The Strategic Industry Supply Curve[1]

Flavio M. Menezes[2]
University of Queensland

John Quiggin[3]
University of Queensland

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[2]f.menezes@uq.edu.au

[3]j.quiggin@uq.edu.au

Abstract

In this paper we develop the concept of the strategic industry supply curve, representing the locus of Nash equilibrium outputs and prices arising from additive shocks to demand. We show that the standard analysis of partial equilibrium under perfect competition, including the graphical representation of supply and demand due to Marshall, can be extended to encompass imperfectly competitive markets. Special cases include monopoly, Cournot and Bertrand oligopoly and competition in linear supply schedules. Our approach permits a unified treatment of monopoly, oligopoly and competition, and that it satisfies the five principles of incidence set out by Weyl and Fabinger (2013).

Keywords: industry supply, cost pass-through, oligopoly.

JEL Code: D4, L1.

1 Introduction

Supply and demand curves, and associated concepts such as elasticities, have been central to partial equilibrium analysis since the 19th century. Supply and demand analysis provides a simple and elegant way of modelling the effects of shifts in consumer preferences, production costs and government interventions such as taxes. The graphical representation of the derivation of equilibrium prices and quantities as the intersection of demand and supply curves is an instantly recognizable, even iconic, representation of economics.

Although commonly attributed to Marshall (1890), supply and demand curves were first presented by Cournot (1838), in the same volume that introduced his famous analysis of duopoly. The theoretical foundations of the demand curve were developed shortly afterwards by Dupuit (1844). Despite this overlap, Cournot and Dupuit worked in very different methodological frameworks, which Weyl (2017) distinguishes as 'reductionist' and 'price theory' respectively. Dupuit addressed institutional and historical factors as well as the purely economic determinants of equilibrium that were the focus of Cournot's analysis. Even more than Cournot's duopoly analysis, these early innovations were neglected, and the ideas were subsequently developed independently by a number of writers before being systematized by Marshall.[1]

Despite this early link with the theory of strategic behavior in imperfectly competitive markets, the supply–demand approach has been confined to the non-strategic case of competitive markets, where both firms and consumers may be regarded as price takers. In this case, supply and demand quantities may be represented as functions of prices, and the associated curves are the graphs of those functions.

In the polar case of monopoly, the standard graphical analysis begins with the demand curve, which permits the derivation of the marginal revenue curve. Profit-maximizing output is determined by the intersection of the marginal revenue and marginal cost curves, and the associated price may then be read off the demand curve. In this standard analysis, there is no analog to the supply curve.

For the more general case of oligopoly, supply–demand analysis is rarely, if ever, used. To the extent that a graphical representation of equilibrium determination is employed, the standard approach is to represent the problem in terms of the reaction functions of the firms involved in a duopoly market, and thereby illustrate the Nash equilibrium solution. That is, in the terminology of Weyl (2017), theoretical analysis of the oligopoly problem is

[1] Ekelund and Hebert (1999) provide a detailed discussion of Marshall's predecessors in the development of the supply–demand diagram.

undertaken almost entirely within a reductionist framework.

The aim of the present paper is to show how the tools of supply and demand analysis, fundamental to the price theory approach advocated by Weyl, may be extended to encompass strategic behavior. We examine the case of a market where producers are not price-takers, but face additive demand shocks, parametrized by a scalar shift variable. Firms compete in supply schedules, with monopoly, Cournot and Bertrand competition as special cases.[2]

In the case of competitive markets, graphical analysis using the supply curve has two desirable features. First, and most importantly, the equilibrium price and quantity are given by the intersection of the demand and supply curves. Second, comparative static analysis can be undertaken both with respect to shifts in the demand curve and with respect to cost shocks.

In this paper, we derive a strategic industry supply curve which maps out the (Nash) equilibrium price–quantity pairs associated with any given realization of the demand shock.[3] Using this setup, we derive equilibrium supply elasticities, and show that the standard partial equilibrium analysis of cost and demand is applicable to the case of imperfect competition. In particular, in the linear case, the standard 'welfare triangle' analysis of consumer surplus and of the deadweight loss from monopoly and oligopoly is applicable.

The standard methods of comparative statics are also applicable. We apply these methods to the analysis of 'cost pass-through'. We consider the 'five principles' proposed by Weyl and Fabinger (2013) and show that our approach permits a unified treatment of monopoly, oligopoly and competition.

2 The strategic industry supply curve

The central focus of this paper is on the implications of strategic behavior for firms' supply decisions. Strategic choices will depend on the state of demand.

[2]Unlike Klemperer and Meyer (1989), we restrict attention to affine supply schedules with each strategy available to a firm represented by the value of a scalar shift parameter. By contrast with the Klemperer–Meyer result that any individually rational outcome can be derived as a Nash equilibrium for competition in supply schedules, our approach allows the derivation of a unique, symmetric Nash equlibrium.

[3]Busse (2012) independently developed, for the cases of monopoly and Cournot oligopoly, a similar concept, which she described as the 'equilibrium locus'.

2.1 Demand

We assume that consumers do not behave strategically, so that the demand curve may be taken as exogenously given. We consider the case where the inverse demand curve (willingness to pay) is subjective to additive shocks, that is

$$P(Q,\varepsilon) = \max\left(P\left(Q,0\right) + \varepsilon, 0\right) \tag{1}$$

where Q is quantity, $P\left(Q,0\right)$ is the inverse demand in the absence of shocks, and $\varepsilon \in \mathbb{R}$ is a shock observed by firms before they make their strategic choices. We will assume in what follows that prices are bounded away from zero.

Turning to the direct demand curve, this implies

$$D\left(p,\varepsilon\right) = D\left(p-\varepsilon,0\right) \equiv D_0\left(p-\varepsilon\right) \tag{2}$$

where $D_0\left(p\right)$ is the demand function for the case $\varepsilon = 0$.

Note that p is the market price, while $P : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}_+$ is the stochastic inverse demand **function.**

2.2 Supply

We turn now to supply. We will consider how the equilibrium strategic choices of firms may be represented by a generalization of the concept of the supply curve, which forms the basis of analysis in the competitive case. In the general case of imperfect competition, the strategic choices of firms will depend on the anticipated responses of other firms as well as on market demand. Hence, there is no uniquely defined relationship between market prices and the quantity supplied by individual firms or by the industry as a whole.

Nevertheless, a form of the supply curve arises naturally when we consider the response to additive shifts in the inverse demand curve, characterized by the shock (ε).[4] For each value of ε, the (Nash) equilibrium strategic choices of firms determine a market equilibrium, that is a price–quantity pair on the demand curve at which the market clears. The locus of such points will be a one-dimensional manifold, upward-sloping in price–quantity space, that is, a (strategic) industry supply curve. The analysis is particularly simple in the

[4]In general, the strategic supply curve derived in this way will depend on the form of the shock. Additive shocks to the direct demand curve will yield a different locus of equilibrium points compared to additive shocks to the inverse demand. In the case of linear demand, this distinction is irrelevant and the strategic supply curve is independent of the form of the shock.

case where the firms' strategy spaces consist of a family of firm-level supply curves, with a single strategic shift parameter.

