

Sharing contests with general preferences

Alex Dickson[†], Ian A. MacKenzie[‡], and Petros G. Sekeris[§]

[†]Department of Economics, University of Strathclyde, Glasgow, UK, G4 0QU.

[‡]School of Economics, University of Queensland, Brisbane, Australia, 4072.

[§]Montpellier Business School, Montpellier, France.

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Abstract

This article investigates contests when heterogeneous players compete to obtain a rent share. We prove the existence and uniqueness of the Nash equilibrium when players have general preferences. Our results show that the conventional wisdom in contests—such as a monotonically increasing relationship between effort and the size of the rent—may no longer hold. We derive the key conditions on preferences under which this is the case. By providing a much broader contest environment, our approach is able to nest conventional contest analysis as well as providing a rich framework that helps to explain many previously puzzling applications.

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1 Introduction

Sayre's law: *"In any dispute, the intensity of feeling is inversely proportional to the value of the stakes at issue. That is why academic politics are so bitter."* (Coleman, 2008)

Contests characterize situations in which individuals seek to appropriate an economic rent. This describes a wealth of economic scenarios—such as rent seeking, litigation, and conflict—where the study of contests has improved our understanding of many fundamental economic interactions. The conventional wisdom borne from the analysis of contests suggests that rent-seeking effort is increasing in the size of the rent. Although this is consistent with many applications, there are, however, many other environments in which we might observe that the reverse is true; *Sayre's law*—quoted at the beginning of this introduction—being a case in point.

In this article we study contests in which contestants compete for a share of a perfectly divisible rent and their preferences are more general than have been studied in the existing literature; namely, we allow for (i) diminishing marginal utility of the contest outcome and (ii) interactions between the evaluation of the contest outcome (rent share) and the effort expended in the contest. By providing a more general framework, we demonstrate that the conventional wisdom—of a monotonically increasing relationship between the size of the rent and contestants' effort—need no longer hold.

Contests in the spirit of Tullock (1980) can be interpreted in two ways: 'winner-take-all' contests; and 'share' contests. In the 'winner-take-all' interpretation there exists a *probability* that a player receives the entire rent based on his relative effort. In a 'share' contest, in contrast,

each individual receives a (deterministic) share of the rent based on their effort relative to that of their rivals. If each contestant has linear preferences over the allocation they receive from the contest, then the two interpretations are strategically equivalent, since every contestants' expected payoff in a winner-take-all contest is equivalent to their payoff in a rent-sharing contest. This equivalence, however, breaks down in all but the simplest of settings; thus, the two types of contest command separate study.

Although major advances have been made in developing the analysis of winner-take-all contests to capture non-linear evaluation of contest outcomes (see, for example, Konrad, 2009; Congleton and Hillman, 2015), the same is not true of share contests: the two are not equivalent under these extensions, and the study of share contests themselves has been neglected in the literature. This lack of inquiry is concerning as many of the common applications of contests are best viewed as individuals sharing a divisible rent. It is common, for example, to observe effort being used to obtain a share of pollution permits, research funds, and parental attention, to name but a few. Moreover, the interactions between sellers and buyers in markets can often be considered as sharing contests (e.g., in bilateral oligopoly (Dickson and Hartley, 2008)). Inspired by their broad applicability, the aim of this article is to advance the study of share contests to account for more general preferences.

The conventional wisdom for share contests developed thus far has identified a monotonically increasing relationship between contestants' effort and the size of the rent: as the stakes increase, individuals invest more in effort (this is also true for winner-take-all contests). Although *prima facie*, this might appear correct, consideration of many applications suggest that the opposite, in fact, may be true: when contested rents are relatively small there may exist intense competition. The quote concerning *Sayre's law* at the beginning of this article provides a leading example. Likewise, parents would not find it strange that their children are more effortful in seeking the attention of the parent that is in the home for a smaller proportion of the day. Further, similar interactions are observed within competition for natural resources (i.e., conflict over land use, water rights, and minerals): as Brunnschweiler and Bulte (2008, 2009) investigate, when the abundance of natural resources increases within a country, there is evidence to suggest that there is *less* rent seeking (conflict) and increased economic growth. These empirical findings contrast with conventional contest frameworks, which would predict the very opposite. Although there may exist numerous ways to rationalize such behavior by appealing to considerations outside of the contest environment, there is a very intuitive rationalization within the contest itself: if agents have diminishing marginal utility over their allocation within a contest then when the allocation is small (large) the incremental benefit from improving that allocation is large (small) which may command more (less) effort.

In this article we consider a conventional contest framework where players exert effort in contesting a perfectly divisible economic rent in which the contest success function—influenced by all contestants' efforts—determines each contestant's share of this rent. We focus initially on simple Tullock contests, and follow the approach of Cornes and Hartley (2003, 2005, 2012) by recognizing and exploiting the aggregative properties of the game that is played. This allows us to consider both the existence and uniqueness of Nash equilibrium with heterogeneous contestants under some rather general restrictions on contestants' preferences. We then study the comparative static properties of equilibrium in a tractable way, particularly considering the effect of a change in the size of the contested rent. Having defined a contestant's utility function over their contest allocation and effort, we then go on to define the marginal rate of substitution, which gives the additional rent required to compensate for an increase in effort. How fast or slow this marginal rate of substitution changes relative to the contest allocation is a key determinant of a contestant's effort response to an increase in the rent: effort will increase or decrease depending on whether it falls short of, or exceeds, a threshold. In the standard model it will never exceed this threshold, but this can easily be the case with more general—and very reasonable—preferences. From a simple Tullock share contest with general preferences,

we then advance our framework by including a more general contest success function as well as providing an analysis where the rent is endogenously determined by aggregate efforts. Throughout all advancements, we observe the rate of change of the marginal rate of substitution as pivotal to the outcome of the contest.

An important factor of our framework is the inclusion of diminishing marginal utility of the contest allocation, as well as allowing for interactions between contest effort and the (marginal) valuation of the rent allocation. Since the contribution of Hillman and Katz (1984), non-linear preferences have been considered in winner-take-all contests—in which the contest success function determines the probability of winning the indivisible rent—by accounting for contestants’ expected utility of engaging in the contest. This extension has commanded substantial attention in the literature (Long and Vousden, 1987; Skaperdas and Gan, 1995; Konrad and Schlesinger, 1997; Treich, 2010; Cornes and Hartley, 2012; Jindapon and Whaley, 2015; Jindapon and Yang, 2016).¹ For a larger rent being contested in a winner-take-all contest, the inclusion of expected utility generates nothing surprising. This formulation, however, does not carry over to the rent-sharing interpretation of contests, where the evaluation of the outcome of the contest should be the share of the rent received with certainty. We thus provide the first analysis of share contests where individuals have heterogeneous and general preferences.

This analysis extends the domain of applicability of this important class of models to situations where contestants have more than a simple linear evaluation of the contest outcome. Our framework is not without consequence for, while we show that as in standard contests reasonable conditions admit a unique equilibrium, a conventional wisdom of the contest literature—that effort is increasing in the size of the rent—does not hold when preferences satisfy some very standard conditions. Understanding this is of key importance: the conventional wisdom from the study of contests should not be applied to economic environments where individuals’ preferences do not satisfy the strict assumptions of the existing literature. Our article shows that this would be misguided and the interactions within contest environments are often a lot richer.

The remainder of the article is structured as follows. Section 2 provides two illustrative examples to highlight the importance of non-linear preferences in contests, and explores the intuition that underpins these examples. In Section 3 we outline sharing contests in which players have general preferences, and we go on to analyze the existence and uniqueness of Nash equilibrium by exploiting the aggregative properties of the game in Section 4. In Section 5 we explore the relationship between the size of the contested rent and contestants’ effort in equilibrium. Section 6 considers the effect on the dissipation ratio of accounting for more general preferences. In Section 7 we provide two extensions: to more general contest success functions; and to situations in which the rent is endogenously determined by aggregate effort. Section 8 provides our concluding remarks. All proofs are contained in the Appendix.

