beginning and are mostly European Union members. Predictions from our model are very close to those produced by the Penn World Table and the later is inside our computed prediction interval for most periods.

Turkey exhibited hyperinflation during most of the sample period. Table 3 shows the explosive increase in the market exchange rate which is reflected in the Benchmark estimates of the ICP. We note the accuracy of our predictions in following this trend, and as mentioned before, the much wider prediction interval estimates for benchmark years obtained from the naïve model.

6. Conclusions

The main objective of the paper is to propose an econometric model using the state-space approach which can be employed in the estimation of a panel of purchasing power parities necessary for constructing a consistent set of internationally comparable real income aggregates. The methodology described here successfully combines data drawn from a number of national and international sources in estimating PPPs. It offers several improvements over the existing PWT approach, which is the only source of such data at the present time. These improvements include a method that: (i) can make use of all the PPP data from the ICP for all the benchmark years since 1970; (ii) can provide optimal predictors for PPPs for ICP-non-participating countries and for non-benchmark years; (iii) produces PPPs that are consistent with observed movements in prices in different countries; and (iv) provides standard errors associated to the PPPs and, therefore, for the estimates of real per capita incomes. To achieve these objectives the paper proposes the use of an econometric model with errors that are spatially correlated cross-sectionally and accounts for measurement error in the available sources of data. The econometric model is re-formulated in a state-space form and estimated using Kalman filtering techniques. The new methodology is applied to an illustrative data set of 23 OECD

countries for the period 1970 to 2000. The results from the illustrative application demonstrate the feasibility of using the model for consistent space-time extrapolation.

The main conclusion from the paper is that it is feasible to develop a more comprehensive econometric approach to the construction of a panel of PPPs than the current practice in constructing extrapolations of PPPs. It is also important to note that the approach proposed here makes optimal use of relevant information from diverse sources and the methodology provides standard errors and, therefore, measures of reliability of predicted values of PPPs.

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Table 1. Countries, Currency and their status of the ICP Participation

COUNTRY		CURRENCY	ICP PARTICIPATION
Australia	AUS	Australian dollar	1985, 1990, 1993, 1996, 1999
Austria	AUT	Euros (1999 ATS euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
Canada	CAN	Canadian dollar	1980, 1985, 1990, 1993, 1996, 1999
Belgium	BEL	Euros (1999 BEF euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
Denmark	DNK	Danish kroner	1980, 1985, 1990, 1993, 1996, 1999
Finland	FIN	Euros (1999 FIM euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
France	FRA	Euros (1999 FRF euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
Germany	DEU	Euros (1999 DEM euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
Greece	GRC	Euros (2001 GRD euro) ^b	1980, 1985, 1990, 1993, 1996, 1999
Hungary	HUN	Forint	1999
Italy	ITA	Euros (1999 ITL euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
Japan	JPN	Yen	1980, 1985, 1990, 1993, 1996, 1999
S. Korea	KOR	South Korean Won	1999
Mexico	MEX	Mexican pesos	1999
Netherlands	NLD	Euros (1999 NLG euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
New Zealand	NZL	New Zealand dollar	1985, 1990, 1993, 1996, 1999
Norway	NOR	Norwegian kroner	1980, 1985, 1990, 1993, 1996, 1999
Portugal	PRT	Euros (1999 PTE euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
Spain	ESP	Euros (1999 ESP euro) ^a	1980, 1985, 1990, 1993, 1996, 1999
Sweden	SWE	Swedish kronor	1985, 1990, 1993, 1996, 1999
Turkey	TUR	Turkish lire	1985, 1990, 1993, 1996, 1999
Great Britain	GBR	Pounds sterling	1980, 1985, 1990, 1993, 1996, 1999
United States	US	US dollars	Reference country

^a Pre-Euro domestic currencies were converted using the *1999 Irrevocable Conversion Rate* (Source: http://www.ecb.int/press/pr/date/1998/html/pr981231_2.en.html)

