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Abstract

In this paper, we introduce a new class of index numbers for international price comparisons. We prove the existence and uniqueness of the new price index. We then propose a stochastic approach to the Ikle (1972) and the new system of index numbers. The advantage of the stochastic approach is that we can derive standard errors for the estimates of the purchasing power parities (PPPs). The PPPs and the parameters of the stochastic model are estimated using a weighted maximum likelihood procedure. Finally we estimate PPPs and their standard errors for OECD countries using the proposed methods.

JEL Classification: E31 and C19

Keywords: Purchasing Power Parities, International Prices, CPD, Gamma Distribution, Maximum Weighted Likelihood

1. Introduction

There is considerable demand for reliable comparisons of real incomes between countries. In order to make incomes comparable across countries it is necessary to convert national income aggregates such as gross domestic product using appropriate currency converters. For many obvious reasons exchange rates are not considered appropriate and as such they do not reflect relative price levels in different countries. Instead measures of spatial price levels in different countries, usually referred to as purchasing power parities (PPPs) of currencies, are employed. Much of the work on the compilation of PPP is principally under the auspices of the International Comparison Project/Program (ICP) undertaken jointly by a number of international organisations including the World Bank, United Nations, OECD and the European Union.

Purchasing power parities are computed using price data collected from the participating countries. PPP compilation within the ICP is undertaken at two levels, viz., at the basic heading level and at a more aggregated level. At the basic heading level price data are aggregated without any weights to yield PPPs for various basic headings. The basic heading PPPs are then aggregated to yield PPPs for higher level aggregates like consumption, investment and gross domestic product. The main focus of the paper is on the step involving the aggregation above the basic heading level where weights for each basic heading are available for all the countries.

A range of methods have been proposed in the literature by different authors to compute purchasing power parities for aggregation above the basic heading level. Some of the more popular ones are Geary-Khamis (Khamis 1970), Ikle (1972), Country-Product-Dummy (CPD) (Rao 1990, 2004, 2005; Diewert, 2005), Elteto-Koves-Szulc (EKS) (see e.g. Rao 2004). Balk (1996) has compared the analytical properties of more than 10 different methods for calculation of PPPs. Diewert (2005) has demonstrated that a number of commonly used formulae can be derived using the CPD method and Rao (2005) established that the Rao (1990) method for computing PPPs is equivalent to the weighted CPD method. Thus a formal link between the stochastic approach to index numbers in the form of the CPD method and some of the

more commonly used multilateral index number formulae has been established through the work of Diewert (2005) and Rao (2005).

The main objective of this paper is to further strengthen this link by showing that the multilateral price index number system introduced by Ikle (1972) can be derived from a stochastic modeling approach. In addition we consider a new variant of Ikle (1972) and Rao (1990) systems and show that it can also be easily incorporated into a stochastic model. These results are derived through the use of “weighted likelihood functions” which are necessary to consider stochastic specifications that involve distributions other than the normal or lognormal distributions implicit in the standard least squares approaches used along with the CPD model.

This paper is organized as follows: In Section 2 we introduce a new method for computing of purchasing power parities and we show its relationship to Rao (1990) and Ikle (1972) methods. We prove the existence and uniqueness of the new price index in Section 3. In Section 4 we introduce a stochastic model incorporating the new system and we provide a maximum likelihood approach to estimate the model. In Section 4 we do the same for Ikle index. The advantage of the stochastic approach is that we can derive standard errors for the estimates of the purchasing power parities (PPPs), this aspect is considered in Section 5. Section 6 presents estimates PPPs and their standard errors for OECD countries using the Rao, Ikle and the new methods of aggregation and the stochastic approach proposed here. The paper is concluded with a few remarks.

2. Notation and Definitions

Let p_{ij} and q_{ij} represent the price and the quantity of the i th commodity in the j th country respectively where $j = 1, \dots, M$ indexes the countries and $i = 1, \dots, N$ indexes the commodities. We assume that all the prices are strictly positive and all the quantities are non-negative with the minimum condition that for each i q_{ij} is strictly positive for at least one j ; and for each j q_{ij} is strictly positive for at least one i . Also define PPP_j as purchasing power parity or the general price level in j -th country

relative to a numeraire country and P_i as the world average price for the i th commodity. We also need the following systems of weights w_{ij} and w_{ij}^* in defining different systems of index numbers. These weights are defined as

$$w_{ij} = \frac{P_{ij}Q_{ij}}{\sum_{i=1}^N P_{ij}Q_{ij}} \quad \text{and} \quad w_{ij}^* = \frac{w_{ij}}{\sum_{j=1}^M w_{ij}} \quad (1)$$

It is evident that $\sum_{i=1}^N w_{ij} = 1$ and $\sum_{j=1}^M w_{ij}^* = 1$.

