Review of basic concepts used in investment appraisal

- interest rate
- discount factor
- net present value
- internal rate of return
- marginal productivity of capital
- benefit/cost ratio
- net benefit stream
- annuities
- perpetuities
- cost of capital
- depreciation
- inflation
- real and nominal (money) rates of interest
- risk premium

How do we appraise this proposed investment?

Compare:
- the world with the investment (represented by point B, with consumption C₁ and C₂); and
- the world without the investment (represented by point A, with consumption Y₁ and Y₂).

Which do you prefer? Point A or point B?
We can’t simply compare Y₁+Y₂ with C₁+C₂ because of the time value of money (represented by the interest rate).

Calculate present values:
PV(Y₁,Y₂) = F;
PV(C₁,C₂) = E;
E>F, hence, prefer E – i.e. undertake the investment

Lending and Borrowing

We have been assuming that if your income stream is Y₁,Y₂, your consumption stream must be the same. And if you invest, your consumption stream must be C₁,C₂.

However, by lending or borrowing at the market rate of interest, you can choose any point on the net present value line through A (if you don’t invest), or through B (if you do invest).

For example, if you do invest (B) you could borrow in period 1 to finance the consumption combination represented by point G. Note that G represents more of both commodities (dollars now and dollars next year) than A.

Applying Investment Decision Rules

NPV = BC[1/(1+r)] - AC > 0, hence undertake project.

The relation BC[1/(1+r)] - AC > 0 can be rearranged in various ways to yield equivalent decision rules:
- Benefit/Cost Ratio, BCR = BC[1/(1+r)]/AC > 1, hence undertake the project;
- Marginal Productivity of Capital, MPK = BC/AC > (1+r), hence undertake the project;
- Internal Rate of Return, IRR = MPK - 1 = [BC/AC] > r, hence undertake the project.

To solve for IRR choose rₚ to set BC[1/(1+rₚ)] - AC = 0 ie. IRR = [BC/AC] - 1.
Benefit-Cost Analysis Course: Investment Appraisal Principles

Figure 2.2: A Country’s Inter-temporal Production Possibilities Curve

Dollars Worth of Consumption Goods Now

Dollars Worth of Consumption Goods Next Year

Figure 2.3: The Inter-temporal Effects of International Trade

Dollars Worth of Consumption Goods Now

Dollars Worth of Consumption Goods Next Year

Figure 2.4: Net Benefit Stream of a Two-Period Investment Project

Year

B1 + B2

K

Figure 2.5: Net Present Value in Relation to the Discount Rate

Net Present Value

Discount Rate (% p.a.)

Figure 2.6: Calculating Internal Rates of Return – One Positive Value

IRR

$K

Discount Rate (% p.a.)

Calculating Net Present Value

\[ \text{NPV} = -K + B_1 \left\{ \frac{1}{1+r_1} \right\} + B_2 \left\{ \frac{1}{1+r_1} \right\} \left\{ \frac{1}{1+r_2} \right\} \]

Assume \( r_1 = r_2 = r \):

\[ \text{NPV} = -K + B_1 \left\{ \frac{1}{1+r} \right\} + B_2 \left\{ \frac{1}{1+r} \right\} \left\{ \frac{1}{1+r} \right\} \]

IRR: Solve a quadratic equation for \( r_P \):

\[ 0 = -K + B_1 \left\{ \frac{1}{1+r_P} \right\} + B_2 \left\{ \frac{1}{1+r_P} \right\} \left\{ \frac{1}{1+r_P} \right\} \]

This quadratic equation could have:

• one positive solution
• two positive solutions
• no solution

Example: \( K = 1.6 \); \( B_1 = 10 \); \( B_2 = 10 \)
Another example of an IRR calculation:

\[ K = 1.6 ; B_1 = 10 ; B_2 = -10 \]

When we solve the quadratic equation in this case, we will get two positive IRRs.

What makes this example different?
There are two changes in the sign of the net benefit stream:
-1.6, +10, -10
(Compare with the earlier example: -1.6, +10, +10)

There will generally be as many positive IRRs as there are changes in sign.