^b The irrevocable conversion rate of the drachma vis à vis the euro was set at GRD 340.750. Source: <http://www.bankofgreece.gr/en/euro/>

Table 2. Estimated Models

Covariance Parameters	Naïve Model		Constrained Model		Unconstrained Model	
	Estimates	Standard Error	Estimates	Standard Error	Estimates	Standard Error
$\hat{\sigma}_\eta^2$	0.098	0.003	0.223	0.008	0.138	0.013
$\hat{\sigma}_u^2$			0.014	0.002	0.004	0.001
$\hat{\sigma}_\varepsilon^2$	0.006	2.99E-04	1E-10	0.000	0.007	0.005
$\hat{\phi}$			0.884	0.017	0.319	0.174
Equation (2) Parameters						
Intercept			-0.759	0.206	-0.760	0.124
D1973			-1.437	1.694	0.376	0.189
D1974			-1.398	0.270	-1.295	0.049
D1975			-1.407	0.261	-1.282	0.044
D1976			-1.426	0.261	-1.314	0.040
D1977			-1.437	0.262	-1.299	0.037
D1978			-1.489	0.263	-1.245	0.034
D1979			-1.476	0.263	-1.228	0.031
D1980			-1.732	0.249	-1.221	0.028
D1981			-1.515	0.265	-1.319	0.032
D1982			-1.487	0.267	-1.392	0.033
D1983			-1.571	0.266	-1.428	0.033
D1984			-1.579	0.264	-1.460	0.031
D1985			-1.662	0.247	-1.487	0.027
D1986			-1.583	0.264	-1.386	0.031
D1987			-1.482	0.266	-1.331	0.033
D1988			-1.437	0.265	-1.328	0.033
D1989			-1.370	0.263	-1.379	0.031
D1990			-1.260	0.244	-1.349	0.027
D1991			-1.365	0.263	-1.371	0.030
D1992			-1.430	0.263	-1.374	0.030
D1993			-1.594	0.245	-1.457	0.028
D1994			-1.619	0.262	-1.440	0.031
D1995			-1.534	0.262	-1.415	0.031
D1996			-1.480	0.244	-1.449	0.027
D1997			-1.545	0.261	-1.520	0.031
D1998			-1.629	0.261	-1.571	0.031
D1999			-1.713	0.243	-1.593	0.028
D2000			-1.799	0.268	-1.664	0.034
CPIREUS			0.041	0.015	0.031	0.011
EURO_DUMMY			0.045	0.028	0.043	0.015
FDI			0.002	0.004	0.001	0.002
LABOR_PROD			1.72E-05	6.65E-07	1.72E-05	4.34E-07
LE			0.003	0.002	0.002	0.001
OPEN			-0.001	3.08E-04	-0.001	0.000
SERV			0.012	0.002	0.011	0.001
lnL	-618.66		422.12		575.88	