We assume that, conditional on the observed demand shock ε, firms $n = 1, ..., N, N \geq 1$, choose strategies

$$S_n(p, \alpha_n) = \alpha_n + \beta p \tag{3}$$

in supply schedules, where p is the market-clearing price, α_n is the strategic variable for firm n, and β is a fixed parameter, common to all firms.[5]

The idea here is that the strategy space for firm n consists of a family of supply curves parametrized by α_n. The competitiveness or otherwise of the market is reflected in the parameter β. Cournot competition is represented by $S_n(p, \alpha_n) = \alpha_n \forall n$, so that $\beta \equiv 0$. The opposite polar case of Bertrand competition is approached as $\beta \to \infty$.

From the viewpoint of any given firm i, the strategic choices of the other firms $n \neq i$, along with the market demand curve and the realization of the demand shock ε, determine the residual demand curve faced by that firm. Nash equilibrium requires that firm i chooses its optimal price–quantity pair from the residual demand curve, holding the strategic choices of the other firms, $\alpha_n, n \neq i$ constant.

It does not matter, however, whether firm i conceives of its own choice as picking the strategic variable α_i, the associated quantity or price, or some other variable such as the markup on marginal cost. All that matters is that the decision variable should uniquely characterize the profit-maximizing price–quantity pair on the residual demand curve for firm i.[6]

The residual demand facing firm i, given the realized value of ε and the strategic choices of other firms, denoted by α_{-i}, is

$$\begin{aligned} D_i(p; \varepsilon, \alpha_{-i}) &= D(p, \varepsilon) - \sum_{n \neq i} S_n(p, \alpha_n) && (4) \\ &= D_0(p - \varepsilon) - \sum_{n \neq i} \alpha_n - (N-1)\beta p && (5) \end{aligned}$$

and firm i can be regarded as a monopolist facing this demand schedule.

The inverse residual demand for firm i is therefore implicitly defined as

[5] It is straightforward to generalize to the case when the parameter β is firm-specific, but this complicates the statement of results.

[6] The observation of a Delta airlines executive, cited by Klemperer and Meyer (1989, footnote 5), that: 'We don't have to know if a balloon race in Albuquerque or a rodeo in Lubbock is causing an increase in demand for a flight' is apposite here. This observation remains true if the increased demand for Delta services is caused by a reduction in the number of flights offered by, say, Southwest.

$$
\begin{aligned}
P_i\left(Q_i;\varepsilon,\alpha_{-i}\right) &= P\left(Q_i+\sum_{n\neq i} S_n\left(p,\alpha_n\right),0\right)-\varepsilon \qquad (6)\\
&= P\left(Q_i+\sum_{n\neq i}\alpha_n+(N-1)\beta p,0\right)-\varepsilon
\end{aligned}
$$

from (1)

Since each firm i acts as a monopolist facing the residual demand curve, the choice of the strategic variable α_i may equivalently be regarded as setting the market-clearing price p or the firm's own quantity Q_i. We will analyze the optimal choice of Q_i, conditional on the demand shock ε and α_{-i} the vector of strategic choices of other firms. Firm i's producer surplus (profit) is:

$$\Pi_i = P_i\left(Q_i;\varepsilon,\alpha_{-i}\right)Q_i - cQ_i \qquad (7)$$

where c, assumed constant, denotes marginal cost.

Profit maximization requires that Q_i satisfies the first-order condition

$$MC = c = MR = P_i\left(Q_i;\varepsilon,\alpha_{-i}\right)+P_i'\left(Q_i;\varepsilon,\alpha_{-i}\right)Q_i \qquad (8)$$

where

$$P_i'\left(Q_i;\varepsilon,\alpha_{-i}\right)=\frac{1}{D_0'\left(p-\varepsilon\right)-(N-1)\beta}. \qquad (9)$$

Rearranging (8) yields

$$Q_i=\frac{P_i\left(Q_i;\varepsilon,\alpha_{-i}\right)-c}{-P_i'\left(Q_i;\varepsilon,\alpha_{-i}\right)}. \qquad (10)$$

For Nash equilibrium, (8) must be satisfied for all firms.

The second-order condition is

$$MR'\left(Q_i\right)=P_i'\left(Q_i;\varepsilon,\alpha_{-i}\right)\left(1+Q_i\right)+P_i''\left(Q_i;\varepsilon,\alpha_{-i}\right)Q_i<0. \qquad (11)$$

As in the monopoly case, this condition will be satisfied if and only if residual demand has elasticity greater than 1. Since the supply of competing firms is linear, residual demand is more elastic than market demand. Hence, (11) is satisfied whenever market demand has elasticity greater than 1.

2.3 Equilibrium

Given the concavity of the objective function, equilibrium is unique. Since all firms have the same objective and strategy space, the unique equilibrium must be symmetric. In this symmetric equilibrium, for given ε, all firms choose the same α_i^* and produce the same output $Q_i^*(\varepsilon)$. Denote the associated aggregate output by

$$Q^*(\varepsilon) = NQ_i^*(\varepsilon).$$

The inverse demand facing each firm is $P_i^*\left(Q_i^*; \varepsilon, \alpha_{-i}^*\right)$ for all i.

From (10), aggregate output must satisfy

$$\begin{aligned} Q^*(\varepsilon) &= N\frac{P_i^*\left(Q_i^*; \varepsilon, \alpha_{-i}^*\right) - c}{-P_i^{*\prime}\left(\left(Q_i^*; \varepsilon, \alpha_{-i}^*\right)\right)} \qquad (12) \\ &= N\left(P_i^*\left(Q_i^*; \varepsilon, \alpha_{-i}^*\right) - c\right)\left((N-1)\beta - D_0'(p-\varepsilon)\right) \end{aligned}$$

and the market-clearing condition is, for all i,

$$\begin{aligned} D\left(p^*(\varepsilon), \varepsilon\right) &\equiv D_0\left(p^*(\varepsilon) - \varepsilon\right) \qquad (13) \\ &= N\left(P_i^*\left(Q_i^*; \varepsilon, \alpha_{-i}^*\right) - c\right)\left((N-1)\beta - D_0'\left(p^*(\varepsilon) - \varepsilon\right)\right) \qquad (14) \\ p^*(\varepsilon) &= P_i^*\left(Q_i^*; \varepsilon, \alpha_{-i}^*\right). \end{aligned}$$

Equations (13) represent the equilibrium price and quantity for a given value of the additive inverse demand shock ε. In terms of the price theory approach described by Weyl (2017), equations (13) represent a description sufficient for the class of phenomena under consideration.

