Virtual Trade between Separated Time Zones and Growth

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ABSTRACT

The purpose of this paper is to propose a model where trade has a direct and positive impact on growth rate of two trading nations beyond the level effect. We use the idea of virtual trade in intermediates induced by non-overlapping time zones and show how trade can increase the equilibrium optimal rate of growth. In this structure the trade impact goes beyond the level effect and directly causes growth. Typically standard models of trade cannot generate an automatic growth impact. Virtual trade may allow production to continue for 24x7 in separated time zones such as between US and India and that can lead to higher growth for both countries. Later we extend the model to incorporate accumulation of skill which becomes necessary for sustaining steady state growth.

Key Words : International Trade, Time Zone, Growth

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ABSTRACT

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Section 1. Introduction

In his well-known monograph on growth Lucas (2002) rightly points out that removing barrier to trade do not necessarily lead to a rise in the growth rate. Standard neo classical models of trade cannot generate a direct and automatic growth impact of a more open trade regime. He was commenting on the pioneering work on trade reform by Krueger (1983) and Harberger (1984). The observation made by Lucas tells us about a key problem in having a growth model where gains from trade naturally lead to a higher growth rate. This is the reason why there is no generic model of trade and growth with clear predictions. Typically trade leads to reallocation of resources in the direction of
efficient use. But such reallocation may not either increase rate of investment or lead to permanent increase in productivity—the two sources through which long run growth can increase. Unfortunately the current literature does not refer to the well-known work of David Ricardo as elaborated in Findlay (1974) which clearly exhibits a direct relation between trade and growth by increasing the rate of profit. In fact Ricardo’s argument for import of corn\(^2\) was related to making a critical input for production, labor, less expensive for the capitalists, the drivers of growth and industrial development. Trade in final goods thus reduced the cost of an input raising the rate of profit, and hence investment and growth. Our paper is also in similar tradition where virtual trade\(^3\) reduces the inefficiency in the process of utilization of intermediate inputs. We will explain this phenomenon in more detail later. Two elegant recent papers on Ricardo and growth are by Naito (2012) and Kaneko (2013). But they do not discuss the separated time zones induced mechanism between the level effect and growth effect of trade that we discuss here. Also they do not consider the problem of skill accumulation and labor market issues that may potentially block growth when intermediates are produced by skill, a focal point of this paper.

\(^2\) Readers may look into ‘corn law’ for further details.

\(^3\) We primarily focus on trade in services or where transportation cost of trading intermediate input is either zero or very negligible both in terms of money and time. This characteristic is predominantly present in case of service trade when transactions are done mainly through information communication technology (ICT) or virtually. In this sense we use the term ‘virtual trade’ though trade is very much real in value and volume. Readers are requested not to confuse with the idea of ‘non-existing’ or ‘unreal’ implication of ‘virtual’.
More recent work on trade and growth such as Grossman and Helpman (1990) and Rivera Batiz and Romar (1991) relate trade to innovation, growth and increasing returns. These two papers primarily focus on how trade leads to increasing flows of good and ideas to end up with accumulation of knowledge capital and creation of an ambience to benefit from possible increasing returns to scale. Interesting papers by Ventura (1997) and Acemoglu and Ventura (2002) discuss the case where small economies when exposed to international capital flow, can grow avoiding diminishing returns to capital accumulation as the lower world rate of return is held fixed. But their work is not related to the standard trade models that are supply side in nature. Summary of the existing literature of this kind, in a pedagogic form, is available in Donaldson (2011). In an interesting paper Baldwin and Robert-Nicoud (2008) also discusses at length the growth implications of recent genre of firm-heterogeneity models. They conclude that freer trade raises productivity in a level sense, but slows measured productivity growth. This set of well-known contributions provides rich insights as to how trade models can be extended to generate a growth impact. But the fact remains that trade is essentially about more efficient allocation of existing resources and trade by itself cannot lead to an increase in growth rate, though it definitely increases real income. Given this backdrop we present a simple framework and extend it to fit the literature on optimal endogenous growth where trade has level as well as growth effects. This is also a departure from Kikuchi and Marjit (2011) where time zone differences
and growth has been discussed, but not in terms of an optimal growth model and more importantly the labor market issue was not discussed at all. Since the possibility of steady state growth hinges very much on the availability of skill, the issue of labor market is of prime importance. Also the labor market story clearly demonstrates the separation of level effect through a one shot rise in the wage rate and a permanent change in the growth rate where subsequently the wage rate remains unchanged.