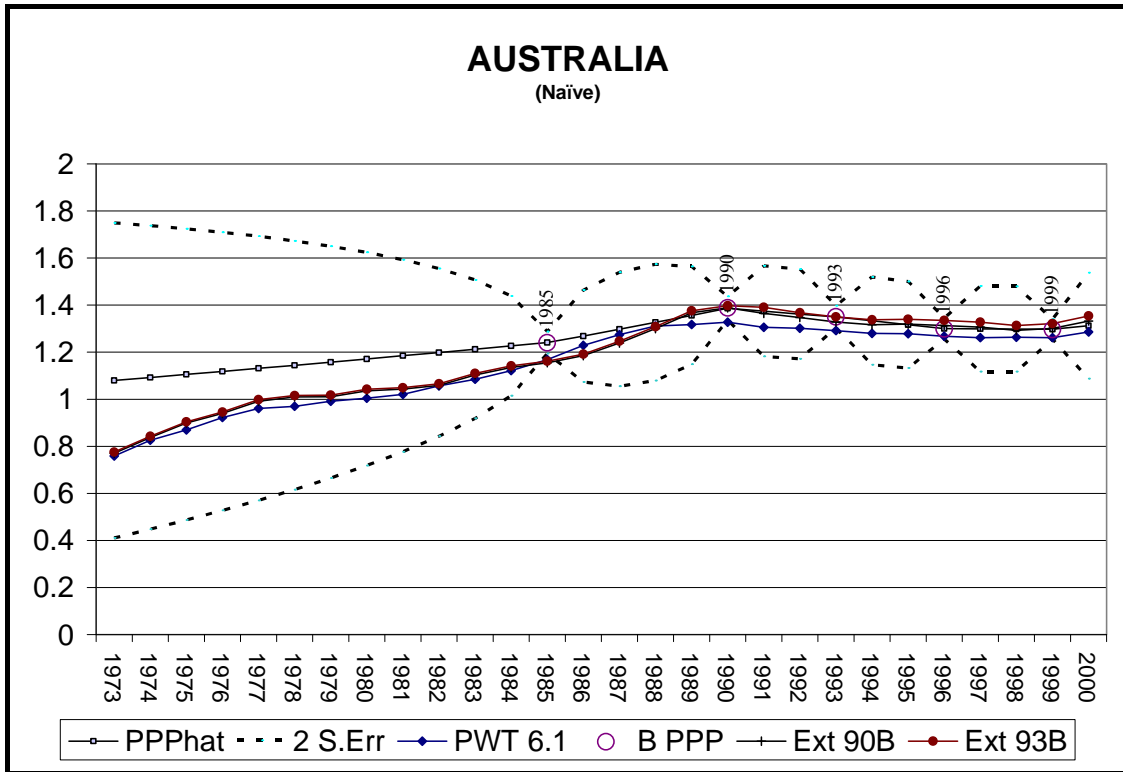


Figure 1. PPP predictions from Extrapolation of the 1990 and 1993 ICP Benchmarks, ICP PPP Benchmarks, Predictions from Penn World Table 6.1 and Prediction (with prediction interval) from naïve model. AUD per \$US.