2 Tullock contests, examples, and intuition

In a contest each of the n contestants can be thought of as choosing a level of (costly) effort $x^i \geq 0$ in contesting a rent of size R . Their success in the contest is determined by the contest success function $\phi(x^i, \mathbf{x}^{-i})$, which is influenced not only by their own effort but also the effort of other contestants, denoted by the vector \mathbf{x}^{-i} . In a simple Tullock contest this takes the form $\phi(x^i, \mathbf{x}^{-i}) = x^i / [x^i + X^{-i}]$ if the aggregate effort $X \equiv x^i + X^{-i}$ is strictly positive, otherwise $\phi(x^i, \mathbf{x}^{-i}) = 1/n$, where X^{-i} is the sum of all contestants’ efforts excluding that of contestant i . Two interpretations of such contests are possible. In the first, the rent is indivisible and the

¹Long and Vousden (1987) consider a model in which individuals each contest a rent that they will ultimately receive a share of, but the share is determined randomly, the process being influenced by all contestants’ choices of efforts. However, this is not a contest as axiomatized by Skaperdas (1996) since there is nothing to tie the shares of contestants together that would ensure the full rent, and only the full rent, is allocated.

contest success function determines the probability that each contestant is awarded the rent; thus, in such winner-take-all contests only one contestant becomes the winner of the economic rent. In the second interpretation, which is the focus of this article, the rent is perfectly divisible and the contest success function determines the share of the rent awarded to each contestant.

In a winner-take-all contest the allocation a typical contestant receives is determined probabilistically. Two states occur: that in which the contestant wins the contest, which occurs with probability $\phi(x^i, \mathbf{x}^{-i})$ with an outcome $R - x^i$; and that in which the contestant does not win the contest with outcome $-x^i$, which occurs with probability $1 - \phi(x^i, \mathbf{x}^{-i})$. In the sharing interpretation the allocation of the rent to a contestant is deterministic: the contest success function determines the share of the rent the contestant receives with certainty. If contestants' evaluation of the contest outcome is linear then, even with a non-linear but additive cost of effort, the expected payoff in the winner-take-all interpretation is exactly the same as in the sharing interpretation, so they are strategically equivalent and conclusions drawn for one apply to the other. However, from very early in the contest literature non-linearities in the evaluation of the contest outcome in the winner-take-all interpretation—in the form of risk aversion—have been considered (Hillman and Katz, 1984). The expected utility in a winner-take-all contest bears no resemblance to a sensible payoff function in a sharing contest where the outcome is deterministic, and therefore none of the advances that account for non-linear evaluation of the contest outcome can be applied to sharing contests: the two require separate study. Despite the wealth of applications where sharing contests are a more appropriate model for determining the allocation of economic rents that we alluded to in the introduction, there has been no attempt to capture more general preferences in such contests.

In a sharing contest the allocation of the rent a contestant receives is $z^i = \phi(x^i, \mathbf{x}^{-i})R$. The best the literature has done in providing an analysis of sharing contests is to consider non-linear cost of effort in which case payoffs take the form $V^i(x^i, X^{-i}) = z^i - c^i(x^i)$; it is from careful study of this model (that could also be deduced from a winner-take-all interpretation since the two are equivalent in this case) that provides the conventional wisdom of a monotonically increasing relationship between the contested rent and contest effort. However, this misses two key aspects of the evaluation of the contest outcome: first, contestants may exhibit diminishing marginal utility over the contest outcome; second, there may be interactions between the effort a contestant exerts in contesting the rent and their (marginal) evaluation of the allocation from the contest, in which case payoffs should not be separable in x^i and z^i . It is not inconceivable that both of these are important.

We now turn to present two examples that illustrate the importance of diminishing marginal utility. In the first, we consider a contest in which there are n identical contestants whose payoffs take the form $V^i(x^i, X^{-i}) = [z^i - k]^\alpha - x^i$, where we assume $0 < \alpha \leq 1$, $k \geq 0$, and $R > nk$. Clearly, if $\alpha = 1$ this is a standard contest with a fixed cost k of competing in the contest. We deduce from investigation of the first-order condition that the equilibrium effort in any symmetric Nash equilibrium, written as a function of the contested rent R , takes the form

$$x^*(R) = \frac{\alpha[n-1]}{n^2} R \left[\frac{R}{n} - k \right]^{\alpha-1}.$$

Consequently, the response of individual effort to a change in the rent is given by

$$x^{*'}(R) \gtrless 0 \Leftrightarrow R \gtrless \frac{nk}{\alpha}.$$

When $\alpha = 1$, $x^{*'}(R) > 0$ for all $R > nk$. However, when $\alpha < 1$, $x^{*'}(R) < 0$ for $nk < R < \frac{nk}{\alpha}$, and $x^{*'}(R) > 0$ for $R > \frac{nk}{\alpha}$; by extension the same conclusions apply to aggregate effort. As such, although the conventional wisdom holds when the rent is large enough, when it is small a reduction in the size of the contested rent will lead to an *increase* in contest effort. This is illustrated in Panel (a) of Figure 1.

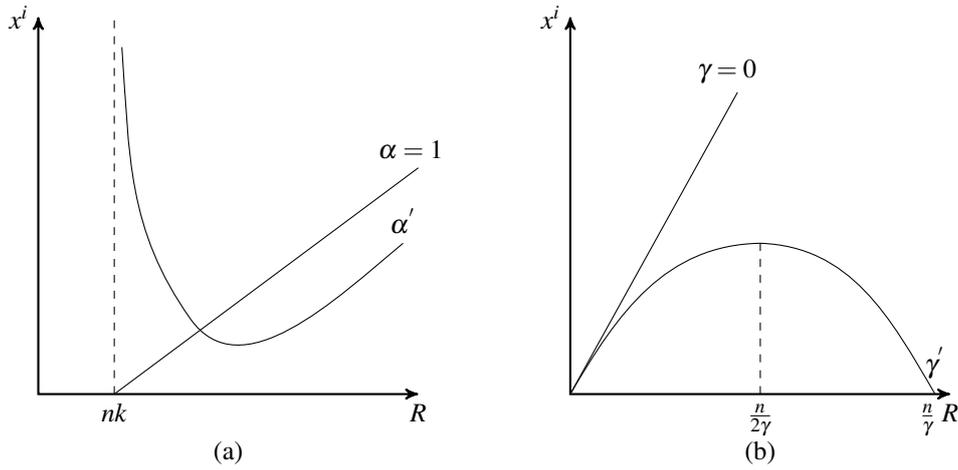


Figure 1: Panel (a) shows equilibrium effort in the first example, with $\alpha' < 1$; Panel (b) shows equilibrium effort in the second example, with $\gamma' > 0$.

Our second example again considers a situation where there are n identical contestants, but where their payoffs take the form $V^i(x^i, X^{-i}) = z^i - \frac{\gamma}{2}[z^i]^2 - x^i$. To ensure positive marginal utility at least in equilibrium, we assume $R < n/\gamma$. If $\gamma = 0$ payoffs are linear in contest allocation and effort, as in a standard contest. Analysis of the symmetric Nash equilibrium in this contest reveals

$$x^*(R) = \frac{n-1}{n^3}R[n - \gamma R]$$

and therefore for $\gamma > 0$ the response of equilibrium effort to a change in the contested rent is given by

$$x^{*'}(R) \geq 0 \Leftrightarrow R \leq \frac{n}{2\gamma}.$$

If $\gamma = 0$ then $x^{*'}(R) = \frac{n-1}{n^2} > 0$ for all R , but when $\gamma > 0$ then $x^{*'}(R) > 0$ for $0 < R < n/2\gamma$, but $x^{*'}(R) < 0$ for $n/2\gamma < R < n/\gamma$. In this example the conventional wisdom holds when the rent is small, but when the rent is large further increases in the size of the rent lead to a reduction in contest effort. Panel (b) in Figure 1 illustrates this case.

In each of these examples—which involve quite standard preferences—contestants exhibit diminishing marginal utility over the contest allocation. In the first example contestants would increase their effort if the rent is small and reduced further. With a small allocation from the contest, marginal utility is high and any reduction in that allocation resulting from a reduction in the contested rent would have a large effect on utility, commanding an increase in effort despite it being costly. In the second example, contestants would reduce their effort if the rent is large and it increased further. With a large allocation from the contest, marginal utility is low and an increase in the contest allocation resulting in an increase in the rent would lead contestants to save the cost of effort for a reduction in their contest allocation, leading to a reduction in effort. These arguments, based on standard microeconomic principles, make clear that the conventional wisdom is at stake when diminishing marginal utility is accounted for in sharing contests.