The rest of the paper is laid out as follows. Motivation for the model is provided in the second section. The third section describes the autarkic equilibrium. Section four deals with trade and growth and five with the labor market issues. The last one concludes.

**Section II. Motivating Environment**

Virtual trade is possibly the most innovative form of market transaction the world has seen in recent times\(^4\). Thanks to the growth of satellite and computer technology, online transactions have flooded the global market. For an overview of this radical phenomenon readers are referred to Lehdonvirta & Castronova (2014). By now it is well recognized that India’s phenomenal growth in the turn of the century outstripping the legacy of the so called Hindu rate of growth\(^5\) owes a great deal to the growth in the service sector of which IT services is extraordinarily important.

\(^4\) For virtual nature of trade please consult Mandal (2015).

\(^5\) Hindu rate of growth basically indicates the slow growth rate of Indian economy during the pre-liberalization period.
Das, Banga and Kumar (2011) describe the rise of the India’s services and software sector in clear terms. Since 1995, a few years after the reform process was initiated in India, more than 60% of the GDP growth rate was contributed by the service sector. The share of the services in GDP grew from 50% in 1995 to around 65% in 2008-2009. The trend continues more or less unabated even in the midst of global financial crisis of 2007-2008. Since 2001-2002 to 2007-2008, the year on year growth of software exports was more than 30% on average. It constitutes more than 50% of India’s service sector exports. It will be difficult not to agree with the claim that India’s rise as an economic power does have a lot to do with its performance in the IT related services. Sahoo (2013) relates the growth in the service sector with the TFP growth of the new firms created near the technology frontier. In fact it is quite clear that India’s agriculture and manufacturing did not perform commendably over the last two decades and never looked impressive. India could exploit the opportunity offered by global revolution in ICT starting in 80s being endowed with a substantial reservoir of low cost skilled labor, reasonably well versed with English language and as we shall argue in this paper, being strategically located in the virtual value chain.

Countries located in separated and non-overlapping time zone have a natural tendency to trade on virtual platform. In the 80s Bangalore in India emerged as the Silicon Valley of the east developing a working relation with the legendary Silicon Valley of California. Tasks generated in USA during working hours could be completed in India when
USA will rest in bed, offices will open in Bangalore and the world effectively worked for 24 hours. Such outsourcing or exchange of tasks according to the static interpretation of Grossman and Rossi-Hansberg (2008) naturally generates gains from trade. Thus for countries exactly identical in all respects but located in separated time zones with non-overlapping normal working hours could gain from trade by completing a task quicker, by exchanging services in processes like intermediates and delivering the final product more efficiently. The benefit could be reaped because technology was available to exploit natural geographic conditions. Hence time became a cause for trade apart from the usual trinity—technology, endowment and preference. This was first posed in terms of a simple Ricardian model of comparative advantage by Marjit (2007) and immediately followed up by many papers of Toru Kikuchi, now collated in Kikuchi (2013) in terms of network formation. One issue that seems to be of critical theoretical importance is the relationship of such a pattern of trade with growth, the focal point of the current paper.