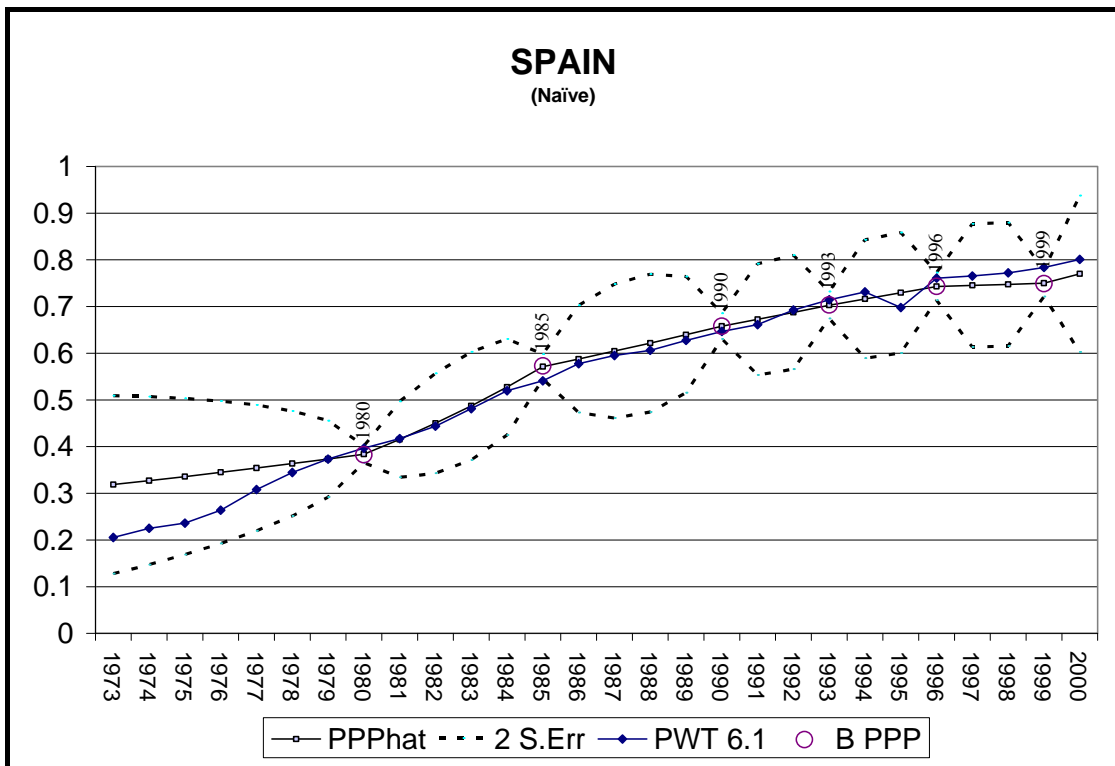


Figure 2. ICP PPP Benchmarks, Predictions from Penn World Table 6.1 and Prediction (with Prediction Interval) from naïve model. Euro per \$US.

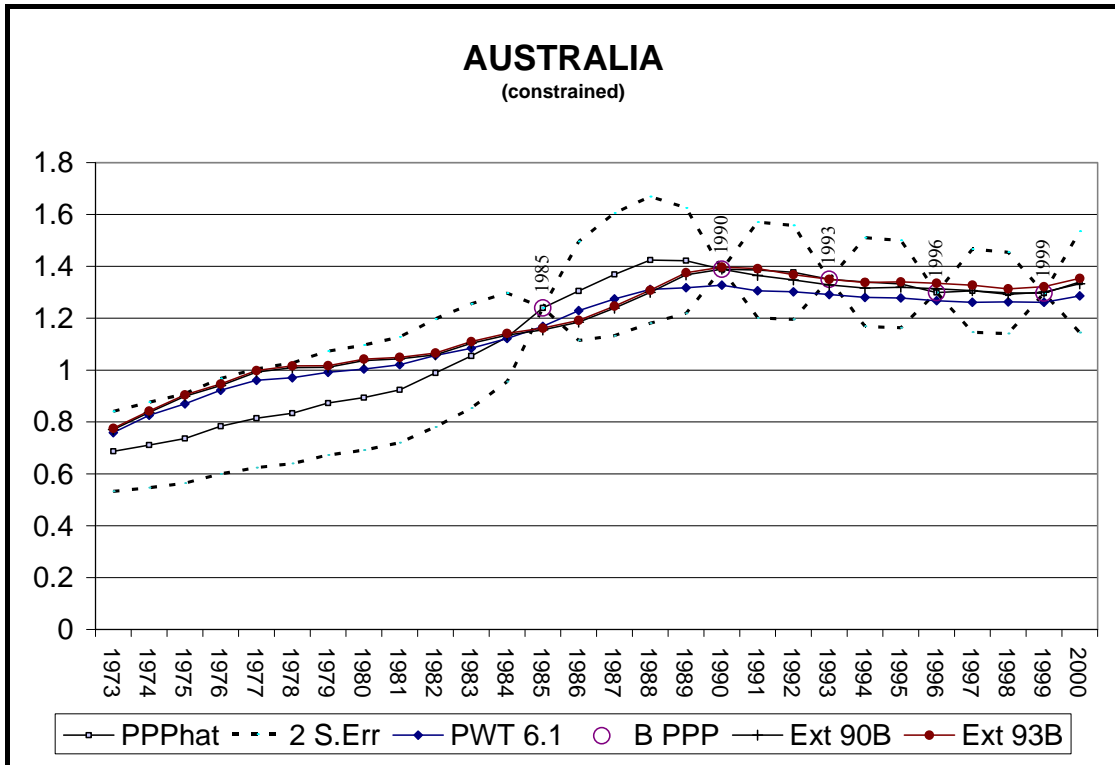


Figure 3. PPP predictions from Extrapolation of the 1990 and 1993 ICP Benchmarks, ICP PPP Benchmarks, Predictions from Penn World Table 6.1 and Prediction (with Prediction Interval) from constrained model. AUD per \$US.