To further explore the intuition behind this finding, consider that a contestant derives utility from the contest allocation z^i and their effort x^i , given by $u^i(z^i, x^i)$. Then in a Nash equilibrium in a simple Tullock contest each contestant may be seen as solving the problem

$$\max_{x^i \geq 0} u^i(z^i, x^i) \text{ s.t. } z^i = \frac{x^i}{x^i + X^{-i}}R.$$

This optimization problem can be represented graphically in the (x^i, z^i) -space by considering the point on the budget constraint—which is an increasing and concave function that starts

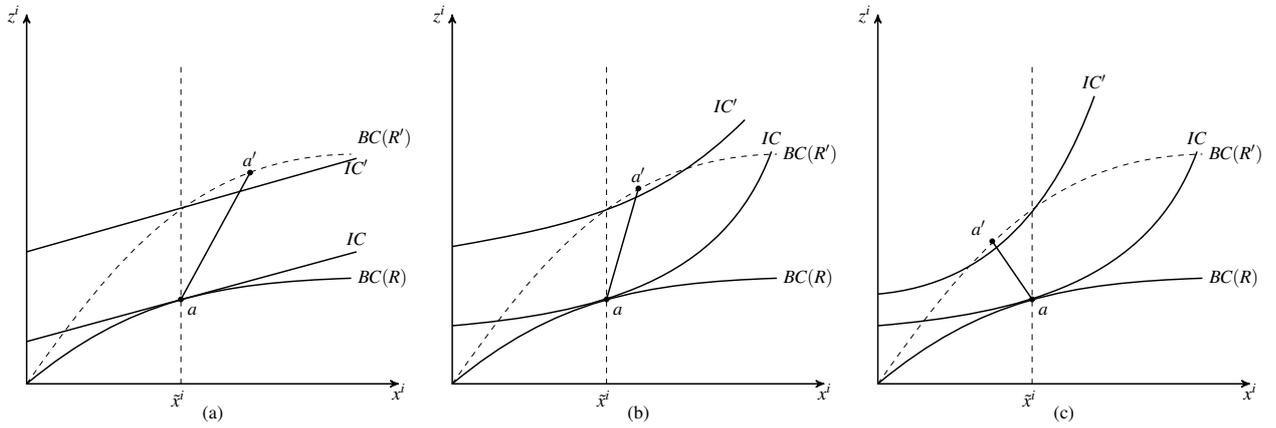


Figure 2: Rent expansion paths aa' with different preferences. In Panel (a) the slope of indifference curves do not depend on z^i , so the rent expansion path is upward-sloping; in Panel (b) the contestant has diminishing marginal utility but the marginal rate of substitution does not increase by as much as the slope of the budget constraint, so the rent expansion path is again positively-sloped; in Panel (c) the marginal rate of substitution increases by more than the slope of the budget constraint, giving a negatively-sloped rent expansion path.

from the origin—that puts the individual on the most north-westerly indifference curve derived from the utility function. Thus, we seek a level of effort—denoted by \tilde{x}^i —where the marginal rate of substitution of contest allocation for effort is equal to the slope of the budget constraint, which is $\frac{X^{-i}}{[x^i + X^{-i}]^2}R$. With a linear payoff function $u^i(z^i, x^i) = z^i - x^i$ and the marginal rate of substitution is everywhere equal to 1; and with linear evaluation of the rent but a convex cost of effort, i.e., $u^i(z^i, x^i) = z^i - c^i(x^i)$, it is $1/c^{i'}(x^i)$. In each of these cases the slope of indifference curves do not depend on the contest allocation: indifference curves are vertical displacements of each other.

Define a *rent expansion path* as the points that trace out the optimal effort-contest allocation combination for a contestant when the rent increases (keeping fixed the actions of all other contestants). This is illustrated in Figure 2 by aa' when the rent increases from R to R' for a variety of preferences. From Figure 2, we can observe that with a higher rent the slope of the budget constraint increases everywhere. With linear preferences (illustrated in Panel (a)) indifference curves are straight parallel lines and so in this case the former optimal effort can no longer be optimal on the new budget constraint since at this allocation the slope of the indifference curve must be less than the slope of the budget constraint. This necessitates an increase in effort to regain tangency, hence tracing out a positively-sloped rent expansion path. The same is true with quasi-linear preferences in which the cost of effort is convex and there is linear evaluation of the contest allocation. In this case (not illustrated) the slope of the indifference curve does not depend on z^i : at the former optimal level of x^i on the new budget constraint the slope of the indifference curve must be less than that of the budget constraint, again necessitating an increase in optimal effort.

With more general preferences the marginal rate of substitution² is $-\frac{u_x^i}{u_z^i}$ and if the marginal utility of the contest allocation is decreasing with z^i , then when z^i increases with x^i fixed, the marginal rate of substitution increases. If, as in Panel (b) of Figure 2, the marginal rate of substitution increases by less than the increase in the slope of the budget constraint then, again, the rent expansion path will be positively-sloped, consistent with the conventional wisdom on contests. Conversely, if the marginal rate of substitution increases by more than the increase in the slope of the budget constraint, then with the higher rent optimality will occur at a *lower*

²Throughout we use subscripts to denote partial derivatives.

level of effort, giving a negatively-sloped rent expansion path, as shown in Panel (c) of Figure 2.

This discussion highlights that taking diminishing marginal utility into account in contests is very important, since it reverses some of the core results of contest theory. Another key factor—not explored within these examples—is the general conditions on preferences that allow for either a positively or negatively sloped rent expansion path in equilibrium. Therefore, a key aim of this article—and what we now turn to—is to understand under what conditions on preferences the conventional wisdom in contests holds, and when it does not.

3 Sharing contests with general preferences

In this section we formally define our model. Consider a set of individual contestants $N = \{1, \dots, n\}$, $n \geq 2$, which participate in a contest to obtain an economic rent R . Their success in the contest is determined by their effort relative to the effort of other contestants and is given by the contest success function $\phi(x^i, \mathbf{x}^{-i})$, where x^i denotes the costly effort of player $i \in N$ and \mathbf{x}^{-i} denotes the vector of all other contestants' effort levels. In this article we focus on contests in which the rent is perfectly divisible and is shared between contestants in accordance with the contest success function, rather than awarded to a single contestant as would be the case in a winner-take-all contest. Define z^i as being contestant i 's allocation of the rent from the contest:

$$z^i \equiv \phi(x^i, \mathbf{x}^{-i})R. \quad (1)$$

We begin by studying a 'simple' Tullock contest for an exogenously-given rent of size R in which

$$\phi(x^i, \mathbf{x}^{-i}) = \begin{cases} \frac{x^i}{x^i + X^{-i}} & \text{if } X > 0 \text{ or} \\ \frac{1}{n} & \text{otherwise,} \end{cases} \quad (2)$$

where, as previously stated, $X \equiv \sum_{j \in N} x^j$ is the aggregate effort of all contestants and $X^{-i} \equiv X - x^i$. Later in the article we consider more general contest success functions, as well as situations in which the size of the rent is endogenously determined by contestants' efforts.

For each contestant i we define a utility function $u^i(z^i, x^i)$ over their contest allocation, z^i , and their effort in contesting the rent, x^i . We denote by $MRS^i(z^i, x^i)$ contestant i 's marginal rate of substitution between z^i and x^i :

$$MRS^i(z^i, x^i) \equiv -\frac{u_x^i}{u_z^i},$$

which gives the amount of additional rent that is required to compensate for an incremental increase in effort. Consider the (x^i, z^i) -space. Since utility is increasing in z but decreasing in effort, indifference curves derived from the utility function defined above will have a positive slope (measured by the marginal rate of substitution just defined) and utility is increasing in a north-west direction.

We allow contestants to be heterogeneous with different utility functions, but assume that all contestants' utility is increasing in their allocation of the rent at a decreasing rate; decreasing in effort at an increasing rate; and if there are complementarities between the allocation of the rent from the contest and effort then these are sufficiently small.

Assumption 1. For each $i \in N$,

- (a) the utility function is twice continuously differentiable with $u_z^i > 0$, $u_{zz}^i \leq 0$, $u_x^i < 0$, $u_{xx}^i \leq 0$;
- (b) $MRS^i(0, 0) < \infty$, and if $MRS^i(0, 0) = 0$ then $MRS_x^i(0, 0) > 0$;
- (c) $u_{zx}^i \leq \min \left\{ MRS^i |u_{zz}^i|, \frac{1}{MRS^i} |u_{xx}^i| \right\}$.