The effect of trade on growth as studied in the literature has turned out to be an empirical question. Recent contributions starting with a critical overview of Fernández & Rodrik (1990) evolve around evaluating the hypothesis whether trade has impacted on growth of developing countries. In an interesting paper reflecting on Indian experience Goldberg et al. (2010) have rigorously demonstrated how imported inputs have contributed to Indian growth rate in the post reform period. In spite of the positive effects of liberal
trade policies on India’s growth, one does not find a significant change in the manufacturing to GDP ratio in India in a regime when the standard measures of openness show profound transformation. For example the trade volume to GDP has increased from around 15% in pre-reform to more than 35 % in the post reform era. Also the remarkable growth of the services and software sectors cannot be explained by increasing import of physical inputs. Rather India’s rise in services had to do with the rise of the virtual world and technologies that facilitated trade across virtual platforms and in particular India’s global location vis a vis USA with normal working hours in each country coinciding with the resting period in the other. Not only this is an interesting story to tell, but it breaks a theoretical impasse between trade and growth by accommodating the level and growth effect of trade in the same model and in simplest possible terms.

Trade induced by separated time zones has a natural impetus for growth because it allows the countries to work double shift with lower cost. In a way there is a permanent change in level of productivity\(^6\). Think of a situation when one unit of the product requires two inputs each of which can be produced over twelve hour cycle. Therefore, if we work for twelve hours, do not work overnight and we start the day at six in the morning, we shall get one unit the next day at six in the evening. Now if we could get another country to produce one of the inputs and that they start working at six in the morning their time

\(^6\) Interested readers are requested to look at Kikuchi et al. (2013) for some relevant explanations for how virtual trade may lead to an increase in productivity.
which is six in the evening our time, one unit will be done when we wake up and start the next day. We get the product twelve hours earlier and get more than one unit for any given amount of time and cost when the opportunity of virtual trade is exploited. Thus, output per unit of time goes up raising the growth rate. This is one type of interpretation.

Another type of interpretation is when we do work overnight at a premium to compensate for the normal resting time. Once we have another trading partner across a non-overlapping time zone, both of us can sleep well. Firms can hire workers without a premium. Inputs are available at a lower cost increasing the incentive for the firms to invest more leading to a higher rate of growth. This is a natural process by which the level effect and growth effect hold simultaneously.

**Section III. Autarkic Equilibrium**

Consider two symmetric countries, identical in all respects, but located in two non-overlapping time zones. Therefore, autarkic equilibrium in one will be replicated in another. We have a single final good $Y$ which uses capital $K$ and intermediates $m_1$ and $m_2$ and they need two twelve hour cycles to be produced. One unit of $Y$ is produced by one unit of $m_1$ and $m_2$ each. Also we choose $Y$ as the numeriare. Hence all prices are unity. Figure–1 depicts the time line of production.
We rule out the nightly production assuming that it is immensely costly. For production of the final good one unit of time is denoted by the phase AB, when \(m_1, m_2\) and \(K\) are used to produce \(Y\) via the following production function. Assume that \(K\) is costless.

\[
Y = A m_1^{1-a} m_2^{1-a} K^a \quad \ldots (1)
\]

Since \(m_1\) is available only in the first half of tomorrow, and it requires another input or phase of processing viz. \(m_2\). \(Y\) can not delivered early and / or there may be carryover costs of \(m_1\) up to \(m_2\). We take up the second interpretation, although the first has been dealt with in Marjit (2007). The delay or carryover cost is denoted by \((1 + \mu)\) with \(\mu > 0\) denoting a premium over unit cost.

It is straight forward to argue that optimal \(m_1\) and \(m_2\) are given by (details is provided in the Appendix I)

\[
m_{10} = \frac{1-a}{2(1+\mu)} Y \quad \text{and} \quad m_{20} = \frac{1-a}{2} Y
\]

Substituting these values in (1) we get

\[
Y_0 = \tilde{A} \left(\frac{1-a}{2}\right)^{\frac{1-a}{\alpha}} \left[\frac{1}{(1+\mu)}\right]^{\frac{1-a}{2\alpha}} K \quad \text{where} \ K \text{ is aggregate capital.} \quad \ldots (2)
\]

Or, \(Y_0 = \tilde{A}_0 \cdot K\), where \(\tilde{A}_0 = \tilde{A} \left(\frac{1-a}{2}\right)^{\frac{1-a}{\alpha}} \left[\frac{1}{(1+\mu)}\right]^{\frac{1-a}{2\alpha}} \quad \ldots (2A)\)
Given $K$ and other parameters (1) and (2) determine $Y_0$. From (2) it is obvious that $\mu$ has a negative effect on $A_0$ and $y_0$.