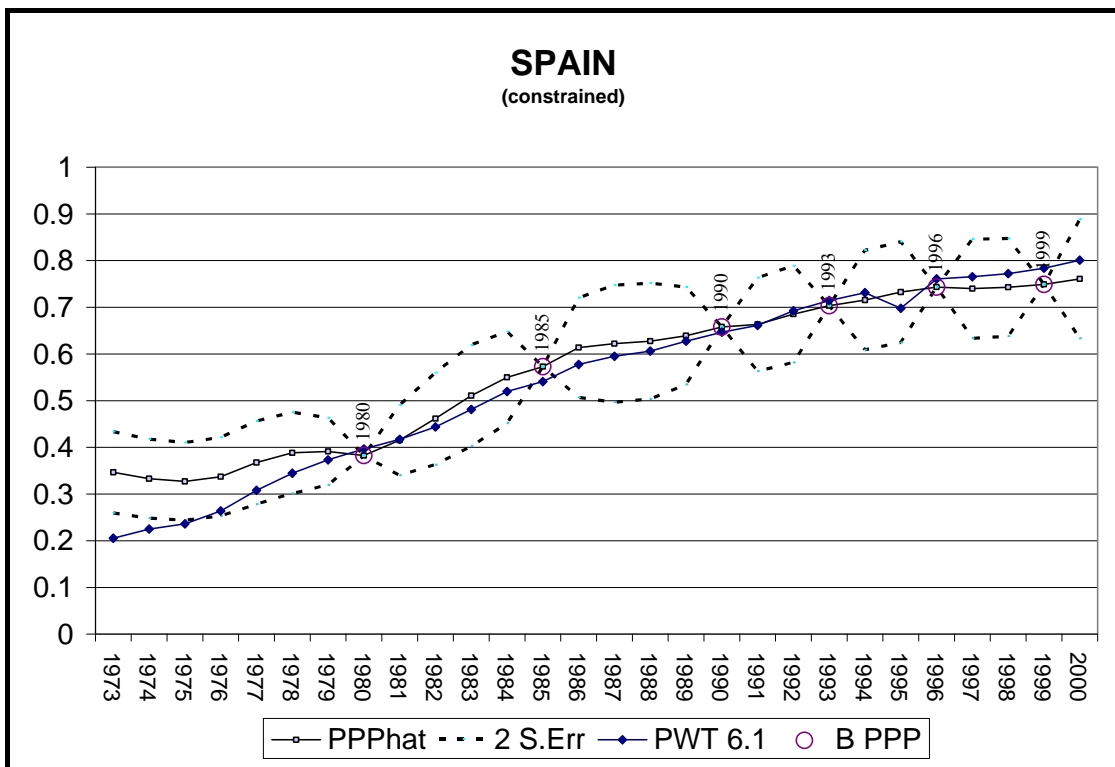


Figure 4. ICP PPP Benchmarks, Predictions from Penn World Table 6.1 and Prediction (with Prediction Interval) from constrained model. Euro per \$US