With the above notation, Rao (1990) defines a system for international price comparisons as follows

$$PPP_j = \prod_{i=1}^N \left(\frac{P_{ij}}{P_i} \right)^{w_{ij}}$$

$$P_i = \prod_{j=1}^M \left(\frac{P_{ij}}{PPP_j} \right)^{w_{ij}^*} \quad (2)$$

The Rao system is conceptually similar to the Geary-Khamis system in that it uses the twin concepts of purchasing power parities (PPP_j's) and international prices (P_i's).

Following Balk (1996) another system proposed by Ikle (1972) can be written as :

$$\frac{1}{PPP_j} = \sum_{i=1}^N \left(\frac{P_i}{P_{ij}} w_{ij} \right)$$

$$\frac{1}{P_i} = \sum_{j=1}^M \left(\frac{PPP_j}{P_{ij}} w_{ij}^* \right) \quad (3)$$

Note that in Rao system, PPPs and world prices are defined as geometric means (Jevons type of price index) of some appropriate prices while in Ikle system harmonic means of the same prices have been used in a similar manner. Here, we propose a similar system of equations but using arithmetic means (Carli type of price index) as follows:

$$\begin{aligned}
PPP_j &= \sum_{i=1}^N \left(\frac{P_{ij}}{P_i} w_{ij} \right) \\
P_i &= \sum_{j=1}^M \frac{P_{ij}}{PPP_j} w_{ij}^*
\end{aligned} \tag{4}$$

3. Existence and Uniqueness of the new Index

For both Rao and Ikle cases it has been shown that there are unique positive solutions for $\mathbf{P}=(P_1, P_2, \dots, P_N)$ and $\mathbf{PPP}=(PPP_1, PPP_2, \dots, PPP_M)$ in their systems (see Rao 1990 and Balk 1996). Following the same tradition we prove the existence and uniqueness of the new system. To do that, we use the following theorem from Nikaido (1968, page 170).

Theorem (1): Let

- (i) functions $G_i(x_1, x_2, \dots, x_n)$ for $i = 1, \dots, n$ be homogenous of degree one;
- (ii) $G_i(\cdot)$ s are defined for non-negative values of the arguments and are continuous except possibly at the origin;
- (iii) for each k, $x_k = y_k$ and $\mathbf{x} \geq \mathbf{y}$
 $(i \neq k) \Rightarrow G_k(x_1, x_2, \dots, x_n) \leq G_k(y_1, y_2, \dots, y_n)$.; and
- (iv) $G_i(u_1, u_2, \dots, u_n) > 0$ ($i = 1, \dots, n$) for some $u_i \geq 0$

then the system of equations

$$G_i(x_1, x_2, \dots, x_n) = a_i \quad (i = 1, \dots, n)$$

has a unique positive solution if $a_i > 0$ ($i = 1, \dots, n$).

Before presenting the main theorem concerning the existence and uniqueness of the proposed index we can easily see that if (P^*, PPP^*) is a solution to the system then

$(\frac{P^*}{\lambda}, \lambda PPP^*)$ is also a solution. So we can normalize the system by setting $P_N = 1$ and

we can rewrite the system as

$$PPP_j = \sum_{i=1}^{N-1} \left(\frac{P_{ij}}{P_i} w_{ij} \right) + p_{Nj} w_{Nj} \quad (j = 1, \dots, M) \quad (5.1)$$

$$P_i = \sum_{j=1}^M \frac{P_{ij}}{PPP_j} w_{ij}^* \quad (i = 1, \dots, N-1) \quad (5.2)$$

If we substitute P_i s from (5.2) in the first set of above equations (5.1) we have

$$PPP_j - \sum_{i=1}^{N-1} \frac{P_{ij}}{\sum_{j=1}^M \frac{P_{ij}}{PPP_j} w_{ij}^*} w_{ij} = p_{Nj} w_{Nj} \quad (j = 1, \dots, M) \quad (6)$$