The simple Autarkic growth problem is described by the following representative agent problem with per capita capital stock $k$.

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t). \quad \beta = \frac{1}{1+\rho}, \quad \rho > 0, 0 < \beta < 1 \text{ is the discount factor}$$

subject to $[y_t - c_t - (k_{t+1} - k_t)] = 0 \quad \ldots (3)$

We further assume zero depreciation of capital.

Using Bellman’s principle we can break the decision problem defined above into smaller sub-problems. Again, taking help of the principle of optimality and simplifying a little bit we can have the Bellman’s equation in the following form.

$$\max Z_t = \max u(c_t) + \beta V(k_{t+1}) + \lambda_t [y_t - c_t - (k_{t+1} - k_t)] \quad \ldots (4)$$

Here $V$ is the Maximum Value Function, $\lambda_t$ is the standard Lagrange multiplier, and $\beta$ indicates discount factor as stated before.

From this growth formulation it is easy to demonstrate the following Euler relationship

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\bar{A} + 1}{1+\rho} \quad (\text{See Appendix II for details}) \quad \ldots (5)$$

With specific structure on $u(c_t)$, such as log-linear $(\bar{A} - \rho)$ approximately captures the steady state endogenous growth as in a typical $AK$ model a la Rebello (1991), etc.

Therefore, $m_{10}, m_{20}, y_0, k$ all will grow at the same rate $(\bar{A} - \rho)$. Note that $\bar{A}$ does not have a time dimension and greater $\mu$ will reduce the growth rate as $\mu$ has negative
connotation for $\tilde{A}$ (see equation (2)). So, in autarky two economies located in non-overlapping time zones will grow at the same rate $(\tilde{A}_0 - \rho)$.

**Section IV. Trade and Growth**

Looking back at figure-1, it is simple to demonstrate that if the other country, say, the foreign, which wakes up at $A_1$, could provide $m_2$ to the home country, $Y$ will be available at $A_2$ instead of at $B$. Similarly at point $A$ when the foreign country is asleep, $m_1$ could be produced and shipped to the foreign country ready to go with $m_2$ at $A_1$. Remember that we are talking of virtual trade through computerized network. This may even entail some trading costs; let’s call this $\tau$. So the effective price of the input/product/service is $(1 + \tau)$ even if we assume away any delay or carryover cost. So long as $\tau < \mu$, home country will import $m_2$ from the foreign country and the foreign country will import $m_1$ from the home country.

(2) now reads as (6) and $\tilde{A}_F$ denotes the free trade level of $\tilde{A}$.

$$\tilde{A}_F = A^\frac{1}{2} \left( \frac{1 - \alpha}{2} \right)^{\frac{1 - \alpha}{\alpha}} \frac{1}{1 + \tau} \left[ \frac{1}{2a} \right]^{1 - \alpha} \cdots (6)$$

Comparing (2) and (6)

And $\tilde{A}_F > \tilde{A}_0$ iff $\tau < \mu$

Higher value of $\tilde{A}_F$ implies more $Y$ and, of course, higher rate of growth denoted by $(\tilde{A}_F - \rho)$. 
Therefore the following proposition is immediate.

**Proposition-I.** If $\tau < \mu$, mutually gainful trade will take place raising $Y$ in both countries. But as $A_F > A_0$, such trade will increase the growth rate also.

*Proof:* See discussion above.

Now let us check what happens to the balance of trade. The trade balance condition is given by (where $*$ signifies Foreign country).

$$\frac{1-\alpha}{2} \cdot \frac{y^F}{1+\tau} = \frac{1-\alpha}{2} \cdot \frac{y^{F*}}{1+\tau} \quad \ldots \ (7)$$

Since $y^F = y^{F*}$, it trivially holds.