This description may be represented by taking the locus of solutions $\left(p^*(\varepsilon), Q^*(\varepsilon)\right)$ for (13) as ε varies over its range. In a standard competitive model, this locus of solutions would trace out the supply curve.

In the more general strategic setting proposed here, we therefore refer to the locus of equilibrium solutions as the *strategic industry supply curve.* We have

Proposition 1 *Under the stated conditions, the strategic industry supply curve has the form*

$$\begin{aligned} \mathbf{S}\left(p(\varepsilon)\right) &= N\frac{\left(p(\varepsilon) - c\right)}{-P_i^{*\prime}\left(\left(Q_i^*; \varepsilon, \alpha_{-i}^*\right)\right)} \qquad (15) \\ &= N\left(p(\varepsilon) - c\right)\left((N-1)\beta - D_0'\left(p(\varepsilon) - \varepsilon\right)\right). \qquad (16) \end{aligned}$$

The strategic industry supply curve reflects the requirement that, for every firm, marginal revenue should equal marginal cost.

We derive by inspection

Corollary 1 *The strategic industry supply at any price is higher, the higher are β and N. Hence, the equilibrium quantity (price) is higher (lower), the higher are β and N.*

Menezes and Quiggin (2012) observe that an increase in the number of competitors N will have similar effects on equilibrium market outcomes as an increase in the competitiveness of the market (higher β). The strategic industry supply curve enables us to make this point more sharply. The more competitive is the strategic structure of the market, the more closely the strategic industry supply curve will resemble that of a competitive market with a large number of firms.

In econometric terms, the standard identification strategy for estimation of the supply curve is to identify variables that affect demand but not supply. In the model presented here, these variables are captured by the stochastic shock ε. As in econometric estimation generally, the form of the estimated strategic supply curve depends on both the functional form of demand and the nature of the shocks and, in particular, whether shocks are modelled as shifting direct demand or inverse demand.

To ensure that the usual Marshallian analysis is applicable, we require that the strategic supply curve should slope upwards. We first derive $\mathbf{S}'(p(\varepsilon))$ by totally differentiating (15) wrt ε, and cancelling terms in $\frac{dp^*}{d\varepsilon}$, yielding:

$$S'(p(\varepsilon)) = N\left((N-1)\beta - D_0'(p(\varepsilon)-\varepsilon)\right) - N(p(\varepsilon)-c)\, D_0''(p(\varepsilon)-\varepsilon) \tag{17}$$

From (17), a sufficient condition for $S'(p(\varepsilon)) > 0$ is that $D_0''(p(\varepsilon)-\varepsilon) < 0$. A more precise characterization may be obtained in terms of $\frac{dp^*}{d\varepsilon}$ as follows. Observe that, for any market-clearing triple (Q, p, ε),

$$\begin{aligned} Q &= D(p, \varepsilon) \\ &= D_0(p-\varepsilon) \end{aligned}$$

Hence, for the equilibrium $(p^*(\varepsilon), Q^*(\varepsilon), \varepsilon)$

$$\frac{dQ^*}{d\varepsilon} = D_0'(p^*(\varepsilon)-\varepsilon)\left(\frac{dp^*}{d\varepsilon} - 1\right) \tag{18}$$

From (18) we derive

Proposition 2 *The strategic industry supply curve is upward sloping if and only if* $0 < \frac{dp^*(\varepsilon)}{d\varepsilon} < 1$.

Proof. Since

$$\frac{d\mathbf{S}}{dp} = \frac{dQ^*/\partial\varepsilon}{dp^*/\partial\varepsilon}$$

we have $\frac{d\mathbf{S}}{dp} > 0 \Leftrightarrow \frac{dQ^*}{d\varepsilon} > 0 \Leftrightarrow \frac{dp^*}{d\varepsilon} < 1$. ■

The condition $0 < \frac{dp^*(\varepsilon)}{d\varepsilon} < 1$ means that an inverse demand shock is partly, but not completely, reflected in an increase in the equilibrium price. Equivalently, as we will discuss below, the condition implies partial pass-through of cost shocks. Although it is intuitively plausible, the partial cost pass-through condition does not hold in some cases. Fabinger and Weyl (2012) give a detailed discussion of this issue. For the remainder of this paper, we will focus on the case where $0 < \frac{dp^*(\varepsilon)}{d\varepsilon} < 1$.

From (13), we obtain

$$D\left(p^*\left(\varepsilon\right), \varepsilon\right) = N\left(p\left(\varepsilon\right) - c\right)\left((N-1)\beta - D_0'\left(p\left(\varepsilon\right)\right)\right).$$

$$p^*\left(\varepsilon\right) = \frac{D\left(p^*\left(\varepsilon\right), \varepsilon\right)}{N\left[(N-1)\beta - D_0'\left(p\left(\varepsilon\right)\right)\right]} + c.$$

The term

$$m = \frac{D\left(p^*\left(\varepsilon\right), \varepsilon\right)}{N\left[(N-1)\beta - D_0'\left(p\left(\varepsilon\right)\right)\right]} \tag{19}$$

represents the markup over marginal cost c.

Inspection of (19) yields

Corollary 2 *The markup over marginal costs is decreasing in N and β.*

2.4 Elasticity of demand and supply

As in the standard Marshallian framework, the elasticity of demand and strategic supply may be used to characterize the comparative static behavior of market equilibrium. The elasticity of market demand is

$$\epsilon_d = -\frac{p}{QP'\left(Q, \varepsilon\right)} \tag{20}$$

The elasticity of residual demand faced by firm i is

$$\begin{aligned} \epsilon_d &= -\frac{1}{P_i'\left(q_i\right)}\frac{p}{q_i} && (21) \\ &= \left(D_0'\left(p-\varepsilon\right) - (N-1)\beta\right)\frac{p}{q_i} && (22) \end{aligned}$$

We define the elasticity of strategic supply as

$$\epsilon_s^* \equiv \frac{\frac{dS}{d\varepsilon}}{\frac{dp}{d\varepsilon}} \frac{p}{S(p)} = \frac{\frac{dS(p)}{d\varepsilon}}{1 - p'(q)\frac{dq}{d\varepsilon}} \frac{p}{S(p)} \tag{23}$$

$$= \frac{p'(q)\frac{dS(p)}{d\varepsilon}}{1 - p'(q)\frac{dq}{d\varepsilon}} \left(q'(p) \frac{p}{S(p)} \right)$$

$$= \frac{\frac{dp}{d\varepsilon}}{\left(1 - \frac{dp}{d\varepsilon}\right)} \epsilon_{di}$$

where $\in_{di}$ is the elasticity of the residual demand facing each firm and is the same for all firms in a symmetric equilibrium.

The analog of the usual Marshallian result may now be derived.