Note that existence of a solution to (6) is equivalent to existence of a solution to the whole system (5.1) and (5.2). To prove that let us define

$$f^j(P_1, P_2, \dots, P_M) = \sum_{i=1}^{N-1} \left(\frac{P_{ij}}{P_i} w_{ij} \right)$$

$$h^i(PPP_1, PPP_2, \dots, PPP_M) = \sum_{j=1}^M \frac{P_{ij}}{PPP_j} w_{ij}^*$$

$$G_j(PPP_1, PPP_2, \dots, PPP_M) = PPP_j - \sum_{i=1}^{N-1} \frac{P_{ij}}{\sum_{j=1}^M \frac{P_{ij}}{PPP_j} w_{ij}^*} w_{ij}$$

Theorem: (i) if $p_{Nj} w_{Nj} > 0$ and (ii) there is at least one $\mathbf{PPP} \geq 0$ such that $G_j(\mathbf{PPP}) > 0$ ($j = 1, \dots, M$), the system of equations (5) has a unique positive solution

As we showed above the system (5.1) and (5.2) can be reduced to

$$G_j(\mathbf{PPP}) = PPP_j - \sum_{i=1}^{N-1} \frac{P_{ij}}{\sum_{j=1}^M \frac{P_{ij}}{PPP_j} w_{ij}^*} w_{ij} = p_{Nj} w_{ij} \quad (7)$$

It is easy to check that G_j satisfy all the three conditions:

- (i) G_j s is homogenous of degree one in \mathbf{PPP}

- (ii) G_j s are defined over the non-negative values and are continuous except at the origin
- (iii) there is at least one $\mathbf{PPP} \geq \mathbf{0}$ such that $G_j \geq 0$ (one of the theorem's assumptions)

We can also show that

$$\frac{\partial G_j}{\partial PPP_k} = - \sum_{i=1}^{N-1} \frac{\partial f_j}{\partial P_i} \frac{\partial h_i}{\partial PPP_k} \quad j \neq k$$

It is easy to see that $\frac{\partial f_j}{\partial P_i} \leq 0$ and $\frac{\partial h_i}{\partial PPP_k} \leq 0$ therefore $\frac{\partial G_j}{\partial PPP_k} \leq 0$ ($j \neq k$) which

proves condition (iv) required from theorem (1).

As we see all the conditions cited in theorem (1) are satisfied. Therefore there is a unique positive \mathbf{PPP} that solves (7).

Q.E.D

In the above theorem we have assumed that there is at least one $\mathbf{PPP} \geq \mathbf{0}$ such that $G_j(\mathbf{PPP}) > 0$ ($j = 1, \dots, M$). Our guess is that this condition is always satisfied but so far we have not been able to prove it. So this theorem is not a perfect existence theorem however it guarantees uniqueness of the solution and that is what is usually necessary for empirical applications. Currently work is in progress to derive necessary and sufficient conditions for the existence and uniqueness of positive solutions for \mathbf{PPP} in the new system.

4. Stochastic Approach to the Ikle Index and the New Index

To obtain the stochastic model incorporating the new index we follow Rao (2005) and Diewert (2005) to postulate that the observed price of i-th commodity in j-th country, p_{ij} , is the product of three components: the purchasing power parity (i.e. PPP_j); the price level of the i-th commodity relative to other commodities (i.e. P_i) and a random disturbance term u_{ij} as follows

$$p_{ij} = P_i PPP_j u_{ij} \quad (8)$$

where u_{ij} s are random disturbance terms which are independently and identically distributed. The model postulated in (8) is essentially the CPD model (for details of the CPD method see Rao, 2004). Rao (2005) has shown that Rao system (2) can be obtained as an estimator from the above model using weighted least squares method after taking logs from both sides of the above equation and using expenditure share weights. The same solution can be obtained by assuming a log-normal distribution for u_{ij} and using a maximum likelihood approach.

In the following discussion, we explore alternative specifications for the distribution of u_{ij} which can be used in modeling the residuals of the CPD model in (8). In particular we use the gamma and inverted-gamma distributions and show that under these two specific distributions the resulting weighted maximum likelihood estimators coincide with the Ikle and the new system of index numbers.