If we work with the delay in delivery of the final good as in Marjit (2007) and the discount rate applied is $\delta \ (< 1)$ on the price of $Y$, then $m_1 = m_2 = \frac{1-\alpha}{2} \cdot \delta y^{\tau}$ and

$$A_F = A^\tau \left( \frac{1-\alpha}{2} \right)^{\frac{1-\alpha}{\alpha}} \cdot [\delta]^{\frac{1-\alpha}{2\alpha}}.$$  If trade is allowed in such situation, people will have the final product early and hence the discount rate would be minimized. Therefore, the value of $\delta$ will tend to be equal to 1, and so will be the level of $y$ and growth rate. Consequently, international trade, by reducing the delay in the production process has a positive impact on the productivity and growth. Note that both $\mu$ and $\delta$ have to be compared with $\tau$. If $\tau$ is very high, as was the case when the virtual platform was not available, growth rate must suffer. Once such platform was made available, $\tau$ dropped substantially and virtual trade became a reality. In the process it radically transformed the way a task is performed.

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*Readers are referred to Marjit (2007) and Kikuchi et al (2013) for better explanation of this argument.*
globally. Thus trade not only raised the value of output but brought onto a higher growth path.

Section V. Growth with Labor Market

In this section we try to introduce labor market in the model we have been discussing so far. This is important in the sense that increasing demand for skill may raise its price and block the possibility of a steady state. Existence of steady state growth path becomes conditional on skill accumulation. Following standard trade arguments it is very convenient to understand that virtual trade in services is actually a skill intensive product or service – services which are traded virtually requires certain skill such computer knowledge, command over English, engineering, software designing etc. These skills are not naturally developed. People need to acquire such skill through proper training. Therefore, an increase in virtual trade demands higher amount of skilled labor and higher levels of skill accumulation as well.8

Consider that the home and the foreign country have same endowments of human capital, call it \( H \). Autarkic equilibrium will be recast under the assumption that to produce one unit of \( m_1 \) and \( m_2 \), labourers have to be paid \( w \) which is the real wage in terms of \( Y \). \( Y \) can be consumed, saved as \( k \) and (now in case of skill accumulation) also can be invested to augment \( H \). That is even if the population size remains fixed, either the skill content can be

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8 Beladi et al. (2011) and Mandal et al. (2015) are two related papers for the issue where skill formation and virtual trade is discussed.
increased to increase $H$ or the proportion of skilled workers needed to produce $m$ can be increased. Here let us focus on the optimum input quantities and output. We will return to the growth question little later.

Now optimum quantity of $m_1$ and $m_2$ are

$$m_1 = \frac{1-a}{2} \cdot \frac{Y}{w(1+\mu)} \quad \ldots \ (8)$$

$$m_2 = \frac{1-a}{2} \cdot \frac{Y}{w} \quad \ldots \ (9)$$

Except the introduction of $w$, (8) and (9) look very similar with what we got from optimizing equation (1). Therefore, $Y = A\tilde{K}$ where $A = \tilde{A} \left(\frac{1-a}{2}\right)^{1-a} \left[\frac{1}{w^2(1+\mu)}\right]^{1-a} \cdot K$

Before we proceed to the labor market, we briefly reflect on the above expression.

It is obvious that $w$ enters with a square in $A$ since two intermediate inputs are put together for the final product. Even if we forget about the time zone effect as such and consider any pair of intermediate goods we are likely to derive a similar expression without delay costs or carryover costs. Now consider that this small open economy faces a rest of the world wage rate, $w^*$. Assume, $w^*(1 + z) < w(1 + a)$ where $z$ is some sort of trading costs or outsourcing costs and $a$ represents production costs at home. So, Home will gain from importing the intermediate from abroad and exporting $Y$. This will immediately increase the long run rate of growth. The underlying reason behind such increase in output and growth rate is the reduction in loss due to delay in delivery or carryover cost. This has been

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9 Alternatively $a$ can be interpreted as managerial cost or something in addition to wage cost/labor cost.
possible primarily due to the natural difference in time zones between two trading
countries and low cost of virtual communication. If $z$ is prohibitively high as in the case
with virtual trade when communication costs were too high, such trade and growth will
not take place. The concept of time zone clearly manifests a situation when $z$ is low enough
to cause trade and growth even if $w$ is the same across borders\(^{10}\). Thus trade in
intermediates leads naturally to an increase in the growth rate.