Proposition 3 *The response of the market clearing price to a demand shock is given by*

$$\frac{dp}{d\varepsilon} = \frac{\epsilon_s^*}{\epsilon_s^* + \epsilon_D}. \tag{24}$$

Proof.

$$\left(1 - \frac{dp}{d\varepsilon}\right) \epsilon_s^* = \frac{dp}{d\varepsilon} \epsilon_D$$

$$\epsilon_s^* = \frac{dp}{d\varepsilon} \left(\epsilon_s^* + \epsilon_D\right).$$

and the result follows. ■

The condition derived in Proposition 3 is formally identical to the standard result for the competitive case. Note, however, that the elasticity of the strategic supply curve is not determined solely by costs, as in the competitive case. Indeed, we have assumed constant marginal costs, which would imply perfectly elastic supply in the case of perfect competition. By contrast, the elasticity of the strategic supply curve is determined by the change in the Nash equilibrium prices and quantities as demand shifts.

As far as market equilibrium and consumer welfare are concerned, it makes no difference whether the elasticity of supply is determined by cost, strategic behavior by firms or some combination of the two. However, for a given elasticity of supply, producer surplus is higher in the imperfectly competitive case. Imperfect competition is analogous to a case where producers engage in 'cost-padding' and recoup both the resulting producer surplus and the spurious costs.

2.5 Welfare

The representation of market equilibrium in terms of demand and strategic supply permits us to apply the standard notions of consumer and producer surplus, with a crucial modification which characterizes the distinction between strategic and competitive equilibrium.

Exactly as in the standard case, consumer surplus (CS) is given by:

$$CS(\varepsilon) = \int_0^{Q(\varepsilon)} \left(P(Q) + \varepsilon - p(\varepsilon)\right) dQ \tag{25}$$

where $p_0(\varepsilon)$ is the price for which $D(p,\varepsilon) = 0$. The maximum value of consumer surplus is attained as $\beta \to \infty$, $p \to c$.

Aggregate producer surplus (PS) is

$$PS(\varepsilon) = (p(\varepsilon) - c) S(p(\varepsilon)). \tag{26}$$

From (15), the definition of the strategic supply curve, we obtain

$$\begin{aligned} (p(\varepsilon) - c) &= \frac{\mathbf{S}(p(\varepsilon))}{N} \left(-P_i^{*\prime}\left(Q_i^*; \varepsilon, \alpha_{-i}^*\right)\right) \\ &= \frac{\mathbf{S}(p(\varepsilon))}{N\left[(N-1)\beta - D_0'(p(\varepsilon))\right]} \end{aligned} \tag{27}$$

Hence,

$$PS(\varepsilon) = \frac{\left(\mathbf{S}(p(\varepsilon))\right)^2}{N\left[(N-1)\beta - D_0'(p(\varepsilon))\right]} \tag{28}$$

Producer surplus for firm i is

$$PS_i(\varepsilon) = \frac{\left(\mathbf{S}(p(\varepsilon))\right)^2}{N^2\left[(N-1)\beta - D_0'(p(\varepsilon))\right]} \tag{29}$$

From Corollary 1, the equilibrium price $p(\varepsilon)$ is lower, the higher is β. Hence, we derive

Corollary 3 *The higher are β and N the lower is producer surplus and the higher are consumer surplus and total surplus.*

This result may be confirmed by direct inspection of (25) and (28).

2.6 Graphical illustration

Our approach to constructing the strategic industry supply curve is illustrated in Figures 1 and 2 below, for the case of a symmetric Cournot duopoly, that is, $\beta \equiv 0$, and linear demand with unit slope[7]:

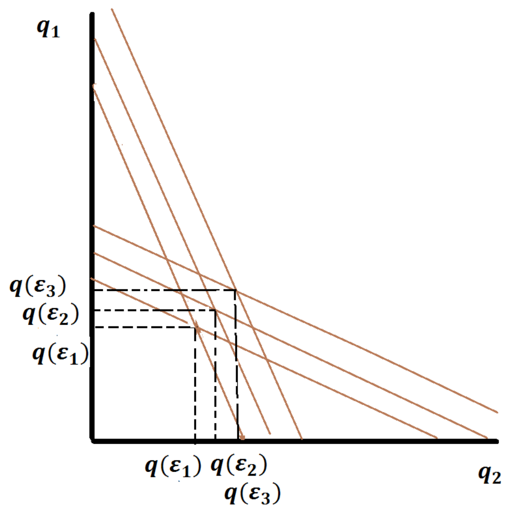
$$\begin{aligned} P(Q,\varepsilon) &= a - Q + \varepsilon. \\ D(p) &= a - p + \varepsilon \end{aligned} \tag{30}$$

Figure 1 shows how the Cournot equilibrium quantity was obtained for three values of ε. For the case of linear demand (30), firm i's reaction function, $i = 1, 2$, $i \neq j$, is given by:

$$\begin{aligned} q_i &= p - c \\ &= \frac{a - q_j - c + \varepsilon}{2}. \end{aligned}$$

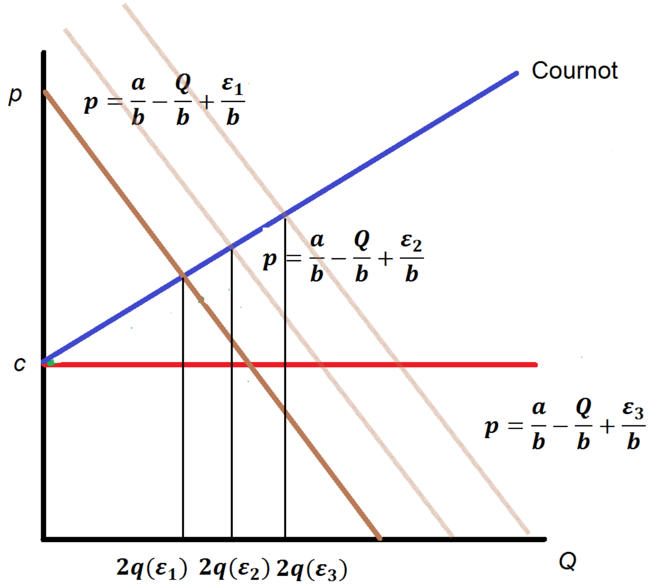
Figure 2 shows the derivation of the strategic industry supply curve, which is obtained by tracing the equilibrium price–quantity supplied pairs as ε varies over its range.

Figure 1: Reaction Curves and the Strategic Industry Supply Curve



[7]An algebraic analysis of this case is presented in Section 4.1.

Figure 2: Strategic Industry Supply Curve (Cournot)



In Figure 3 below, we show how the standard supply–demand graphical approach can be extended to the analysis of symmetric oligopoly and to the case of monopoly, drawn below for constant marginal cost c.