Gamma distribution and the new Index

Here we assume that u_{ij} s follow a gamma distribution as follows:

$$u_{ij} \sim \text{Gamma}(r, r) \quad (9)$$

where r is a parameter to be estimated. We combine (8) and (9) to write²

$$\frac{P_{ij}}{P_i PPP_j} \sim \text{Gamma}(r, r) \quad (10)$$

Our purpose here is to estimate parameters (i.e. P_i , PPP_j and r) using a maximum likelihood procedure. From the definition of the gamma density function we can easily show that

$$P_{ij} \sim \frac{r^r}{\Gamma(r)} \frac{P_{ij}^{r-1}}{P_i PPP_j^r} e^{-r \frac{P_{ij}}{P_i PPP_j}} \quad (11)$$

² One may notice the close association of the proposed model to what is known as a generalized linear model with gamma distribution. A generalized linear gamma regression may be defined as (see McCullagh and Nelder 1989) $\frac{y_i}{\mathbf{x}_i \boldsymbol{\beta}} \sim \text{Gamma}(r, r)$. Our model is a nonlinear version of such a model.

Therefore the log of density function can be written as

$$LnL_{ij} \propto r \ln r - \ln \Gamma(r) + (r-1) \ln p_{ij} - \ln P_i - r \ln PPP_j - r \frac{P_{ij}^r}{P_i PPP_j} \quad (12)$$

We can proceed with this (log-) density function and obtain estimates of the parameters of interest using the standard maximum likelihood procedure but we would like to incorporate the weights into the model as well. Use of weights is consistent with standard index number approach of weighting price relatives by their expenditure shares. This is also the approach used by Rao (2005) and Diewert (2005) where weighted least squares method is employed.

One way of doing this is to use a weighted likelihood estimation procedure. We define the weighted likelihood function as

$$WL = \prod_{i=1}^N \prod_{j=1}^M L_{ij}^{w_{ij}/M} \quad (13)$$

and therefore the log of weighted-likelihood function becomes

$$LnWL = \sum_{i=1}^N \sum_{j=1}^M \frac{w_{ij}}{M} LnL_{ij} \quad (14)$$

Then our weighted log- likelihood function becomes

$$\begin{aligned} \ln WL \propto (r-1) \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln p_{ij} - r \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln P_i - r \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln PPP_j - \\ r \sum_{i=1}^N \sum_{j=1}^M \frac{P_{ij} w_{ij}}{P_i PPP_j} + r \ln r \left(\sum_{i=1}^N \sum_{j=1}^M w_{ij} \right) - \ln \Gamma(r) \sum_{i=1}^N \sum_{j=1}^M w_{ij} \end{aligned} \quad (15)$$

Note that the above function may not represent a density function therefore we do not interpret the estimation procedure as a maximum likelihood procedure. We rather interpret it as an M-estimation procedure (for more on M-Estimators and their properties see Chapter 12 of Wooldridge 2002 or Chapter 5 of Cameron and Trivedi 2005).

Maximization of this objective function is not particularly difficult. The only potential problem is the presence of a gamma function in the likelihood function however most of the existing software such as LIMDEP and GAUSS can handle maximization of the functions containing gamma functions fairly easily.

We can also derive the first order conditions from maximization of the above likelihood function as follows

$$-\frac{r \sum_{j=1}^M w_{ij}}{P_i} + \frac{r}{P_i^2} \sum_{j=1}^M \frac{p_{ij} w_{ij}}{PPP_j} = 0$$

$$-\frac{r \sum_{i=1}^N w_{ij}}{PPP_j} + \frac{r}{PPP_j^2} \sum_{i=1}^N \frac{p_{ij} w_{ij}}{P_i} = 0$$

$$\sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln p_{ij} - \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln P_i - \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln PPP_j - \sum_{i=1}^N \sum_{j=1}^M \frac{p_{ij} w_{ij}}{P_i PPP_j} + M + M \ln r - M \frac{\partial}{\partial r} \ln \Gamma(r) = 0$$

From the above sets of equations we obtain the following system of equations in the unknowns:

$$P_i - \sum_{j=1}^M \frac{p_{ij} w_{ij}^*}{PPP_j} = 0 \tag{16}$$

$$PPP_j - \sum_{i=1}^N \frac{p_{ij} w_{ij}}{P_i} = 0$$

$$\frac{\partial}{\partial r} \ln \Gamma(r) - \ln r = \frac{1}{M} \left(\sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln p_{ij} - \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln P_i - \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln PPP_j - \sum_{i=1}^N \sum_{j=1}^M \frac{p_{ij} w_{ij}}{P_i PPP_j} + M \right)$$

As we can see the first two equations are the same as the system of equations we introduced as the new system and these equations do not depend upon the value of r .