Now we turn to the equilibrium in the labor market. Our model does not consider
the existence of unemployment of labor. Hence, labor market must clear

Therefore,

$$H = \frac{1-a}{2w} \cdot A \cdot \left[1 + \frac{1}{1+\mu}\right] \quad \cdots \text{(10)}$$

Substituting the value of $A = A^{\frac{1}{\alpha}} \left(\frac{1-a}{2}\right)^{\frac{1-a}{\alpha}} \cdot \left[\frac{1}{w^2(1+\mu)}\right]^{\frac{1-a}{2\alpha}} \cdot K$

Or, $w = \frac{1-a}{2} \cdot \frac{\tilde{A}K}{H} \cdot \left[1 + \frac{1}{1+\mu}\right] = \frac{A^{\frac{1}{\alpha}}(1-a)^{\frac{1-a}{\alpha}}}{H} \cdot \left(\frac{1}{1+\mu}\right)^{\frac{1-a}{2\alpha}} \cdot (1 + \frac{1}{1+\mu}) \cdot (w)^{\frac{1-a}{\alpha}} \cdot K$

Simple mathematical manipulation gives

$$w = \left(\frac{B}{K}\right)^{\alpha} \quad \cdots \text{(11)}$$

Where $B = A^{\frac{1}{\alpha}} \left(\frac{1-a}{2}\right)^{\frac{1}{\alpha}} \cdot \left[\frac{1}{(1+\mu)}\right]^{\frac{1-a}{2\alpha}} \cdot (1 + \frac{1}{1+\mu}) \quad \cdots \text{(12)}$

Note that a decline in $\mu$ will surely increase $w$ given $K$ and $H$.

\(^{10}\) Our results would be further strengthened if one introduces differences in wage rates in different countries. In
fact, introduction of wage differential may add an interesting dimension to the literature in that the optimum
distance related time zone difference for mutually beneficial trade would crucially depend on the value of
absolute wage difference.
Following the argument in the last section it is straightforward to argue that trade will increase $w$ in both countries.

We now turn to the growth issue. It is obvious that steady state growth is impossible if $H$ does not increase over time. To elaborate the decision making process at the micro level, we invoke the following.

A representative agent is endowed with $k$ and $h$, the per capita endowments. She faces the following dynamic choice problem expressed through Bellman’s equation (see equation (3) and (4))

$$\max_{c_t, h_{t+1}} Z (c_t, k_t, h_t) = \max u(c_t) + \beta V (k_{t+1}, h_{t+1}) + \lambda_t [y_t - (k_{t+1} - k_t) - (h_{t+1} - h_t)] \quad \ldots (13)$$

We could have introduced the special effect of $h_t$ on the accumulation process of $h_t$ as per Lucas (1988). But here we focus on the trade induced impact and abstract from more complex human capital issue. Also note that in equilibrium

$$Y = \bar{A}K$$

Where $\bar{A} = A^\frac{1}{\alpha} \left( \frac{1-a}{2} \right)^{\frac{1-a}{a}} \left[ \frac{1}{B} \left( \frac{1}{(1+\mu)^2} \right) \right]^{\frac{1-a}{2a}} . K$ as $w = \left( \frac{B}{H} K \right)^\alpha \ldots (14)$

Therefore,$y = \bar{B}K^{\alpha}H^{1-\alpha} \ldots (15)$

Where, $\bar{B} = A^{\frac{1}{\alpha}} \left( \frac{1-a}{2} \right)^{\frac{1-a}{a}} . \left[ B^{2a(1+\mu)} \right]^{-\frac{1-a}{2a}}$

So, for the representative agent $y = \bar{B}k^\alpha h^{1-\alpha} \ldots (16)$
Here, $\mu$ has a negative relation with $\tilde{B}$ if the following condition holds true.