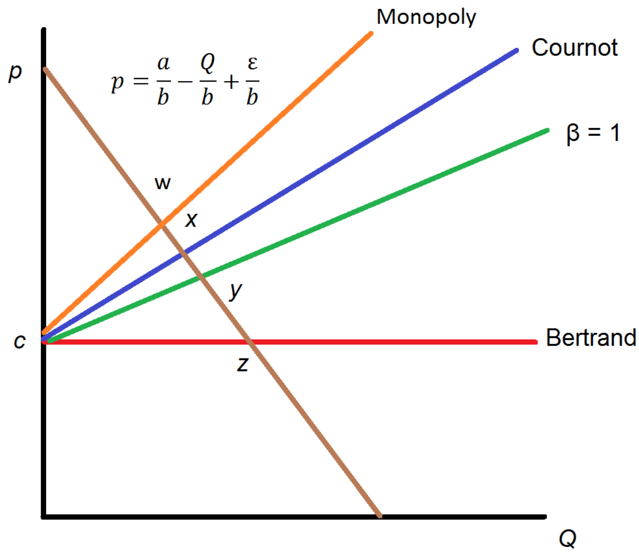
As it is clear from Figure 3, the strategic industry supply curve is an equilibrium concept in the sense that it is derived from the firms' profit maximization for each realization of the demand shock.

The construction of a strategic industry supply curve also allows us to undertake the standard graphical analysis of welfare using a supply–demand diagram. This is illustrated in Figure 4 below for the case of linear demand and constant marginal costs c. Figure 4 depicts consumer surplus (CS), producer surplus (PS), total surplus (TS) and deadweight loss (DWL) for given values of β and ε.

In Figure 4, consumer surplus is represented by the area of the triangle ABF. The maximum value of consumer surplus is ACE, arising as $\beta \to \infty$, $p \to c$.Note that producer surplus is not equal to the area under the supply curve, represented by the triangle BEF between the price and the supply curve in Figure 4.[8] Rather, in the case of constant marginal cost examined

[8] We are indebted to Glen Weyl for this observation.

Figure 3: Industry Strategic Supply Curve: Constant Marginal Cost



here, producer surplus for given ε is equal to $(p(\varepsilon) - c)Q(\varepsilon)$ represented by the rectangle BDEF. The total surplus is represented by the area ABDEF in Figure 4.

In the special case of linear demand, shown in Figure 4, the producer surplus associated with a linear strategic supply curve and constant marginal cost is exactly twice the surplus that arises in a competitive market with the same supply curve resulting from increasing marginal cost. More generally, if the strategic supply curve is convex (concave) the associated producer surplus will be more (less) than twice the corresponding competitive consumer surplus.

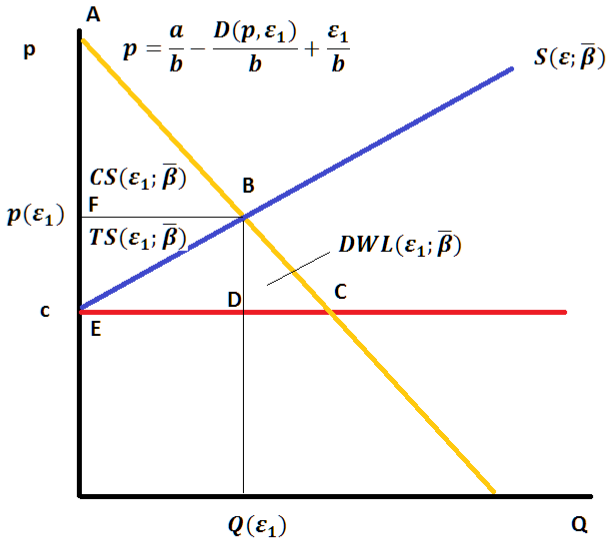
3 Special cases

3.1 Monopoly

For the monopoly case, where $N = 1$, the supply curve becomes

$$S(p(\varepsilon)) = \frac{(p(\varepsilon) - c)}{-D_0'(p)}.$$

Figure 4: Welfare analysis with the strategic industry supply curve



The market-clearing value $p^*(\varepsilon)$ satisfies $Q(\varepsilon) = D(p,\varepsilon)$ so that $p^*(\varepsilon)$ is implicitly determined by

$$\begin{aligned} p^*(\varepsilon) &= \frac{D(p^*(\varepsilon),\varepsilon)}{D'(p^*(\varepsilon),\varepsilon)} + c \\ &= \frac{D_0(p^*(\varepsilon)-\varepsilon)}{D_0'(p^*(\varepsilon)-\varepsilon)} + c. \end{aligned}$$

The monopoly markup is given by the first term on the RHS.

Formally, we can derive

$$\alpha^*(\varepsilon) = Q^*(\varepsilon) - \beta p^*(\varepsilon),$$

which depends on the strategic parameter β. Note, however, that

$$Q^*(\varepsilon) = \alpha^*(\varepsilon) + \beta p^*(\varepsilon).$$

is the same for any finite β. This is an illustration of the more general point that players conceive their situation as that of a monopolist, picking a point on a demand curve, independently of the strategies available to them. The choice of β only affects the players' understanding of their opponents' strategies and is therefore irrelevant in the case of monopoly.

3.2 Cournot

For Cournot ($\beta \equiv 0$), the market clearing price is

$$p^*(\varepsilon) = \frac{1}{N}\frac{D_0(p^* - \varepsilon)}{D_0'(p^* - \varepsilon)} + c.$$

and the equilibrium value of $\alpha^*(\varepsilon)$ is $\frac{D(p^*(\varepsilon),\varepsilon)}{N}$. The markup on marginal cost is additive and is given by $\frac{1}{N}\frac{D_0(p^*-\varepsilon)}{D_0'(p^*-\varepsilon)}$.

The strategic supply curve is given by

$$\mathbf{S}(p(\varepsilon)) = N\frac{(p-c)}{-D_0'(p)}.$$

The Cournot solution coincides with the monopoly solution for the case $N = 1$. As shown in the remark above, this coincidence holds for all finite values of β, and is not specific to the Cournot solution.

In particular, for the case of Cournot oligopoly, the strategic industry supply curve is strictly upward sloping even though the firms' equilibrium supply schedules are all vertical. This reflects the fact that the strategic industry supply curve is derived from a locus of equilibria, one for each value of ε.

3.3 Bertrand/perfect competition

In the limit as $(N-1)\beta \to \infty$, $p^*(\varepsilon) \to c$. Hence the slope of the industry supply curve approaches 0 as $(N-1)\beta$ approaches ∞. For fixed $N > 1$, as $\beta \to \infty$, the industry supply curve converges to the marginal cost curve for the representative firm. As would be expected, ϵ_S approaches infinity for the Bertrand case $\beta \to \infty$.

4 Linear demand

In the special case of linear demand with unit slope, given by (30), we can derive a closed-form solution, noting that $D_0' \equiv 1$

From (15), the industry supply curve is

$$S(p(\varepsilon)) = (N + N(N-1)\beta)(p-c).$$

Note that

$$S'(p(\varepsilon)) = (N + N(N-1)\beta) > N\beta.$$

with equality only for the monopoly case $N = 1$. That is, the industry strategic supply is more price-responsive than the sum of the schedules $\alpha_i^* + \beta p$, which define the strategies of individual firms.