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**Annuities and Perpetuities**

An **annuity** is a stream of equal annual payments, \( B \), starting one year from the present and terminating after \( n \) payments.

\[
PV(A) = B/(1+r) + B/(1+r)^2 + B/(1+r)^3 + \ldots + B/(1+r)^n
\]

An **annuity due** is simply an annuity that starts right now:

\[
PV(D) = B + PV(A) - B/(1+r)^n
\]

Treating \( PV(A) \) as a geometric progression, we can write:

\[
PV(A) = B[(1+r)^n - 1]/[r(1+r)^n]
\]

A **perpetuity** is an annuity that goes on for ever. The \( PV \) of a perpetuity is obtained by setting \( n \) go to infinity in the above expression:

\[
PV(A) = B/r
\]

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**Annual Cost of Capital**

Suppose you have just bought a machine (e.g. a car) that cost \( K \) and will last for \( n \) years. The annual cost of capital (sometimes called its rental price) is given by \( C \), where:

\[
K = \sum_{t=1}^{n} \frac{C}{(1+r)^t} = \frac{C[(1+r)^n - 1]}{r(1+r)^n} = C \cdot A(r,n)
\]

where \( A(r,n) \) is the annuity factor.

We also know that annual capital cost consists of interest plus depreciation: \( C = rK + D \); assuming that depreciation, \( D \), is treated as a constant annual cost.
From the preceding discussion we have:

\[ C = \frac{K}{A(r,n)} - rK + D, \]

where

\[ A(r,n) = \frac{(1+r)^n - 1}{r(1+r^n)} \]

We can now solve for the annual depreciation cost:

\[ D = K\left(\frac{1}{A(r,n)} - r\right) = \frac{rK}{(1+r)^n - 1} \]

To check this calculation, plug the expression for \( D \) into

\[ C = rK + D \]

to get:

\[ C = K/A(r,n) \]

We have seen that there are two equivalent methods of dealing with capital cost:

1. Include it in the net benefit stream as a cost, \( K \), at the point at which it occurs (usually the present). This is what we usually do in cost-benefit analysis;
2. Include it in the net benefit stream as the annual cost of interest plus depreciation. This is how firms usually treat capital cost (for tax reasons).

Since the present value of the annual costs included under method 2 is equal to the initial cost accounted for under method 1, it is **important not to use both methods** or you will double-count capital cost.

In calculating NPV we use method 1 and we ignore any annual interest or depreciation costs.

The Role of Inflation in Benefit-Cost Analysis

The interest rates quoted in the financial press are nominal (or money) rates of interest (denoted by \( m \)). The money rate of interest is the real rate, \( r \), plus the expected rate of inflation, \( i \).

\[ m = r + i \]

The net benefit stream is a flow of goods and services valued at a set of prices.

There are two sets of prices which are commonly used:

- today’s prices can be used to value commodities at all points in time (i.e. valued at constant prices);
- prices at the time the commodity is produced or used can be used (i.e. valued at current prices).

Two important rules to remember when discounting to calculate net present value:

- If the net benefit stream is valued at constant prices, use a real rate of interest to compute the discount factors;
- If the net benefit stream is valued at current prices, use a nominal interest rate to compute the discount factors.

Suppose we want to calculate the PV of a ton of coal in year \( t \). We can value the coal at constant prices (today’s price), \( P_0 \); or we can value it at current prices (the price in year \( t \), \( P_t \)).

The relationship between these two prices is:

\[ P_t = P_0 (1+i)^t \]

where \( i \) is the annual inflation rate.

The two approaches to discounting and inflation are equivalent:

\[ P_0\left(\frac{1}{(1+r)^t}\right) = P_t\left(\frac{1}{(1+m)^t}\right) = P_0(1+i)^t/(1+m)^t \]

For this equivalence to hold it must be the case that:

\[ \frac{1}{(1+r)^t} = \frac{(1+i)^t}{(1+m)^t} \]

By cross-multiplying, it can be seen that this implies:

\[ m = r + i + ri \]

And since \( ri \) can be ignored as very small, it implies that the money rate equals the real rate, plus the rate of inflation.

How should we deal with inflation in benefit-cost analysis?

The easiest approach is to ignore it: value costs and benefits at constant prices and use a real rate of interest as the discount rate.

An exception to this approach is when the price of a significant output or input is inflating at a very different rate from the rate of general price inflation. In that case we would want to value all inputs at current prices and use the nominal interest rate as the discount rate.
The Risk Premium on the Discount Rate

Suppose you have built a dam which yields an annual benefit of $B$ (valued at constant prices), but which, in any year, may burst with probability, $p$. The expected benefit in year $t$ is: $E(B) = B(1-p)^t$, and the present value of expected benefit is: $PV[E(B)] = B(1-p)^t/(1+r)^t$, where $r$ is the real rate of interest.

It is easy to show that $(1-p)/(1+r)^t$ is approximately equal to $[1/(1+r+p)]$. In other words the “risk” of the dam bursting can be accounted for by adding a risk premium to the discount rate.

This procedure does not really deal with risk, which economists define as “variation around the expected value”.

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