Note that $(h, k)$ are variables relevant for the representative agent, and $\frac{d[B^2u(1+\mu)u]}{d(1+\mu)} > 0$ (See Appendix III).

Again, (15) can be rewritten as

$$y = \tilde{B}.\left(\frac{h}{k}\right)^{1-\alpha}k \quad \ldots (17)$$

If $k$ and $h$ grow at the same rate effectively (17) assumes the ‘Ak’ form.

If we solve the problem specified by (13), we get the following conditions, as before,

$$\frac{\tilde{B}kh^{\alpha-1}h_{t+1}^{\alpha+1}}{1+\rho} = \frac{\lambda_t}{\lambda_{t+1}} = \frac{u'(c_t)}{u'(c_{t+1})} = \frac{\tilde{B}(1-\alpha)kh^{1-\alpha}h_{t+1}^{\alpha+1}}{1+\rho} \quad \ldots (18)$$

From (18) it is obvious that

$$\frac{k_t}{h_t} = \frac{\alpha}{1-\alpha} \quad \ldots (19)$$

As before with log linear contemporaneous utility function

$$\tilde{B}ah^{1-\alpha}k_{t+1}^{\alpha-1} - \rho = \tilde{B}(1 - \alpha)h^{-\alpha}k^\alpha - \rho \approx \frac{c_{t+1}-c_t}{c_t} = g_c \quad \ldots (20)$$

c$_t$, k$_t$, h$_t$ all will grow at the same rate, so will $y$. From (11) we also know that $w$ will remain same over time. Following similar logic as in earlier section, we know a drop in $\mu$ upto $\tau$ with $\tau < \mu$, will increase $B$ and $\tilde{B}$, raising the rate of growth. In the primitive production function $m_1, m_2, k$ all will grow at the same rate as in (20). Hence $Y$ will also grow. Growth in $H$, however, will keep $w$ in check. $MP_k$ and $MP_H$ will remain constant.
along the path of steady state growth. As we shift from $\mu$ to $\tau$, $w$ will go up. This is the level effect, but then it remains the same. Thus we have the following proposition.

**Proposition 2** – Trade across time zones will lead to higher steady state growth. The level effect will lead to one shot rise in $w$, but in the new growth path $w$ remains constant guaranteeing steady state growth. ■

*Proof: See discussion above.*

**Section VI. Conclusion**

The purpose of this paper has been to demonstrate how time zone led virtual trade can naturally and easily generate both level and growth effects of international trade. Typically trade generates level effect on income but not growth effects which need further structure on the static model. Virtual trade made it possible through separated time zones. It increases productivity permanently by allowing the world to work round the clock. Thus gains from trade and increase in the growth rate happen simultaneously. Availability of skilled labor can be a problem and the growth impact of trade needs skill accumulation at the same rate, which may not be forthcoming. Thus balanced growth path we discuss in this paper might get disturbed if two countries have skill constraints.
References


Appendices

Appendix I. Optimum demand for \( m_1 \) and \( m_2 \)

Total input cost to produce one unit of \( Y \) is \{\( m_1(1 + \mu) + m_2 \)\} as \( m_1 \) needs to be carried over for final use which has cost of \((1 + \mu)\). Note that \( P_y \) is normalized to unity. Therefore, the profit equation becomes:

\[
\pi = P_y Y - \{m_1(1 + \mu) + m_2\} = A m_1^{1-\alpha} m_2^{1-\alpha} K^\alpha - \{m_1(1 + \mu) + m_2\}
\]

Setting \( \pi_{m_1} \) and \( \pi_{m_2} = 0 \) (First Order Condition for profit maximization):

\[
\frac{1-\alpha}{2} Y m_1^{-1} = (1 + \mu) \text{ Or, } m_1 = \frac{1-\alpha}{2(1+\mu)} Y \text{ (we have denoted it by } m_{10} \text{ in the main text).}
\]

Similarly, \( \frac{1-\alpha}{2} Y m_2^{-1} = 1 \) Or, \( m_2 = \frac{1-\alpha}{2} Y \) (we have denoted it by \( m_{20} \) in the main text).