Inverse Gamma Distribution and the Ikle Index

A similar procedure can be followed to obtain the stochastic model leading to Ikle index. In order to use the inverse-Gamma distribution, we rewrite the CPD model slightly differently. We use the reciprocal of the price and obtain:

$$\frac{1}{p_{ij}} = \frac{1}{P_i PPP_j} u_{ij} \quad (17)$$

where u_{ij} s are random disturbance terms which are independently and identically and as before they are assumed to follow a gamma distribution

$$u_{ij} \sim \text{Gamma}(r, r) \quad (18)$$

where r is a parameter to be estimated. Model in equation (17) differs from the model in equation (4) mainly in the specification of the disturbance term and how it enters the equation. We combine (17) and (18) to write

$$\frac{1}{p_{ij}} \propto \frac{r^r}{\Gamma(r)} \frac{(P_i PPP_j)^r}{p_{ij}^{r-1}} e^{-r \frac{P_i PPP_j}{p_{ij}}} \quad (19)$$

Following the same procedure as we used earlier in this section, we may obtain the likelihood function as

$$\begin{aligned} \ln L \propto & -(r-1) \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln p_{ij} + r \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln P_i + r \sum_{i=1}^n \sum_{j=1}^n w_{ij} \ln PPP_j - \\ & r \sum_{i=1}^N \sum_{j=1}^M \frac{P_i PPP_j w_{ij}}{p_{ij}} + r \ln r \left(\sum_{i=1}^N \sum_{j=1}^M w_{ij} \right) - \ln \Gamma(r) \sum_{i=1}^N \sum_{j=1}^M w_{ij} \end{aligned} \quad (20)$$

Taking derivative with respect to **PPP** and **P** yields the Ikle system of equations

$$\begin{aligned} \frac{1}{PPP_j} &= \sum_{i=1}^N \left(\frac{P_i}{P_{ij}} w_{ij} \right) \\ \frac{1}{P_i} &= \sum_{j=1}^M \frac{PPP_j}{P_{ij}} w_{ij}^* \end{aligned} \quad (21)$$

Thus the difference between Ikle and our newly proposed system in this paper is essentially in the specification of the disturbance term.

5. Computation of Standard Errors

We have emphasized that the advantage of the stochastic approach to index numbers is to obtain standard errors for estimated indices. One might think that standard errors from conventional weighted least square or weighted maximum likelihood provided by standard software can be used for this purpose. But such standard errors may not be valid if proper formulae are not used in deriving them.

To elaborate the point let us start with a general discussion of M- estimators and their variances. An M-Estimator $\hat{\boldsymbol{\theta}}$ is defined as an estimator that maximizes an objective function of the following form (See e.g. Cameron and Trivedi 2005)

$$Q_N(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^N h_i(y_i, \mathbf{x}_i; \boldsymbol{\theta}) \quad (22)$$

where y_i and \mathbf{x}_i represent dependent and independent variables respectively. $\boldsymbol{\theta}$ is the vector of parameters to be estimated. It has been shown that $\hat{\boldsymbol{\theta}}$ has the following asymptotic distribution

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \xrightarrow{d} \mathcal{N}[\mathbf{0}, \mathbf{A}_0^{-1} \mathbf{B}_0 \mathbf{A}_0^{-1}]$$

where

$$\mathbf{A}_0 = \text{plim} \frac{1}{N} \sum_{i=1}^N \left. \frac{\partial^2 h_i}{\partial \boldsymbol{\theta}' \partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}_0} \quad (23)$$

$$\mathbf{B}_0 = \text{plim} \frac{1}{N} \sum_{i=1}^N \left. \frac{\partial h_i}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}_0} \left. \frac{\partial h_i}{\partial \boldsymbol{\theta}'} \right|_{\boldsymbol{\theta}_0}$$

In practice, a consistent estimator can be obtained as

$$\mathbf{VAR}(\hat{\boldsymbol{\theta}}) = \frac{1}{N} \hat{\mathbf{A}}^{-1} \hat{\mathbf{B}} \hat{\mathbf{A}}^{-1} \quad (24)$$

where

$$\hat{\mathbf{A}} = \frac{1}{N} \sum_{i=1}^N \frac{\partial^2 h}{\partial \boldsymbol{\theta}' \partial \boldsymbol{\theta}} \Big|_{\hat{\boldsymbol{\theta}}}$$