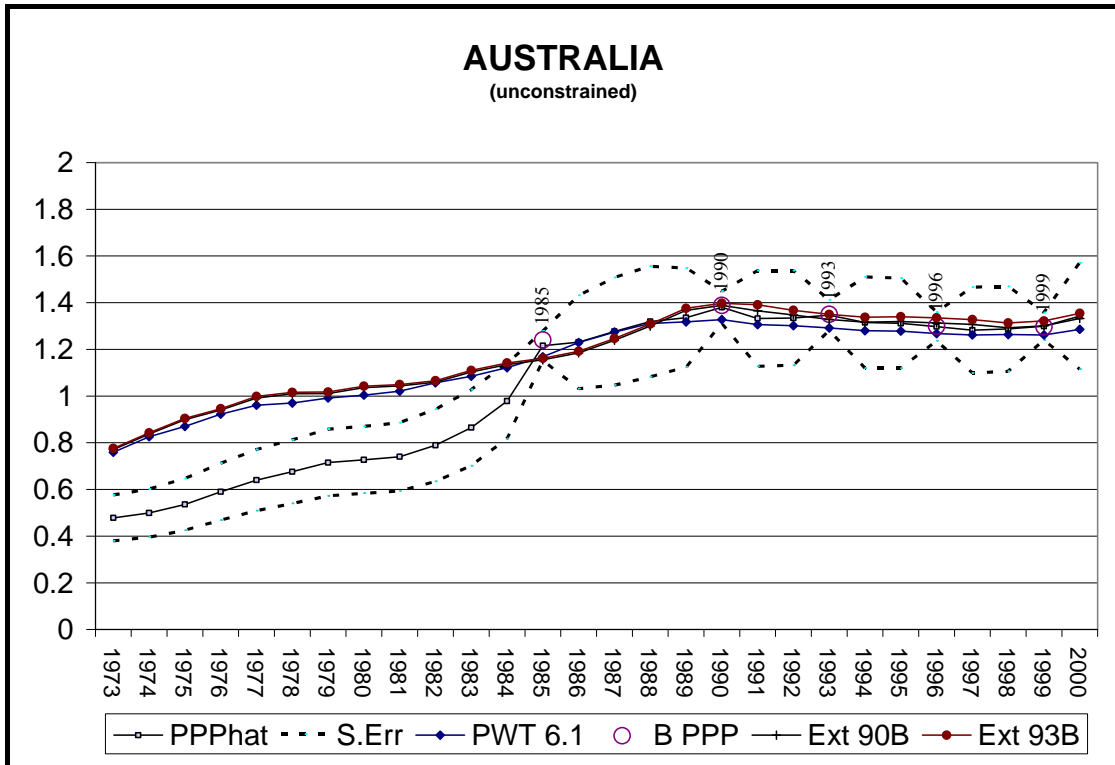


Figure 5. PPP predictions from Extrapolation of the 1990 and 1993 ICP Benchmarks, ICP PPP Benchmarks, Predictions from Penn World Table 6.1 and Prediction (with Prediction Interval) from unconstrained model. AUD per \$US.