Concavity of the utility function is, of course, standard. The first part of condition (b) rules out contestants always being inactive and the second part rules out contestants always wanting to exert infinite effort. The last condition in the assumption—which ensures complementarities between z^i and x^i are sufficiently small—implies that $MRS_z^i \geq 0$ and $MRS_x^i \geq 0$; to observe this note that

$$MRS_z^i = -\frac{u_z^i u_{zx}^i - u_x^i u_{zz}^i}{(u_z^i)^2} \quad \text{and} \quad MRS_x^i = -\frac{u_z^i u_{xx}^i - u_x^i u_{zx}^i}{(u_z^i)^2}$$

and therefore $MRS_z^i \geq 0 \Leftrightarrow u_{zx}^i \leq -MRS^i u_{zz}^i$ and (noting that $u_x^i < 0$) $MRS_x^i \geq 0 \Leftrightarrow u_{zx}^i \leq -\frac{1}{MRS^i} u_{xx}^i$.

Assumption 1, which we suppose is satisfied in the remainder of the analysis, allows for a very broad class of preferences. For instance, this framework nests the standard linear case where $u^i(z^i, x^i) = z^i - x^i$, which is the dominant structure used within the contest literature; and we can also capture a convex cost of effort if we specified $u^i(z^i, x^i) = z^i - c^i(x^i)$ with $c_x^i > 0, c_{xx}^i \geq 0$. As well as being able to capture diminishing marginal utility, the fact that we do not assume a separable utility function also allows us to capture situations in which there are interactions between the level of effort a contestant uses, and their (marginal) valuation of the contest allocation. Existing studies have assumed that preferences are (quasi-)linear, with utility being linear in the share of the rent received. Thus, by considering general preferences, our framework can not only provide an analysis that nests previous studies of share contests but also provides a tractable methodology by which to consider alternative and novel preferences, which can be used to advance and expand the understanding and applicability of contests.

4 Characterizing equilibria in Tullock contests with general preferences

We now turn to characterize equilibria in a simple Tullock contest over an exogenously-given perfectly divisible rent R . We seek a Nash equilibrium in the simultaneous-move game of complete information in which the player set is the contestants $N = \{1, \dots, n\}$; their strategies are their choice of effort $x^i \geq 0$; and their payoffs are given by their utility of the contest outcome $u^i(z^i, x^i)$ that we assume satisfies Assumption 1, where $z^i = \phi(x^i, \mathbf{x}^{-i})R$ with $\phi(x^i, \mathbf{x}^{-i})$ specified in (2).

First, we note that at a Nash equilibrium of the contest each player may be seen as solving the problem

$$\max_{x^i \geq 0} u^i \left(\frac{x^i}{x^i + X^{-i}} R, x^i \right).$$

The necessary first-order condition for x^i to maximize utility given $X^{-i} = \sum_{j \neq i \in N} x^j$, i.e. identify a best response, is

$$\frac{X^{-i}}{[x^i + X^{-i}]^2} R u_z^i + u_x^i \leq 0, \quad (3)$$

with equality if $x^i > 0$.

Lemma 1. *The first-order condition is both necessary and sufficient for identifying a best response.*

Contestant i 's best response is thus given by $b^i(X^{-i}; R) = \max\{0, x^i\}$ where x^i is the unique solution to

$$MRS^i \left(\frac{x^i}{x^i + X^{-i}} R, x^i \right) = \frac{X^{-i}}{[x^i + X^{-i}]^2} R,$$

and we seek a Nash equilibrium in which players use mutually consistent best responses.

Rather than working directly with best responses, we turn to analyze the contest using an extension of the 'share function' approach that exploits the aggregative nature of the game,

as developed by Cornes and Hartley (2003, 2005, 2012), but where we allow for more general preferences in sharing contests. This approach differs from pursuing study of best responses in the following way: rather than asking what value of a contestant's effort is consistent with a Nash equilibrium in which the aggregate effort of all other contestants is X^{-i} (which is the best response), it asks what value of individual effort is consistent with a Nash equilibrium in which the aggregate effort of *all* contestants, including contestant i , is X . This gives individual consistency, and to identify a Nash equilibrium aggregate consistency is required, where the sum of individual efforts is exactly equal to the aggregate effort. Rather than working with effort *levels*, it is natural in sharing contests to work with *shares* of the aggregate effort, in which case the aggregate consistency condition requires the sum of the shares to be equal to 1.

For each contestant we define a 'share function' that gives their share of the rent that is consistent with a Nash equilibrium in which the aggregate effort of all contestants is $X > 0$. By replacing X^{-i} with $X - x^i$ in the first-order condition (3), letting $\sigma^i \equiv x^i/X$ and then replacing x^i with $\sigma^i X$, we deduce that contestant i 's share function is given by $s^i(X; R) = \max\{0, \sigma^i\}$ where σ^i is the solution to

$$l^i(\sigma^i, X; R) \equiv MRS^i(\sigma^i R, \sigma^i X) - [1 - \sigma^i] \frac{R}{X} = 0. \quad (4)$$

Share functions shed light on individual behavior consistent with a Nash equilibrium: $Xs^i(X; R)$ is the effort of contestant i consistent with a Nash equilibrium in which the aggregate effort of all contestants is $X > 0$. As noted, identification of a Nash equilibrium requires aggregate consistency, that is, the sum of individual share functions to be equal to unity. Letting

$$S(X; R) \equiv \sum_{j \in N} s^j(X; R),$$

we have the following equivalence statement.

Lemma 2. *In a contest with rent R , there is a Nash equilibrium with aggregate effort $X^* > 0$ if and only if*

$$S(X^*; R) = 1.$$

Questions of the existence and uniqueness of Nash equilibrium now rest on consideration of the behavior of the aggregate share function $S(X; R)$, whose properties are derived from individual share functions, and its intersection with the unit line. The following lemma sets out the properties of individual share functions.

Lemma 3. *For each contestant $i \in N$,*

1. $s^i(X; R)$ is a continuous function defined for all $X > 0$ and R ;
2. (a) $s^i(X; R) \rightarrow 1$ as $X \rightarrow 0$; and (b) either $s^i(X; R) = 0$ for all $X \geq \bar{X}^i(R) \equiv R/MRS^i(0, 0)$ if $MRS^i(0, 0) > 0$ or, if $MRS^i(0, 0) = 0$, $s^i(X; R) \rightarrow 0$ as $X \rightarrow \infty$; and
3. $s^i(X; R)$ is strictly decreasing in X for $0 < X < \bar{X}^i(R)$.

Note that if $MRS^i(0, 0) > 0$ there is some 'drop-out' value of aggregate effort $\bar{X}^i(R)$ where the contestant would become inactive in the contest; whereas if $MRS^i(0, 0) = 0$ for all $i \in N$ then all contestants will be active in any contest.

The properties of individual share functions imply that in a contest in which the rent is R the aggregate share function $S(X; R)$, being constructed from a sum of at least two individual share functions, exceeds 1 when X is small enough, is less than one when X is large enough, and is continuous and strictly decreasing in X implying there is exactly one value of X where $S(X; R) = 1$.

Proposition 1. *In a contest with rent R there is a unique Nash equilibrium with aggregate effort X^* such that*

$$S(X^*; R) = 1$$

in which the equilibrium effort of contestant i is $x^{i} = X^* s^i(X^*; R)$.*

As such, we confirm that in rent-sharing contests where players can have more general preferences over their allocation of the rent and the effort exerted in contesting the rent (but that satisfy Assumption 1), the uniqueness of Nash equilibrium—as found in simple Tullock contests with linear separable preferences—is preserved.

5 The effect of changing the size of the contested rent

We now turn to investigate how contestants' equilibrium behavior depends on the size of the rent they are contesting. We write $\mathcal{X}(R)$ for the equilibrium aggregate effort in a contest where the size of the rent is R , which is implicitly defined by

$$S(\mathcal{X}(R); R) = 1. \quad (5)$$

Having exploited the aggregative properties of the game, this becomes a relatively straightforward task, since understanding the effect of an increase in the rent on equilibrium aggregate effort boils down to understanding how share functions vary with R , which we consider in the following lemma.

Lemma 4.

$$s_R^i \gtrless 0 \Leftrightarrow z^i MRS_z^i - MRS^i \lesseqgtr 0.$$

As we demonstrate in the following proposition, this allows us to understand the effect on the equilibrium aggregate effort expended in contesting a larger rent, one of the main objectives of our study.

Proposition 2. *If $z^i MRS_z^i - MRS^i \lesseqgtr 0$ for all $i \in N$ then in a contest with $R' > R$, $\mathcal{X}(R') \gtrless \mathcal{X}(R)$.*

Although it is tempting to implicitly differentiate (5) to understand the effect of a change in the contested rent on equilibrium effort, it is not correct to do so in general because if there are contestants for whom $MRS^i(0,0) > 0$ then there will be values of X where these contestants 'drop out' of the contest and the aggregate share function is not differentiable at these drop-out values. This necessitates us to assume that the sign of $z^i MRS_z^i - MRS^i$ is the same for all contestants. If, however, we are willing to assume $MRS^i(0,0) = 0$ for all contestants then they will always be active in any contest, implying implicit differentiation of (5) can be used. Proposition 2 can then be re-stated without assuming the sign of $z^i MRS_z^i - MRS^i$ is the same for all contestants.