(25)

$$\hat{\mathbf{B}} = \frac{1}{N} \sum_{i=1}^N \frac{\partial h_i}{\partial \boldsymbol{\theta}} \Big|_{\hat{\boldsymbol{\theta}}} \frac{\partial h_i}{\partial \boldsymbol{\theta}'} \Big|_{\hat{\boldsymbol{\theta}}} \quad (26)$$

In some special cases like the maximum likelihood or nonlinear least squares estimator with homoscedastic errors it can be shown that $\mathbf{A}_0^{-1} = -\mathbf{B}_0$. In such cases the variance formula can be simplified to

$$\mathbf{VAR}(\hat{\boldsymbol{\theta}}) = -\frac{1}{N} \hat{\mathbf{A}}^{-1} \quad (27)$$

Many software packages use this formula as their default standard error formula. But in cases similar to those encountered in this paper this formula leads to incorrect standard errors for the estimated parameters and we must use the more general formula given in (23).

For example if we apply formula (27) to the estimates from a weighted least square regression we obtain the following formula

$$\mathbf{VAR}(\hat{\boldsymbol{\theta}}) = \hat{\sigma}^2 (\mathbf{X}' \boldsymbol{\Omega} \mathbf{X})^{-1} \quad (28)$$

where $\boldsymbol{\Omega}$ is a diagonal matrix with weights on its diagonal which coincide the standard formula for weighted least square when there is heteroscedasticity in error term. However the correct formula for the variance estimator to be used in the case where we used weighted least squares when the disturbances are homoskedastic, is given by:

$$\mathbf{VAR}(\boldsymbol{\theta}) = \hat{\sigma}^2 (\mathbf{X}' \boldsymbol{\Omega} \mathbf{X})^{-1} (\mathbf{X}' \boldsymbol{\Omega}' \boldsymbol{\Omega} \mathbf{X}) (\mathbf{X}' \boldsymbol{\Omega} \mathbf{X})^{-1} \quad (29)$$

where $\hat{\sigma}^2$ is obtained from the un-weighted regression. This formula is similar to that suggested in Rao (2004) for the computation of standard errors used the weighted CPD method.

6. Empirical Application using OECD data

In this section we present estimated PPPs and their standard errors derived using the three methods of aggregation discussed in the paper and the 1996 OECD data. The price information that we have is in the form of PPPs at the basic heading level for 158 basic headings, with US dollar used as the numeraire currency. In addition we have expenditure, in national currency units, for each basic heading in all the OECD countries. These nominal expenditures provide the expenditure share data used in deriving the weighted maximum likelihood estimators under alternative stochastic specification of the disturbances.

For weighted CPD estimates we have used the weighted least squares methodology as explained in Rao (2005). For Ikle and the new index we used the weighted maximum likelihood approach described in Section 4.

The estimates of PPPs based on the new index, Ikle's and the standard CPD for 24 OECD countries along with their standard errors are presented in the following table.

Table: MLE estimates of PPPs and SE's

Country	MLE Estimates					
	New Index		Weighted CPD		Ikle	
	PPP	S.E	PPP	S.E	PPP	S.E.
GER	1.887	0.136	2.034	0.144	2.187	0.147
FRA	6.092	0.429	6.554	0.455	7.035	0.466
ITA	1425.96	109.727	1504.02	115.509	1584.381	119.196
NLD	1.921	0.150	2.056	0.155	2.205	0.156
BEL	35.491	2.577	37.890	2.698	40.450	2.728
LUX	33.578	2.488	35.816	2.618	38.191	2.700
UK	0.603	0.043	0.642	0.044	0.682	0.045
IRE	0.637	0.051	0.669	0.055	0.696	0.060
DNK	8.525	0.586	9.131	0.615	9.762	0.631
GRC	180.470	13.452	188.482	13.891	196.640	14.005

SPA	112.414	8.304	118.546	8.606	124.799	8.738
PRT	126.043	10.400	129.037	10.994	130.317	12.002
AUT	12.770	0.881	13.730	0.928	14.728	0.948
SUI	2.050	0.168	2.183	0.177	2.320	0.180
SWE	9.424	0.686	10.075	0.720	10.758	0.742
FIN	6.159	0.432	6.598	0.453	7.070	0.462
ICE	86.828	7.000	89.541	6.975	92.329	6.810
NOR	8.807	0.684	9.238	0.736	9.642	0.764
TUR	6304.23	579.128	6321.42	544.907	6357.003	506.991
AUS	1.264	0.099	1.333	0.103	1.407	0.104
NZL	1.464	0.111	1.530	0.113	1.596	0.115
JAP	182.031	13.622	187.429	14.282	192.392	14.780
CAN	1.168	0.090	1.229	0.094	1.295	0.096
USA	1.0		1.0		1.0	