The market-clearing condition is

$$a - bp(\varepsilon) + \varepsilon = (N + N(N-1)\beta)(p(\varepsilon) - c).$$

Solving for the market-clearing values, we obtain

$$\begin{aligned} p(\varepsilon) &= \frac{(a+\varepsilon)}{(N + N(N-1)\beta) + 1} + \frac{(N + N(N-1)\beta)c}{(N + N(N-1)\beta) + 1} \\ Q(\varepsilon) &= \frac{N(a - c + \varepsilon)}{(N + (1 + (N-1)\beta))} \\ \alpha^*(\varepsilon) &= \frac{Q(\varepsilon)}{(1 + (N-1)\beta)} - \beta p. \end{aligned} \tag{31}$$

For comparative statics

$$\begin{aligned} \frac{dQ}{d\varepsilon} &= \frac{N}{(N + (1 + (N-1)\beta))} \\ \frac{dP}{d\varepsilon} &= 1 - \frac{dQ}{d\varepsilon} \\ &\quad \frac{(1 + (N-1)\beta)}{(N + (1 + (N-1)\beta))} \\ \frac{dP}{dQ} &= \frac{dP}{d\varepsilon} \Big/ \frac{dQ}{d\varepsilon} = \frac{(1 + (N-1)\beta)}{N}. \end{aligned}$$

The elasticity of industry supply with respect to price is simply

$$\epsilon_S = \frac{p}{p - c}.$$

This expression does not contain β explicitly, but the equilibrium price p depends on β.

As noted in Corollary 2 above, increases in N and β have similar effects. We can sharpen this point for the linear case. Consider as a benchmark the Cournot case with linear demand, where $S'(p) = N$. Now, for any $2 \leq M < N$, let $\beta(M) = \frac{(N-M)}{M(M-1)} > 0$. Then, for all p

$$S(p; N, 0) = S(p; M, \beta(M)).$$

More generally, for any initial β and $M < N$, we can find $\beta(M)$ such that, for all p,

$$S(p; N, \beta) = S(p; M, \beta(M)).$$

The elasticity of strategic supply is

$$\epsilon_S = 1 + \frac{Nbc}{a - bp + \varepsilon}$$

In particular, for the case of zero costs, $\epsilon_S = 1$.

4.1 Linear demand with Cournot oligopoly

The results are particularly simple for the canonical case of Cournot oligopoly when combined with linear demand. As noted above, the monopoly solution may be derived by setting $N = 1$.

Since $D'(p) \equiv 1$, the strategic industry supply curve is simply

$$\mathbf{S}(p(\varepsilon)) = N(p - c) \tag{32}$$

The market clearing conditions are

$$\begin{aligned} p(\varepsilon) - c &= \frac{(a + \varepsilon - c)}{N + 1} \\ Q(\varepsilon) &= \frac{N(a + \varepsilon - c)}{N + 1} \end{aligned}$$

which reduce to the familiar $p = \frac{1}{N+1}$, $Q = \frac{N}{N+1}$ for zero cost and $\varepsilon = 0$. Note that $p(\varepsilon) - c = \frac{Q(\varepsilon)}{N}$

As noted above, strategic supply under oligopoly may be thought of as a kind of 'cost padding'. The linear case graphically illustrated in subsection 2.6 provides a clear demonstration of this point.

The linear strategic industry supply curve (32) is the same as the standard supply curve for a competitive industry with quadratic costs. To be more precise, consider a competitive industry with N firms, each facing the cost function $c_i(q_i) = cq + \frac{1}{2}q_i^2$, so that, in equilibrium $p = c_i'(q_i) = c + q_i$. Hence, industry supply is given by

$$Q = N(p - c),$$

exactly as in (32).

For welfare,we have

$$
\begin{aligned}
CS(\varepsilon) &= \frac{Q^*(\varepsilon)^2}{2} \\
&= \frac{1}{2}\left(\frac{N}{N+1}\right)^2 (a+\varepsilon-c)^2 \\
PS(\varepsilon) &= Q(\varepsilon)(p(\varepsilon)-c) \\
&= \frac{Q^*(\varepsilon)^2}{N} \\
&= \frac{N}{(N+1)^2}(a+\varepsilon-c)^2 \\
TS(\varepsilon) &= \frac{2N+N^2}{2(N+1)^2}(a+\varepsilon-c)^2 \\
&= \frac{1}{2}\left(1-\frac{1}{(N+1)^2}\right)(a+\varepsilon-c)^2 .
\end{aligned}
$$

The competitive benchmark has

$$
\begin{aligned}
p &\equiv c, \\
PS(\varepsilon) &\equiv 0, \\
CS(\varepsilon) &\equiv \frac{(a+\varepsilon-c)^2}{2}.
\end{aligned}
$$

Hence, deadweight loss is

$$
DWL = \frac{1}{2}\left(\frac{1}{(N+1)^2}\right)(a+\varepsilon-c)^2
$$

5 Cost Pass-through

The problem of cost pass-through is a special case of the comparative statics of Marshallian partial equilibrium analysis. The analysis begins with a market equilibrium disturbed by a shock to suppliers' input prices or technology, which may be represented as an increase of Δc in unit costs. The problem is to determine the resulting change in the equilibrium price Δp, and, more particularly, the ratio $\rho = \frac{\Delta p}{\Delta c}$, which measures the proportion of the cost increase passed through to consumers. Although input prices and technology are subject to constant change, the term 'pass-through' is most commonly used in contexts where the change in equilibrium prices is seen to be of policy concern.

The problem of cost pass-through was recently examined by Weyl and Fabinger (2013), who draw on a long tradition of work on tax incidence, going back to Dupuit (1844), Jenkin (1871-72) and Marshall (1890). Like Weyl and Fabinger, we extend the standard analysis of incidence under competition to the case of imperfectly competitive markets. The concept of the elasticity of strategic supply allows a unified treatment of monopoly, oligopoly and Bertrand competition.

Since firms are concerned only with the margin $p-c$, an additive increase in c is equivalent to an equal and opposite shock to the inverse demand function (1). Hence, we may apply Proposition 3 to obtain

Corollary 4 *Cost pass-through is given by*

$$\rho = \frac{\epsilon_S}{\epsilon_D + \epsilon_S},$$

where ϵ_D denotes the price elasticity of (residual) demand and ϵ_S the price elasticity of the strategic industry supply curve.

Thus, using the concept of the strategic industry supply curve, the standard analysis of cost pass-through in the competitive case may be extended to cover monopoly and oligopoly.

For the linear case, we can derive a closed-form solution.