Proposition 3. *Suppose that $MRS^i(0,0) = 0$ for all $i \in N$. Then*

$$\mathcal{X}'(R) \gtrless 0 \Leftrightarrow \sum_{j \in N} w^j [z^j MRS_z^j - MRS^j] \lesseqgtr 0,$$

where $w^i = [R[R MRS_z^i + X MRS_x^i + \frac{R}{X}]]^{-1} > 0$.

With a larger contested rent, whether effort increases, as the conventional wisdom suggests, or decreases depends on the sign of $z^i MRS_z^i - MRS^i$. If this is positive then the conventional wisdom need not hold, as demonstrated in the previous propositions. Since $z^i MRS_z^i - MRS^i = [z^i]^2 \frac{\partial}{\partial z^i} \left\{ \frac{MRS^i}{z^i} \right\}$, this condition can be interpreted as requiring that contestants' marginal utility increases sufficiently as z^i increases, indeed at a rate that is larger than the increase in z^i . In

conventional analysis of contests this does not happen since the marginal rate of substitution is assumed to be constant in z^i , but as demonstrated through study of our examples, it can happen with very reasonable preferences.

Returning to our diagrammatic exposition in Section 2, this condition makes perfect sense: when the rent increases the change in the slope of the budget constraint is given by $[1 - \sigma^i][1/X]$ which, using the first-order condition, is equal to MRS^i/R ; the change in the slope of indifference curves is $\frac{\partial}{\partial R}\{MRS^i(\sigma^i R, \sigma^i X)\} = \sigma^i MRS_z^i$; as such, at the original optimal effort on the new budget constraint, the slope of the indifference curve increases by more (less) than the slope of the budget constraint—giving rise to a reduction (increase) in effort—precisely if $z^i MRS_z^i - MRS^i > (<)0$.

In terms of derivatives of the utility function,

$$z^i MRS_z^i - MRS^i = \frac{z^i [u_x^i u_{zz}^i - u_z^i u_{xz}^i] + u_z^i u_x^i}{[u_z^i]^2}.$$

In the standard case studied in the literature the term in square brackets in the numerator is equal to zero so, noting that $u_x^i < 0$, $z^i MRS_z^i - MRS^i$ is always negative. The above expression reveals that a negative relationship between the size of the contested rent and contest effort can stem from either sufficiently strong diminishing marginal utility of the contest allocation (i.e. u_{zz}^i sufficiently negative); or a sufficiently negative interaction between effort and the marginal utility of the contest outcome (i.e. u_{zx}^i sufficiently negative); or indeed a combination of both.

6 The dissipation ratio

The early contest literature (e.g., Tullock, 1980) had a keen interest in studying the dissipation ratio, which is the ratio of aggregate effort to the contested rent. Understanding the properties of this reveals whether the social cost of rent seeking can be approximated by knowing the size of the contested rent, even if rent-seeking efforts are not themselves observable. In a simple Tullock contest with linear evaluation of the contest outcome and linear costs, it is well-known (see, for example, Konrad, 2009) that the dissipation ratio is invariant to the size of the contested rent.

We now consider how the dissipation ratio responds to a change in the contested rent in a Tullock contest with general preferences. Recall that the share function satisfies the first-order condition (4). Let $D = \frac{X}{R}$ be the dissipation ratio. Then, we can write share functions as being dependent on the dissipation ratio by changing variables since $X = DR$. As such, $\sigma^i = s^i(DR; R)$ will satisfy

$$l^i(\sigma^i, DR; R) \equiv MRS^i(\sigma^i R, \sigma^i DR) - [1 - \sigma^i] \frac{1}{D} = 0, \quad (6)$$

and the equilibrium dissipation ratio, written $\mathcal{D}(R)$, will satisfy

$$S(\mathcal{D}(R)R; R) \equiv \sum_{j \in N} s^j(\mathcal{D}(R)R; R) = 1. \quad (7)$$

When preferences are such that aggregate effort decreases in the rent then, of course, the dissipation ratio also reduces. In the following proposition we demonstrate that this is true whenever preferences satisfy the conditions stated in Assumption 1 that we have been working under thus far: although aggregate effort may increase in the size of the contested rent, it never increases faster than the increase in the rent.

Proposition 4. *If $R' > R$, then $\mathcal{D}(R') \leq \mathcal{D}(R)$.*

We have so far limited our attention to cases where contestants' marginal utility of the contest outcome is declining. If we allow for marginal utility to be *increasing*, then it is not inconceivable that in a contest with a higher rent the increase in effort by contestants with such preferences may be greater than the increase in the rent, which will mean the dissipation ratio increases. To explore this we need to be careful that when we allow convexity in the utility function we maintain concavity of the optimization problem contestants face. For the purpose of illustration we assume that all contestants are homogeneous and focus on symmetric equilibria in which each contestant's equilibrium share of the rent is $1/n$. When all contestants are symmetric we can use implicit differentiation of (7) to understand how the dissipation ratio changes with the contested rent. This is given by

$$\mathcal{D}'(R) = -\frac{\frac{ds^i}{dR}}{\frac{ds^i}{dD}}.$$

Now, when contestants are symmetric $z^i = R/n$ and $x^i = X/n$, so

$$l_\sigma^i = n \left[\frac{R}{n} MRS_z^i + \frac{X}{n} MRS_x^i + \frac{1}{nD} \right],$$

$$\frac{dl^i}{dD} = \frac{R}{n} MRS_x^i + \frac{n-1}{nD^2}, \text{ and}$$

$$\frac{dl^i}{dR} = \frac{1}{R} \left[\frac{R}{n} MRS_z^i + \frac{X}{n} MRS_x^i \right].$$

To ensure concavity of the optimization problem ($l_\sigma^i > 0$) we must assume that $MRS_z^i > -\frac{X}{R} MRS_x^i - \frac{1}{X}$ (noting that $D = X/R$). It is also the case that $\frac{dl^i}{dD} > 0$, which implies $\frac{ds^i}{dD} < 0$, and therefore the sign of $\mathcal{D}'(R)$ will be the same as the sign of $\frac{ds^i}{dR}$. This is positive if $\frac{R}{n} MRS_z^i + \frac{X}{n} MRS_x^i < 0$, which requires $MRS_z^i < -\frac{X}{R} MRS_x^i$. Therefore, when preferences are such that $-\frac{X}{R} MRS_x^i - \frac{1}{X} < MRS_z^i < -\frac{X}{R} MRS_x^i$, which is a non-empty range, the dissipation ratio will be *increasing* in the size of the contested rent.

7 Extensions

In this section we pursue two generalizations of our model of contests with general preferences: the first allows for a more general contest success function; the second allows for the rent to be endogenously determined by contestants' efforts.

7.1 General contest success functions

So far, we have considered a simple Tullock contest in which the contest success function has taken the form $\phi(x^i, \mathbf{x}^{-i}) = \frac{x^i}{x^i + X^{-i}}$. Of course, Tullock contests can be more general than this with a contest success function given by $\phi(x^i, \mathbf{x}^{-i}) = \frac{[x^i]^r}{[x^i]^r + \sum_{j \neq i \in N} [x^j]^r}$, where r captures a contest technology. To study more general contest success functions we follow Szidarovszky and Okuguchi (1997) and specify that

$$\phi(x^i, \mathbf{x}^{-i}) = \frac{p^i(x^i)}{\sum_{j \in N} p^j(x^j)}, \quad (8)$$

where we assume that $p^i(0) = 0$, $p^{i'} > 0$ and $p^{i''} \leq 0$.³

We now need to re-consider our analysis with this more general contest success function. So that the share function approach can be utilized, let us change the variable of consideration

³Cornes and Hartley (2005) studied contests with general technologies but assumed linear payoffs for contestants.

and rather than focus on effort, x^i , think of contestants choosing their 'input' $y^i = p^i(x^i)$, from which effort can be derived since $x^i = p^{i-1}(y^i)$. The assumptions on the contest technology just stated imply $p^{i-1}' > 0$ and $p^{i-1}'' \geq 0$. With this change of variable, contestants can be seen as choosing their input to maximize their payoff $u^i(z^i, p^{i-1}(y^i))$, where their share of the rent is $z^i = \frac{y^i}{y^i + Y^{-i}}R$ in which $Y^{-i} = Y - y^i$ and Y is the aggregate input $\sum_{j \in N} y^j$.