Results shown in the table clearly demonstrate the feasibility and comparability of the new approaches to the estimation of PPPs. As it can be seen, PPPs and their standard errors based on CPD, Ikle and the new index are all numerically close to each other. An additional phenomenon to note is that the PPPs based on the weighted CPD (or from the log-normal specification for the disturbances) appear to be bounded by PPP estimates from the new index and the Ikle index. This phenomenon needs further investigation.

7. Concluding remarks

The paper has proposed a straightforward extension to two known multilateral methods due to Ikle (1972) and Rao (1990). The new index uses weighted arithmetic averages to define PPPs and international prices, P_i 's, instead of harmonic and geometric averages used respectively in Ikle and Rao specifications. The paper has also established that all the three indexes can be shown to be the weighted maximum likelihood estimators of the CPD model when the disturbances follow lognormal, gamma or the inverse gamma distributions. Derivation of the indices using the stochastic approach makes it possible to derive appropriate standard errors for the Ikle and the new index proposed here. Further, given that all these indexes are generated

by the same CPD model but with alternative disturbance specifications it allows us to test for the distributional assumptions underlying these three methods and use such specification tests to choose between alternative methods. Further work is necessary to see if it is possible to explore other specifications for the distribution of the disturbance and the index number formulae resulting from such specifications. The paper also outlines the approach necessary to compute the true standard errors of PPPs when weighted maximum likelihood methods are used.

References

- Balk, Bert M. (1996) "A Comparison of Ten Methods for Multilateral International Price and Volume Comparison" *Journal of Official Statistics*, 12, 199-222.
- Cameron, C. and P. Trivedi. (2005) *Microeconometrics: Methods and Applications.*, New York, Cambridge University Press.
- Diewert , E.W. (2005) "Weighted Country Product Dummy Variable Regressions and Index Number Formulae" *The Review of Income and Wealth*, 51, 561-570
- Diewert , E.W. (2004) "On the Stochastic Approach to Linking Regions in the ICP" Paper prepared for the DECDG of the World Bank, Washington DC.
- Ikle, D.M. (1972), "A New Approach to the Index Number Problem" *Quarterly Journal of Economics*", 86, 188-211.
- Rao, D.S. Prasada (1990), "A System of Log-Change Index Numbers for Multilateral Comparisons", in *Comparisons of Prices and real Products in Latin America* (eds. Salazar-Carrillo and Rao), Contributions to Economic Analysis Series, Amsterdam: North-Holland Publishing Company.
- Rao, D.S. Prasada (2001) "Weighted EKS and Generalized CPD Methods for Aggregation at Basic Heading Level and above Basic Heading Level" Paper presented at the Joint World Bank-OECD Seminar on Purchasing Power Parities: real Advances in Methods and Applications, 30 January-1 February, 2001, Washington DC.
- Rao, D.S. Prasada (2004), "The Country-Product-Dummy- (CPD) Method A Stochastic Approach To The Computation Of Purchasing Power Parities In The ICP" *Paper presented to SSHRC International Conference on Index Number Theory and the Measurement of Productivity and Prices*, Vancouver, Canada, 30 June- 3July.
- Rao, D.S. Prasada (2005) "On the Equivalence Of Weighted Country-Product-Dummy (CPD) Method and The Rao-System for Multilateral Price Comparisons" *The Review of Income and Wealth*, 51, 571-580
- Selvanathan, E.A. and D.S. Prasada Rao (1992) "An Econometric Approach to the Construction of Generalized Theirl-Tornqvist Indices for Multilateral Comparisons" *Journal of Econometrics*, 54, 335-346.
- Selvanathan, E.A. and D.S. Prasada Rao (1994), *Index Numbers: A Stochastic Approach*, Ann Arbor: The University of Michigan Press.
- Summers, R. (1967), *Economics and Information Theory*, Amsterdam, North-Holland.
- Wooldridge, J. (2002) *Econometric Analysis of Cross Section and Panel Data*. Cambridge, MA: MIT Press.