Proposition 4 *Cost pass-through for symmetric oligopoly with linear demand and constant marginal cost is given by:*

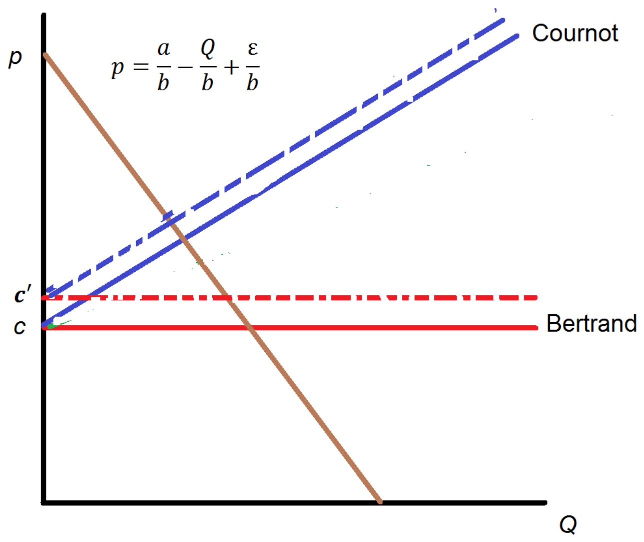
$$\rho = \frac{N + N(N-1)\beta}{(N+1) + N(N-1)\beta}. \tag{33}$$

Proof. Follows from differentiation of (31) with respect to c ■

From (33), we can recover the standard pass-through expression for Cournot models with linear demand and constant marginal cost ($\rho = \frac{N}{N+1}$). For Bertrand, cost pass-through is equal to 1, as for perfect competition. As observed above, an increase in the number of competitors N has the same effect as an appropriately chosen increase in β. In particular, for any fixed β, as $N \to \infty$, $\rho \to 1$. The minimum value of ρ is $\rho = \frac{1}{2}$, attained in the monopoly case $N = 1$.

The Bertrand and Cournot examples are shown in Figure 5 below.

Figure 5: Cost Pass Through for Cournot and Bertrand



5.1 Incidence

Now consider the case when a cost increase arises from the imposition of a tax. In this case, we are interested in the tax burden, that is, the ratio of the loss in producer and consumer surplus to the revenue raised by the tax.

Consider the case when a tax t is imposed. For notational convenience we will focus on derivatives evaluated at $t = 0$.

We have, for producer surplus,

$$\begin{aligned}
PS(\varepsilon) &= (p(\varepsilon) - c - t)S(p(\varepsilon)) \\
\frac{\partial PS(\varepsilon)}{\partial t} &= S(p(\varepsilon))\frac{\partial(p(\varepsilon) - c - t)}{\partial t} + (p(\varepsilon) - c - t)S'(p(\varepsilon))\frac{\partial(p(\varepsilon) - c - t)}{\partial t} \\
&= (\rho - 1)\left[S(p(\varepsilon)) + (p(\varepsilon) - c - t)S'(p(\varepsilon))\right] \\
&= (1 + \eta)(\rho - 1)Q(\varepsilon) \\
&= (1 + \eta)(\rho - 1)\frac{\partial R}{\partial t}
\end{aligned}$$

where $R = tQ(\varepsilon)$ is tax revenue.

$$\eta = \frac{(p(\varepsilon) - c - t)S'(p(\varepsilon))}{S(p(\varepsilon))} > 0$$

under the conditions of Theorem 1.

The change in consumer surplus is $-\rho Q$, so the total burden of the tax is given by

$$\begin{aligned}
\left|\frac{\partial PS(\varepsilon)}{\partial t} + \frac{\partial CS}{\partial t}\right| &= \left[(1+\eta)(1-\rho)+\rho\right]\frac{\partial R}{\partial t} \\
&= \left[(1+\eta)-\eta\rho\right]\frac{\partial R}{\partial t} \\
&\geq \frac{\partial R}{\partial t},
\end{aligned}$$

where equality holds only for the Bertrand case $\rho = 1$.

For the Bertrand case, we have

$$\begin{aligned}
\frac{\partial PS(\varepsilon)}{\partial t} &= 2(\rho-1)\frac{\partial R}{\partial t} = 0 \\
\frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{\partial R}{\partial t}
\end{aligned}$$

That is, in the limit, the burden of a small tax is entirely borne by consumers, and there is no deadweight loss.

5.1.1 Incidence with linear demand

In the case of linear demand, the strategic supply curve is linear, so $\eta = 1$ and we have

$$\begin{aligned}
\frac{\partial PS(\varepsilon)}{\partial t} &= 2(\rho-1)\frac{\partial R}{\partial t} \\
\left|\frac{\partial PS(\varepsilon)}{\partial t} + \frac{\partial CS}{\partial t}\right| &= (2-\rho)\frac{\partial R}{\partial t}
\end{aligned}$$

For monopoly, $\rho = \frac{1}{2}$, and we have

$$\begin{aligned}
\frac{\partial PS(\varepsilon)}{\partial t} &= -\frac{\partial R}{\partial t} \\
\frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{1}{2}\frac{\partial R}{\partial t}.
\end{aligned}$$

Thus, we obtain the well known result that the full tax revenue is paid by the monopolist, and an additional burden is borne by consumers. In the linear case considered here, this additional burden is equal to half of the revenue raised by the tax.

For Cournot $\rho = \frac{N}{N+1}$, we have

$$\begin{aligned}
\frac{\partial PS(\varepsilon)}{\partial t} &= -\frac{2}{N+1}\frac{\partial R}{\partial t} \\
\frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{N}{N+1}\frac{\partial R}{\partial t} \\
\left|\frac{\partial PS(\varepsilon)}{\partial t} + \frac{\partial CS(\varepsilon)}{\partial t}\right| &= \frac{N+2}{N+1}\frac{\partial R}{\partial t} \\
I &= \frac{N}{2}
\end{aligned}$$

where I (incidence) is the ratio of the burden borne by consumers to the burden borne by producers.

For the general oligopoly case, with $\beta < \infty$ and $N > 1$, we have

$$\begin{aligned}
\frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{Nb + N(N-1)\beta}{(N+1)b + N(N-1)\beta}\frac{\partial R}{\partial t} \\
\frac{\partial CS(\varepsilon)}{\partial t} &= -\frac{2}{(N+1)b + N(N-1)\beta}\frac{\partial R}{\partial t} \\
\left|\frac{\partial PS(\varepsilon)}{\partial t} + \frac{\partial CS(\varepsilon)}{\partial t}\right| &= \frac{(N+2) + N(N-1)\beta}{(N+1) + N(N-1)\beta}\frac{\partial R}{\partial t} \\
I &= \frac{Nb + N(N-1)\beta}{2}.
\end{aligned}$$

Once again, the total burden exceeds revenue and is shared between producers and consumers. Producers bear less than the full burden of the tax.