The first-order condition of this optimization problem that characterizes a contestant's input best response is

$$\frac{Y^{-i}}{[y^i + Y^{-i}]^2} R u_z^i + p^{i-1}' u_x^i \leq 0$$

with equality if $y^i > 0$.⁴ Replacing Y^{-i} with $Y - y^i$ and letting $\hat{\sigma}^i = y^i/Y$, this can be used to define a contestant's share function as $\hat{s}^i(Y; R) = \max\{0, \hat{\sigma}^i\}$, where $\hat{\sigma}^i$ is the solution to

$$\hat{l}^i(\hat{\sigma}^i, Y; R) \equiv MRS^i(\hat{\sigma}^i R, p^{i-1}(\hat{\sigma}^i Y)) p^{i-1}'(\hat{\sigma}^i Y) - [1 - \hat{\sigma}^i] \frac{R}{Y} = 0. \quad (9)$$

As with our previous analysis, we can use share functions to shed light on the properties of Nash equilibrium in the contest, since there is a Nash equilibrium with aggregate input Y^* if and only if $\sum_{j \in N} \hat{s}^j(Y^*; R) = 1$, in which contestant i 's input is $y^{i*} = Y^* \hat{s}^i(Y^*; R)$ and therefore their contest effort is $x^{i*} = p^{i-1}(Y^* \hat{s}^i(Y^*; R))$.

In this more general framework an analog of Lemma 3 applies to individual share functions by treating X as Y ; letting $\bar{Y}^i(R) = R/[MRS^i(0,0)p^{i-1}'(0)]$; and noting that share functions are monotonically decreasing in Y since

$$\begin{aligned} \hat{s}_Y^i &= -\frac{\hat{l}_Y^i}{\hat{l}_\sigma^i} \\ &= -\frac{\hat{\sigma}^i [p^{i-1}']^2 MRS_x^i + \hat{\sigma}^i MRS^i p^{i-1}'' + [1 - \hat{\sigma}^i] \frac{R}{Y^2}}{p^{i-1}' [R MRS_z^i + Y p^{i-1}' MRS_x^i] + Y MRS^i p^{i-1}'' + \frac{R}{Y}} \\ &< 0. \end{aligned}$$

The aggregate equilibrium input as a function of the contested rent is denoted $\mathcal{Y}(R)$, which will satisfy

$$\sum_{j \in N} \hat{s}^j(\mathcal{Y}(R); R) = 1.$$

To consider how the aggregate input varies with the size of the contested rent let us assume, for convenience, that all contestants are active in equilibrium, which will be true if either $MRS^i(0,0) = 0$ for all contestants; or at least each contestant's drop-out value exceeds the equilibrium aggregate input. This allows us to simplify the analysis by using implicit differentiation. Since

$$\mathcal{Y}'(R) = -\frac{\sum_{j \in N} \hat{s}_R^j}{\sum_{j \in N} \hat{s}_Y^j},$$

and as we just showed $\hat{s}_Y^i < 0$ for all $i \in N$, the important property is how share functions vary with the contested rent, \hat{s}_R^i . Implicit differentiation of the first-order condition (9) gives $\hat{s}_R^i = -\hat{l}_R^i/\hat{l}_\sigma^i$, and since $\hat{l}_\sigma^i > 0$ it follows that

$$\begin{aligned} \hat{s}_R^i \geq 0 &\Leftrightarrow \hat{\sigma}^i MRS_z^i p^{i-1}' - [1 - \hat{\sigma}^i] \frac{1}{Y} \leq 0 \\ &\Leftrightarrow \frac{p^{i-1}'}{R} [z^i MRS_z^i - MRS^i] \leq 0, \end{aligned}$$

⁴Under our restrictions on the contest technology it is straightforward to show, given Lemma 1, that the second-order condition is satisfied.

where the last line follows from utilizing the first-order condition. As such, the conclusions we made in the simple contest for aggregate effort in Proposition 3 equally apply to aggregate input in more general contests; that is, if $z^i MRS^i - MRS^i \leq 0$ for all $i \in N$ then $\mathcal{Y}'(R) \geq 0$.

However, these conclusions apply to the aggregate contest input, not to effort. To conclude that aggregate contest effort exhibits the same properties, we must consider the effect on individual contest input for each contestant, since this will then imply the change in their individual effort, from which conclusions about aggregate effort can be drawn. Writing $\hat{y}^i(\mathcal{Y}(R); R) = \mathcal{Y}(R)\hat{s}^i(\mathcal{Y}(R); R)$, it follows that

$$\begin{aligned} \frac{d\hat{y}^i(\mathcal{Y}(R); R)}{dR} &= \hat{s}^i \mathcal{Y}' + \mathcal{Y}(\hat{s}_Y^i \mathcal{Y}' + \hat{s}_R^i) \\ &= \mathcal{Y}'(\hat{s}^i + \mathcal{Y}\hat{s}_Y^i) + \mathcal{Y}\hat{s}_R^i. \end{aligned}$$

Now,

$$\begin{aligned} \hat{\sigma}^i + \mathcal{Y}\hat{s}_Y^i &= \hat{\sigma}^i - \frac{Y[\hat{\sigma}^i [p^{i-1}]^2 MRS_x^i + \hat{\sigma}^i MRS^i p^{i-1}'' + [1 - \hat{\sigma}^i] \frac{R}{Y^2}]}{p^{i-1}' [RMRS_z^i + Y p^{i-1}' MRS_x^i] + Y MRS^i p^{i-1}'' + \frac{R}{Y}} \\ &= \frac{\hat{\sigma}^i p^{i-1} RMRS_z^i + \frac{R}{Y}}{p^{i-1}' [RMRS_z^i + Y p^{i-1}' MRS_x^i] + Y MRS^i p^{i-1}'' + \frac{R}{Y}} \\ &> 0 \end{aligned}$$

by gathering terms over a common denominator and simplifying. As such, since the signs of \hat{s}_R^i and \mathcal{Y}' are the same, it follows that

$$\frac{d\hat{y}^i(\mathcal{Y}(R); R)}{dR} \geq 0 \Leftrightarrow z^i MRS_z^i - MRS^i \leq 0,$$

and therefore the same is true of individual, and by extension aggregate, effort.

7.2 Endogenous rent

We next turn to consider the case of endogenous determination of the rent, first studied by Chung (1996), where we revert to a simple Tullock contest. Suppose that the rent being contested is influenced by the aggregate effort of contestants according to some technology $f(X; \alpha)$, that depends in a positive way on a parameter α that will allow us to consider the effect of an improvement in the rent generation technology. If $f_X < 0$ then rent-seeking activity deteriorates the size of the rent, while if $f_X > 0$ it enhances it: with this specification sharing contests embody a wide variety of applications including, for instance, Cournot competition (in which the rent is total revenue).

We make the following assumption concerning the rent generation technology.

Assumption 2. $f(X; \alpha)$ is bounded and twice continuously differentiable with $f_\alpha > 0$ and $f_{XX} \leq 0$. Further, we assume that $\lim_{X \rightarrow \infty} f_X \leq 0$ and if $f_X < 0$ then $f_X > \frac{f_{XX} - MRS_x^i}{MRS_z^i}$ for all $i \in N$.

With this specification, $z^i = \frac{x^i}{x^i + X^{-i}} f(x^i + X^{-i}; \alpha)$, and the first-order condition that characterizes a contestant's best response is given by

$$u_z^i \left[\frac{X^{-i}}{[x^i + X^{-i}]^2} f(x^i + X^{-i}; \alpha) + \frac{x^i}{x^i + X^{-i}} f_X(x^i + X^{-i}; \alpha) \right] + u_x^i \leq 0$$

with equality if $x^i > 0$. The second-order condition, after some manipulation, requires that

$$2MRS^i u_{zx}^i + MRS^i \left[MRS^i u_{zz}^i + \frac{1}{MRS^i} u_{xx}^i \right] - u_z^i \left[\frac{2[X - x^i]}{X^2} \left[\frac{f}{X} - f_X \right] - \frac{x^i}{X} f_{XX} \right] < 0.$$

Noting that concavity of the rent generation function implies $f/X - f_X \geq 0$ and $f_{XX} \leq 0$ allows the steps of Lemma 1 to be followed to conclude that the second-order condition is indeed satisfied.

