Risk and Uncertainty

In risk analysis different forms of subjectivity need to be addressed in deciding:

- what the degree of uncertainty is;
- whether the uncertainty constitutes a significant risk;
- whether the risk is acceptable.

Risk Modeling

Risk modeling is the use of discrete probability distributions to compute expected value of variable rather than point estimate.

Sensitivity Analysis

Establishing the extent to which the outcome is sensitive to the assumed values of the inputs:

- it tells how sensitive the outcome is to changes in input values;
- it doesn’t tell us what the likelihood of an outcome is.

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Table 9.1: Sensitivity Analysis Results: NPVs for Hypothetical Road Project ($ millions at 10% discount rate)

<table>
<thead>
<tr>
<th>Construction Costs</th>
<th>NPV</th>
<th>E(NPV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$50</td>
<td>$30</td>
</tr>
<tr>
<td>Medium</td>
<td>$47</td>
<td>$25</td>
</tr>
<tr>
<td>Low</td>
<td>$43</td>
<td>$20</td>
</tr>
</tbody>
</table>

The expected cost of road construction can be derived as:

\[
E(C) = 10 + 60 + 25 = 95
\]

And the expected NPV as:

\[
E(NPV) = 17.2 + 21.6 + 2.2 = 41
\]
Joint Probability Distributions

- Usually uncertainty about more than one input or output;
- The probability distribution for NPV depends on aggregation of probability distributions for individual variables;
- Joint probability distributions for correlated and uncorrelated variables.

**Joint Probability Distributions**

<table>
<thead>
<tr>
<th>Probability (P)</th>
<th>Cost ($)</th>
<th>Benefits ($)</th>
<th>Net Benefits ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>30 (10)</td>
<td>70 (14)</td>
<td>40 (20)</td>
</tr>
<tr>
<td>Best Guess</td>
<td>100 (30)</td>
<td>125 (35)</td>
<td>25 (15)</td>
</tr>
<tr>
<td>High</td>
<td>150 (50)</td>
<td>205 (45)</td>
<td>55 (35)</td>
</tr>
<tr>
<td>Dependent Value</td>
<td>155 (45)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 9.4: Joint Probability Distribution: Correlated Variables**

<table>
<thead>
<tr>
<th>Probability (P)</th>
<th>Probability (P)</th>
<th>Cost ($)</th>
<th>Benefits ($)</th>
<th>Net Benefits ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>20%</td>
<td>50 (10)</td>
<td>70 (14)</td>
<td>20 (4)</td>
</tr>
<tr>
<td>Best Guess</td>
<td>60%</td>
<td>100 (60)</td>
<td>125 (75)</td>
<td>25 (15)</td>
</tr>
<tr>
<td>High</td>
<td>20%</td>
<td>125 (25)</td>
<td>205 (41)</td>
<td>80 (16)</td>
</tr>
<tr>
<td>Dependent Value</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 9.5: Joint Probability Distribution: Uncorrelated Variables**

**Continuous Probability Distributions**

An example is the normal distribution represented as a bell-shaped curve. This distribution is completely described by two parameters:
- the mean
- the standard deviation

Degree of dispersion of the possible values around the mean is measured by the variance ($s^2$) or, the square root of the variance – the standard deviation ($s$).

**Correlated and Uncorrelated Variables**

Assume that if road usage increases, so do road maintenance costs. There is a 20% chance of road maintenance costs being $50 and road user benefits being $70; a 60% chance of road maintenance costs being $100 and road user benefits being $125, and so on.

**Figure 9.1: Triangular probability distribution**

- triangular or ‘three-point’ distribution offers a more formal risk modeling exercise than a sensitivity analysis;
- the distribution is described by a high (H), low (L) and best-guess (B) estimate;
- provide the maximum, minimum and modal values of the distribution respectively.

**Figure 9.2: Cumulative Probability Distribution**

- The cumulative distribution indicates what the probability is of the NPV lying below (or above) a certain value;
- There is a 50% chance that the NPV will be below $28 million, and a 50% chance it will above it;
- There is an 80% chance that the NPV will be less than $48 million and a 20% chance that it will more than this.
Using Risk Analysis in Decision Making

- Choice depends on decision-maker’s attitude towards risk;
- B has higher expected NPV, but is riskier than A;
- final choice depends on how much the decision-maker is risk averse or is a risk taker.

Using @RISK© with Spreadsheets

- Add-on for spreadsheet allowing for Monte Carlo simulations;
- Instead of entering single point estimate in each input cell, analyst enters information about the probability distribution of variable;
- Program then re-calculates NPV or IRR many times over, using a random sample of input data;
- Output results (NPVs or IRRs) are then compiled and presented in form of a probability distribution in:
  - statistical tables
  - graphical format