6 The five principles

Weyl and Fabinger (2013) analyze the problem of cost pass-through, drawing on the literature on tax incidence. Their analysis is organized around five principles, drawing on the analysis of tax incidence under perfect competition. These principles are extended, with appropriate modifications, to the cases of monopoly and oligopoly.

In the case of symmetric oligopoly with a homogenous product, the competitiveness of the market is represented by a parameter R, which varies between 0 (for Cournot) and -1 (for Bertrand). Weyl and Fabinger use the derived parameter $\theta = \frac{1+R}{N}$, which varies between 0 (Bertrand) and $\frac{1}{N}$ (Cournot). A straightforward manipulation shows that, translating to the terms of our model, we can express the competitiveness parameter θ in terms of the strategic parameter β as $\theta = \frac{1}{N+N\beta(N-1)}$. Thus, our model provides an explicit game-theoretic foundation for the derivation of the parameters R and θ.

The idea of the strategic industry supply curve allows for a more unified treatment of the Weyl–Fabinger principles, with a single statement of the principles applicable to competition, monopoly and oligopoly. We now consider the Weyl–Fabinger principles in turn:

Principle of incidence 1 (Economic versus physical incidence)

The physical incidence of taxes is neutral in the sense that a tax levied on consumers, or a unit parallel downward shift in consumer inverse demand, causes nominal prices to consumers to fall by $1 - \rho$.

This principle of neutrality is fundamental. The same principle underlies the crucial observation that from the viewpoint of any individual producer, a shock to residual demand is identical whether it arises from a shock to market demand or from the (equilibrium) supply of other producers. Weyl and Fabinger (2013) attribute this insight to Jeremy Bulow.

Principle of incidence 2 (Split of tax burden)

(i) Under competition, the total burden of the infinitesimal tax beginning from zero tax is equal to the tax revenue and is shared between consumers and producers.

(ii) Under monopoly, the total burden of the tax is more than fully shared by consumers and producers. While the monopolist fully pays the tax out of her welfare, consumers also bear an excess burden.

(iii) Under homogenous products oligopoly, the total burden of the tax is more than fully shared by consumers and producers. Producers bear less than the full burden of the tax.

As shown above, Principle 2 is satisfied by our model.

Principle of incidence 3 (Local incidence formula)

The ratio of the tax borne by consumers to that borne by producers, the incidence, I, equals:

(i) $\frac{\rho}{1-\rho}$ in the case of perfect competition;

(ii) ρ in the case of monopoly; and

(iii) $\frac{\rho}{1-(1-\theta)\rho}$ in the case of oligopoly.

Our results coincide with those of Weyl and Fabinger in all cases.

Principle of incidence 4 (Pass– through**)**

To analyze pass-through, Weyl and Fabinger introduce the elasticity of the inverse marginal surplus curve ϵ_{ms}. The pass-through rate for constant marginal cost is: $\rho = \frac{\epsilon_{ms}}{\epsilon_{ms}+\theta}$ which becomes

(i) $\rho = 1$ in the case of perfect competition;

(ii) $\rho = \frac{\epsilon_{ms}}{\epsilon_{ms}+1}$ in the case of monopoly; and

(iii) $\rho = \frac{1}{1+\frac{\theta}{\epsilon_{ms}}+\frac{\theta}{\epsilon_{\theta}}}$ in the case of oligopoly.

For the case of linear supply schedules, β, and therefore also θ, are constant. Hence ϵ_θ is also infinite, so $\rho = \frac{\epsilon_{ms}}{\epsilon_{ms}+\theta}$. Substituting $\theta = \frac{1}{N+N\beta(N-1)}$ into this expression, we obtain

$$\rho = \frac{\epsilon_{ms}\left[N + N\beta(N-1)\right]}{1 + \epsilon_{ms}\left[N + N\left(N-1\right)\beta\right]}.$$

In the case of linear demand, $\epsilon_{ms} = 1$, and we obtain $\rho = \frac{N+[(N+1)+N(N-1)\beta]}{[(N+1)+N(N-1)\beta]}$, as in Corollary 4

Finally, we have

Principle of incidence 5 (Global incidence)

Weyl and Fabinger derive global incidence as a weighted average of the pass-through rate which is, in general, variable. For the case of linear demand, the pass-through rate is constant, and therefore global incidence is the same as local incidence. An analysis of the case of non-linear demand can be undertaken using the tools provided by Weyl and Fabinger.

7 Concluding comments

We have shown that, using the concept of the strategic industry supply curve, the standard analysis of partial equilibrium under perfect competition, including the graphical representation of supply and demand due to Marshall, can be extended to encompass imperfectly competitive markets. The class of market structures encompasses monopoly and competition, as well as an entire class of oligopoly models represented by competition in linear supply schedules, with Cournot and Bertrand as polar cases. For the oligopoly case, the results show the interaction between the number of firms N and the competitiveness of the market structure, characterized by the parameter β.

Furthermore, this representation of supply allows for a unified treatment of comparative static problems such as cost pass-through, which have previously required separate treatments for competition, monopoly and oligopoly. In particular, we provide both a game-theoretic foundation for, and a simple derivation of, the Weyl–Fabinger principles of incidence. The tools used here could be applied to a wide range of problems, such as the analysis of mergers. Similarly, we can extend the diagrammatic tools of welfare analysis, such as the representation of deadweight losses as welfare triangles.

References

[1] Busse, M. (2012), 'When supply and demand just won't do: using the equilibrium locus to think about comparative statics', mimeo, North-

western University.

[2] Cournot, A. (1838) *Recherches Sur Les Principes Mathématiques De La Théorie Des Richesses*, Librairie des Sciences Politiques et Sociales. M. Rivière et Cie., Paris.

[3] Dupuit, A. J. E. J. (1844), *De la mesure de l'utilite ́ des travaux publics.* Paris.

[4] Fabinger, M. and Weyl, G. (2012), Pass-Through and Demand Forms, Working Paper,DOI: 10.2139/ssrn.2194855

[5] Ekelund, R.B. and Hebert, R.F. (1999) *Secret Origins of Modern Microeconomics: Dupuit and the Engineers*, University of Chicago Press, Chicago.

[6] Jenkin, H. C. F. (1871–72), 'On the Principles Which Regulate the Incidence of Taxes', *Proc. Royal Soc. Edinburgh,* 7, 618–31.

[7] Klemperer, P.D. and Meyer, M.A. (1989), 'Supply function equilibria in oligopoly under uncertainty', *Econometrica*, 57(6), 1243–77.

[8] Marshall, A. (1890), *Principles of Economics.* New York: Macmillan.

[9] Menezes, F. and Quiggin, J. (2012), 'More competitors or more competition? Market concentration and the intensity of competition', *Economics Letters*, 117(3), 712–714.

[10] Weyl, E. G. and Fabinger, M. (2013), 'Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition,' *Journal of Political Economy*, 123(3), 528-583.

[11] Weyl (2017), Price Theory, working paper, Microsoft Research New York City.