We now need to ensure that share functions satisfy an analog of Lemma 3. With endogenous determination of the rent a contestant's share function will be given by $\tilde{s}^i(X; \alpha) = \max\{0, \sigma^i\}$ where σ^i satisfies

$$\tilde{l}^i(\sigma^i, X; \alpha) \equiv MRS^i(\sigma^i f(X; \alpha), \sigma^i X) - (1 - \sigma^i) \frac{f(X; \alpha)}{X} - \sigma^i f_X(X; \alpha) = 0. \quad (10)$$

First, note that

$$\tilde{l}_\sigma^i = f MRS_z^i + X MRS_x^i + \frac{f}{X} - f_X > 0$$

since $f/X - f_X \geq 0$ by virtue of our assumption that f is concave. As such, share functions are single valued. Next, note that a contestant's drop-out value, denoted $\tilde{X}^i(\alpha)$, is now implicitly defined, if it exists, as the value of X where $\tilde{l}^i(0, X; \alpha) \equiv MRS^i(0, 0) - f(X; \alpha)/X = 0$, which is unique since again concavity of f implies f/X is decreasing in X . If such a $\tilde{X}^i(\alpha)$ exists then $\tilde{s}^i(X; \alpha) = 0$ for all $X \geq \tilde{X}^i(\alpha)$. Otherwise $\tilde{s}^i(X; \alpha)$ is defined for all $X > 0$, and we must ensure it vanishes in the large X limit. Since $\lim_{X \rightarrow \infty} f_X \leq 0$ by Assumption 2, for the first-order condition to be satisfied as $X \rightarrow \infty$ the conditions in Assumption 1 imply that we require $\sigma^i = 0$, confirming $\lim_{X \rightarrow \infty} \tilde{s}^i(X; \alpha) = 0$. Finally, we deduce that

$$\begin{aligned} \tilde{s}_X^i &= - \frac{\tilde{l}_X^i}{\tilde{l}_\sigma^i} \\ &= - \frac{\sigma^i [f_X MRS_z^i + MRS_x^i - f_{XX}] + \frac{1}{X} [1 - \sigma^i] \left[\frac{f}{X} - f_X \right]}{f MRS_z^i + X MRS_x^i + \frac{f}{X} - f_X} \\ &< 0 \end{aligned}$$

since in Assumption 2 we specify that if $f_X < 0$ then $f_X > \frac{f_{XX} - MRS_x^i}{MRS_z^i}$.

In a contest with rent technology $f(X; \alpha)$, the equilibrium aggregate effort will be $\tilde{X}(\alpha)$, such that

$$\sum_{j \in N} \tilde{s}^j(\tilde{X}(\alpha); \alpha) = 1.$$

Since the sum of share functions is strictly decreasing in X by our previous deductions, how the equilibrium aggregate effort responds to a shift in the rent generation technology will depend on how individual share functions respond to a change in α . Implicit differentiation of (10) gives

$$\tilde{s}_\alpha^i = - \frac{\tilde{l}_\alpha^i}{\tilde{l}_\sigma^i},$$

and having already established that $\tilde{l}_\sigma^i > 0$ we know that $\text{sgn}\{\tilde{s}_\alpha^i\} = -\text{sgn}\{\tilde{l}_\alpha^i\}$. Now,

$$\begin{aligned} \tilde{l}_\alpha^i &= \sigma^i f_\alpha MRS_z^i - [1 - \sigma^i] \frac{f_\alpha}{X} - \sigma^i f_{X\alpha} \\ &= \frac{1}{f} \left[f_\alpha z^i MRS_z^i - f_\alpha \left[[1 - \sigma^i] \frac{f}{X} + \sigma^i f_X \right] + \sigma^i f_X f_\alpha - \sigma^i f f_{X\alpha} \right] \\ &= \frac{1}{f} \left[f_\alpha [z^i MRS_z^i - MRS^i] + \sigma^i [f_X f_\alpha - f f_{X\alpha}] \right]. \end{aligned}$$

Analogous to our previous analysis, if $\tilde{s}_\alpha^i \geq 0$ for all $i \in N$ then for $\alpha' > \alpha$ we will have that $\tilde{X}(\alpha') \geq \tilde{X}(\alpha)$. As such, we can conclude that for $\alpha' > \alpha$,

$$f_\alpha [z^i MRS_z^i - MRS^i] + \sigma^i [f_X f_\alpha - f f_{X\alpha}] \leq 0 \text{ for all } i \in N \Rightarrow \tilde{X}(\alpha') \geq \tilde{X}(\alpha).$$

With endogenous determination of the rent the condition that governs the direction of change in equilibrium aggregate effort in the contest when the rent generation technology improves is, as would be expected, more complicated than in our previous analysis. Our condition on the change in the ratio of the marginal rate of substitution to z^i as z^i changes features in this condition, but the properties of the rent generation technology are also important. Note, however, that if $f(X; \alpha) = \alpha g(X)$ then $f_X f_\alpha - f f_{X\alpha} = 0$, and so how equilibrium aggregate effort changes when α changes is governed by exactly the same conditions as in the simple contest with exogenous rent that is the benchmark model of this article.

8 Conclusions

In microeconomic analysis, constant marginal utility and separable preferences are generally seen as a very special case. Yet, the study of sharing contests in the existing literature has assumed this as standard. In this article, we have extended the theory of contests to allow for general preferences that include diminishing marginal utility and interactions between contest effort and the evaluation of the allocation from the contest. With more general preferences the conventional wisdom of a monotonically increasing relationship between the contested rent and effort expended in the contest—which proxies the social cost of rent-seeking—need no longer hold.

We take an aggregative games approach to study sharing contests with heterogeneous contestants that have general preferences. This allows us to deduce the uniqueness of Nash equilibrium in this more general framework and to undertake a tractable analysis of the properties of equilibrium. We show that the direction of change in the ratio of the marginal rate of substitution to contest allocation is crucial in determining whether aggregate effort increases or decreases when the contested rent increases. If contestants have either sufficiently strong diminishing marginal utility, or there are sufficiently strong interactions between effort and the (marginal) valuation of the contest allocation—or a combination of both—the conventional wisdom does not hold and aggregate effort decreases when the contested rent increases. We also study cases with more general contest technologies, and where the contested rent is endogenously determined by contestants' effort.

Our framework opens up the applicability of contests to economic environments where the basic economic interaction corresponds to a sharing contest, but individuals' preferences are more sophisticated than the simple form that has so far been investigated within the contest literature. Imposing linear utility functions is highly restrictive and inconsistent with most real-world situations. It is important to recognize that the conventional analysis that assumes linear utility functions is highly restrictive and inconsistent with most real-world situations. Thus it would be misleading to draw conclusions about the relationship between the strategic variable (effort) and the contested economic rent based on the existing results in the contest literature. Our analysis offers a framework within which to understand the precise nature of this relationship.

Appendix

Proof of Lemma 1. The second-order sufficient condition is

$$u_{zx}^i 2 \frac{X^{-i}}{[x^i + X^{-i}]^2} R + u_{zz}^i \left[\frac{X^{-i}}{[x^i + X^{-i}]^2} R \right]^2 + u_{xx}^i - u_z^i 2 \frac{X^{-i}}{[x^i + X^{-i}]^3} R < 0.$$

For any $x^i > 0$, the first-order condition implies $\frac{X^{-i}}{[x^i + X^{-i}]^2} R = -\frac{u_x^i}{u_z^i} = MRS^i$. As such, the second derivative can be re-written

$$\begin{aligned} & 2MRS^i u_{zx}^i + MRS^i \left[MRS^i u_{zz}^i + \frac{1}{MRS^i} u_{xx}^i \right] - \frac{2}{x^i + X^{-i}} MRS^i u_z^i \\ & < 2MRS^i u_{zx}^i - MRS^i \left[MRS^i |u_{zz}^i| + \frac{1}{MRS^i} |u_{xx}^i| \right] \\ & < 2MRS^i u_{zx}^i - 2MRS^i \min \left\{ MRS^i |u_{zz}^i|, \frac{1}{MRS^i} |u_{xx}^i| \right\} \\ & = 2MRS^i \left[u_{zx}^i - \min \left\{ MRS^i |u_{zz}^i|, \frac{1}{MRS^i} |u_{xx}^i| \right\} \right] \\ & \leq 0, \end{aligned}$$

since $u_{zx}^i \leq \min \left\{ MRS^i |u_{zz}^i|, \frac{1}{MRS^i} |u_{xx}^i| \right\}$ under Assumption 1. \square

Proof of Lemma 2. We seek to show that X^* is a Nash equilibrium if and only if $S(X^*; R) = 1$. First, the ‘if’ part. If X^* is a Nash equilibrium then $x^{i*} = b^i(X^{-i*}; R)$ for all $i \in N$. This implies $x^{i*} = b^i(X^* - x^{i*}; R)$ which in turn implies $x^{i*} = X^* s^i(X^*; R)$ for all $i \in N$, and therefore that $X^* = X^* \sum_{j \in N} s^j(X^*; R)$, and consequently $S(X^*; R) = 1$. For the ‘only if’ part, note that for each $i \in N$, $X^* s^i(X^*; R) = b^i(X^* - X^* s^i(X^*; R); R)$. If $S(X^*; R) = 1$ then $X^* = X^* S(X^*; R)$ and so for each $i \in N$, $X^* s^i(X^*; R) = b^i(X^* S(X^*; R) - X^* s^i(X^*; R); R) = b^i(X^{-i*}; R)$, thus allowing us to conclude that $x^{i*} = X^* s^i(X^*; R)$ for all $i \in N$ constitutes a Nash equilibrium. \square

Proof of Lemma 3. We address each property in turn.