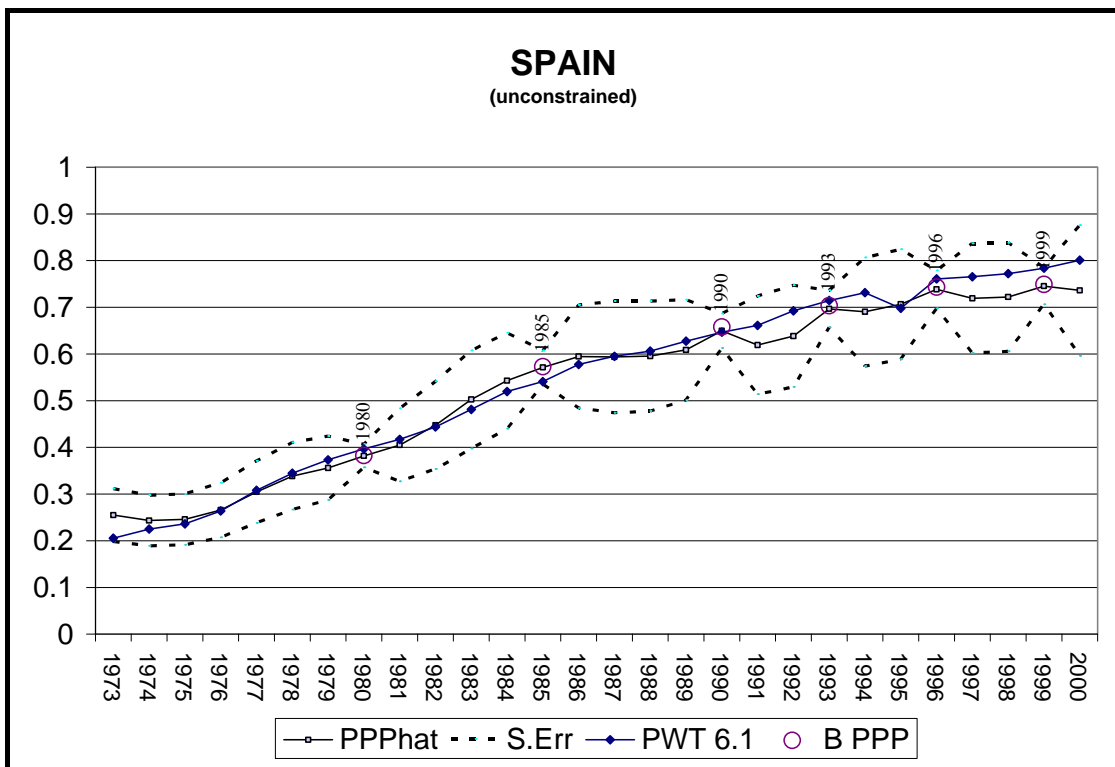


Figure 6. ICP PPP Benchmarks, Predictions from Penn World Table 6.1 and Prediction (with Prediction Interval) from unconstrained model. Euro per \$US

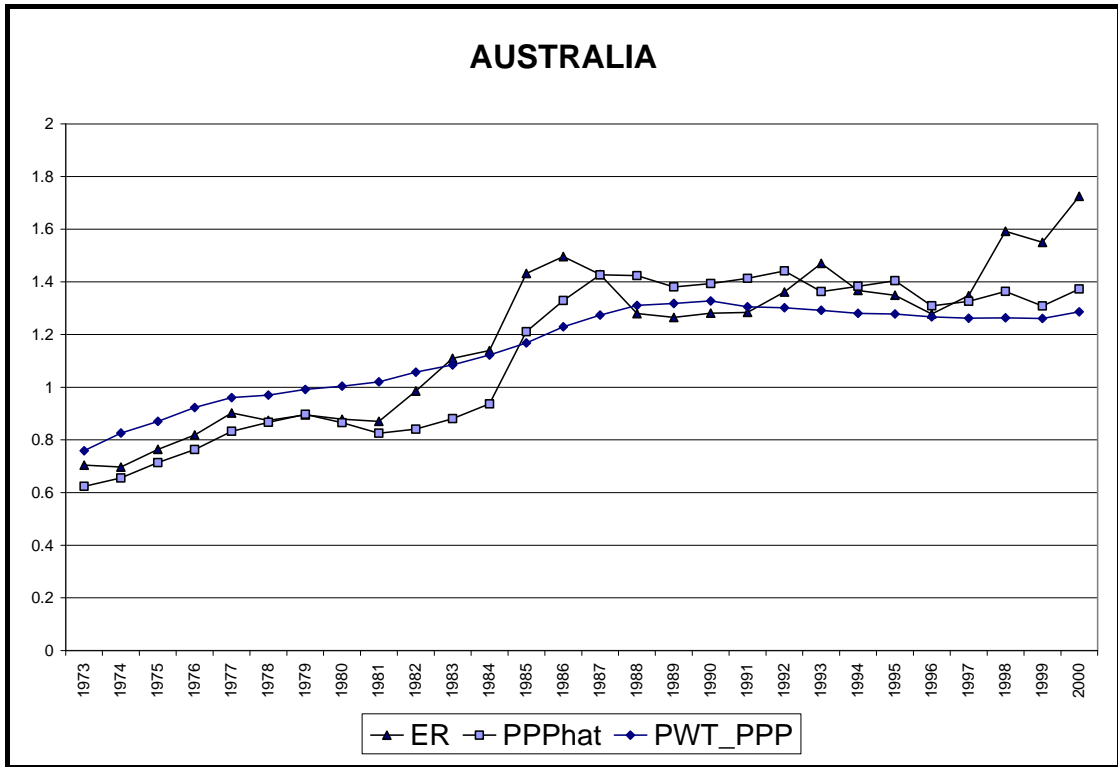


Figure 7. Exchange Rate, PWT6.1 PPP predictions, Unconstrained Model PPP Predictions. AUD per \$US.

Table 3. PPP Estimates for Turkey

	<i>ER</i>	<i>B PPP</i>	<i>PWT</i>	NAÏVE		CONSTRAINED		UNCONSTRAINED	
				<i>P\hat{P}P</i>	<i>se(P\hat{P}P)</i>	<i>P\hat{P}P</i>	<i>se(P\hat{P}P)</i>	<i>P\hat{P}P</i>	<i>se(P\hat{P}P)</i>
1973	14.15		7.41	40.30	57.23	8.66	1.68	7.52	0.46
1974	13.93		8.76	45.11	58.64	10.13	1.95	7.76	0.48
1975	14.44		9.54	50.48	59.85	10.91	2.08	8.29	0.51
1976	16.05		10.47	56.49	60.78	11.73	2.22	8.78	0.54
1977	18.00		12.28	63.23	61.36	13.66	2.58	10.22	0.63
1978	24.28		17.22	70.76	61.48	19.04	3.60	14.69	0.90
1979	31.08		28.03	79.19	60.98	31.19	5.92	20.71	1.27
1980	76.04		47.58	88.62	59.66	61.32	11.65	48.95	3.00
1981	111.22		61.12	99.18	57.22	85.02	16.11	64.47	3.96
1982	162.55		74.49	111.00	53.19	105.82	19.93	83.89	5.15
1983	225.46		89.61	124.22	46.71	125.51	23.09	109.40	6.71
1984	366.68		129.82	139.02	35.81	155.11	25.84	163.32	9.98
1985	521.98	153.00	190.56	155.58	8.28	153.00	0.00	185.03	7.66
1986	674.51		252.48	244.54	55.61	306.78	49.88	336.00	20.43
1987	857.22		326.44	384.38	107.79	477.70	84.48	489.65	29.77
1988	1422.35		534.32	604.17	169.30	806.15	142.28	819.41	49.85
1989	2121.68		918.91	949.64	215.30	1219.22	193.19	1184.77	71.82
1990	2608.64	1491.84	1379.11	1492.65	72.69	1491.84	0.01	1548.11	61.07
1991	4171.82		2103.75	2376.82	488.60	2618.83	403.39	2522.95	152.52
1992	6872.42		3374.94	3784.71	777.33	4197.36	634.04	4184.06	252.11
1993	10984.60	5990.00	5449.74	6026.55	280.72	5990.00	0.04	6069.78	233.22
1994	29608.70		11089.46	11244.21	2308.52	13877.53	2117.75	15537.59	937.52
1995	45845.10		20673.12	20979.18	4306.19	24883.42	3722.77	26924.19	1619.11
1996	81404.90	39274.65	36673.27	39142.46	1801.45	39274.65	0.23	42558.82	1619.55
1997	151865.00		64580.30	66745.02	13699.56	73221.15	10655.65	83545.40	4994.43
1998	260724.00		111920.10	113812.42	23362.91	132297.02	19475.88	121286.86	7265.15
1999	418783.00	197156.62	168768.00	194070.92	8986.11	197156.62	1.16	204111.61	7768.07
2000	625218.00		252139.00	217192.00	55599.36	234179.69	40539.48	348295.01	21650.39