1. Recall from (4) that a contestant’s share function is implicitly defined as the value of σ^i where

$$l^i(\sigma^i, X; R) \equiv MRS^i(\sigma^i R, \sigma^i X) - [1 - \sigma^i] \frac{R}{X} = 0,$$

if σ^i is positive, otherwise the share function takes the value zero. Note that

$$l_\sigma^i = R MRS_z^i + X MRS_x^i + \frac{R}{X} > 0 \quad (11)$$

since Assumption 1 implies $MRS_z^i \geq 0$ and $MRS_x^i \geq 0$, as we noted shortly after stating the assumption. This allows us to conclude that there is at most one value of $\sigma^i > 0$ where $l^i(\sigma^i, X; R) = 0$, so $s^i(X; R)$ is a function. Continuity of the share function is established from the assumed differentiability of the utility function.

2. For part (a), as $X \rightarrow 0$, $X l^i(\sigma^i, X; R) \equiv X MRS^i(\sigma^i R, \sigma^i X) - [1 - \sigma^i] R \rightarrow -[1 - \sigma^i] R$, so $\sigma^i = 1$ is the only possibility to achieve $l^i(\sigma^i, X; R) = 0$, implying $\lim_{X \rightarrow 0} s^i(X; R) = 1$.

To prove part (b), note that when $\sigma^i = 0$, $l^i(0, X; R) = MRS^i(0, 0) - R/X$. The fact just deduced that $l_\sigma^i > 0$ implies that if $l^i(0, X; R) \geq 0$ then $l^i(\sigma^i, X; R) > 0$ for all $\sigma^i > 0$, and therefore $s^i(X; R) = 0$. If $MRS^i(0, 0) > 0$, $\bar{X}^i(R) \equiv R/MRS^i(0, 0)$ is well-defined

and we can conclude that $s^i(X; R) = 0$ for all $X \geq \bar{X}^i(R)$. If $MRS^i(0, 0) = 0$ then as $X \rightarrow \infty$, $[1 - \sigma^i][R/X] \rightarrow 0$ and so for the first-order condition (4) to be satisfied we need $MRS^i(\sigma^i R, \sigma^i X) = 0$. If $\sigma^i X > 0$ then part (b) of Assumption 1 implies $MRS^i(\sigma^i R, \sigma^i X) > 0$ and therefore by necessity we must have $\lim_{X \rightarrow \infty} \sigma^i X = 0$, which requires $\sigma^i = 0$. As such, $\lim_{X \rightarrow \infty} s^i(X; R) = 0$.

3. Finally, to understand how share functions vary with X we apply implicit differentiation to (4) to deduce that

$$s_{X}^i = -\frac{l_X^i}{l_{\sigma}^i} = -\frac{\sigma^i MRS_x^i + [1 - \sigma^i] \frac{R}{X^2}}{R MRS_z^i + X MRS_x^i + \frac{R}{X}} < 0, \quad (12)$$

confirming the strict monotonicity claimed. □

Proof of Proposition 1. From Lemma 2 we know that Nash equilibria are identified by intersections of $S(X; R)$ with the unit line. From Lemma 3 we also know that individual share functions are single-valued, continuous and strictly decreasing in $X > 0$, and have the property $s^i(X; R) \rightarrow 1$ as $X \rightarrow 0$ and either $s^i(X; R) = 0$ for all $X \geq R/MRS^i(0, 0)$ or, if $MRS^i(0, 0) = 0$, $s^i(X; R) \rightarrow 0$ as $X \rightarrow \infty$. As such, there exist two values of X , \underline{X} and $\bar{X} > \underline{X}$, such that $S(\underline{X}; R) > 1$ and $S(\bar{X}; R) < 1$. Combined with the fact that $S(X; R)$ is continuous and strictly decreasing in $X > 0$, this implies there is a single value of X where $S(X; R) = 1$, and so a single Nash equilibrium exists. □

Proof of Lemma 4. Recall from (4) that a contestant's share function is implicitly defined as the value of σ^i where

$$l^i(\sigma^i, X; R) \equiv MRS^i(\sigma^i R, \sigma^i X) - [1 - \sigma^i] \frac{R}{X} = 0.$$

As such,

$$\begin{aligned} s_R^i &= -\frac{l_R^i}{l_{\sigma}^i} \\ &= -\frac{\sigma^i MRS_z^i - [1 - \sigma^i] \frac{1}{X}}{R MRS_z^i + X MRS_x^i + \frac{R}{X}}. \end{aligned}$$

The denominator (as deduced in (11)) is positive. Noting that $\sigma^i R = z^i$ and that $[1 - \sigma^i] \frac{R}{X} = MRS^i$ from the first-order condition, gives

$$s_R^i = -w^i [z^i MRS_z^i - MRS^i],$$

where $w^i = [R[R MRS_z^i + X MRS_x^i + \frac{R}{X}]]^{-1} > 0$, from where the statement in the lemma follows. □

Proof of Proposition 2. Lemma 4 demonstrated that if $z^i MRS_z^i - MRS^i \leq 0$ for all $i \in N$ then each individual's share function has the property that $s_R^i \geq 0$. This implies that $S(\mathcal{X}(R); R') \geq S(\mathcal{X}(R); R) = 1$, where the final equality holds by definition of equilibrium in Lemma 2. The fact that individual share functions, and therefore the aggregate share function, are strictly decreasing in X then implies that the value of X where $S(X; R')$ equals 1, which is precisely $\mathcal{X}(R')$, must satisfy $X \geq \mathcal{X}(R)$, which concludes the proof. □

Proof of Proposition 3. Implicit differentiation of (5) gives

$$\mathcal{X}'(R) = -\frac{\sum_{j \in N} s_R^j}{\sum_{j \in N} s_X^j}$$

We deduced in Lemma 3 that $s_X^i < 0$ for all $i \in N$, and therefore $\text{sgn}\{\mathcal{X}'(R)\} = \text{sgn}\{\sum_{j \in N} s_R^j\}$. From Lemma 4 we know that $s_R^i = -w^i[z^i MRS_z^i - MRS^i]$, from where the statement in the proposition follows. \square

Proof of Proposition 4. We first show how share functions defined in (6) vary in D and R . Implicit differentiation of (6) reveals

$$\begin{aligned} \frac{ds^i}{dD} &= -\frac{dl^i}{l_\sigma^i} \\ &= -\frac{\sigma^i R MRS_x^i + [1 - \sigma^i] \frac{1}{D^2}}{R MRS_z^i + D MRS_x^i + \frac{1}{D}} \\ &< 0 \end{aligned}$$

and

$$\begin{aligned} \frac{ds^i}{dR} &= -\frac{dl^i}{l_\sigma^i} \\ &= -\frac{\sigma^i D MRS_x^i}{R MRS_z^i + D MRS_x^i + \frac{1}{D}} \\ &\leq 0 \end{aligned}$$

Now the second of the deductions just made implies that for $R' > R$ we have $S(\mathcal{D}(R)R'; R') \leq S(\mathcal{D}(R)R; R) = 1$. The first of the above deductions implies that $S(DR; R)$ is strictly decreasing in D which implies that the value of D that makes $S(DR'; R')$ equal to 1, which is precisely $\mathcal{D}(R')$, must not exceed $\mathcal{D}(R)$. \square

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