Appendix II.

The Lagrangian for Utility maximization is:

\[
L(c_t, k_{t+1}) = u(c_t) + \beta V(k_{t+1}) + \lambda_t [y_t - c_t - (k_{t+1} - k_t)]
\]

First order conditions for optimization, i.e. \( \frac{\delta L}{\delta c_t} = \frac{\delta L}{\delta k_{t+1}} = \frac{\delta L}{\delta \lambda_t} = 0 \) yield

\[
u'(c_t) = \lambda_t \quad (1A)
\]

\[
\beta V'(k_{t+1}) = \lambda_t \quad (2A)
\]

\[
y_t = c_t + (k_{t+1} - k_t) \quad (3A)
\]
Also by definition (from the budget constraint)

\[ V'(k_t) = \lambda_t(\tilde{A} + 1) \]  \hspace{1cm} (4A)

Updating (4A), (2A) is modified as

\[ \beta \lambda_{t+1}(\tilde{A} + 1) = \lambda_t \]  \hspace{1cm} (5A)

Substituting the value of \( \beta = \frac{1}{1+\rho} \), and comparing (1A) and (5A)

\[ \frac{(\tilde{A}+1)}{1+\rho} = \frac{u'(c_t)}{u'(c_{t+1})} \]  \hspace{1cm} (6A)

Given \( u'' < 0 \), it is obvious that \( c_{t+1} > c_t \) if \( \tilde{A} > \rho \).

We shall work with a log-linear utility function. Therefore, \( \frac{u'(c_t)}{u'(c_{t+1})} = \frac{c_{t+1}}{c_t} \).

From (6A) \[ \frac{c_{t+1}-c_t}{c_t} = g = \frac{(\tilde{A}-\rho)}{1+\rho} \approx (\tilde{A} - \rho) \]  \hspace{1cm} (7A)

Therefore, \( \tilde{A} - \rho \) means the growth rate, \( g \). We will work with this formulation for the rest of this paper. It is the distance between \( \tilde{A} \) and \( \rho \) that determines the magnitude of the growth rate.

**Appendix III**

From equation (12), \( B = A^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{2} \right)^{\frac{1}{\alpha}} \cdot \left( \frac{1}{1+\mu} \right)^{\frac{1-\alpha}{2\alpha}} \cdot (1 + \frac{1}{1+\mu}) \)

Simple algebraic manipulation provides

\[ B^{2\alpha} = A^2 \left( \frac{1-\alpha}{2} \right)^2 \cdot \left( \frac{1}{1+\mu} \right)^{1-\alpha} \cdot (1 + \frac{1}{1+\mu})^{2\alpha} \]

Assuming \( A^2 \left( \frac{1-\alpha}{2} \right)^2 = X \) and \( (1 + \mu) = \varphi \)

\[ B^{2\alpha}(1 + \mu) = X \left( \frac{1}{\varphi} \right)^{1-\alpha} \cdot (1 + \frac{1}{\varphi})^{2\alpha} \cdot \varphi \]
Differentiating we get

\[
\frac{d [B^2 (1 + \mu)]}{d(1 + \mu)} = \frac{d (B^2 \varphi)}{d \varphi} > 0
\]

iff \[2 \alpha (\varphi + 1)^{2 \alpha - 1} X \varphi^{-\alpha} - \alpha X \varphi^{-\alpha - 1} (\varphi + 1)^{2 \alpha} > 0\]
or, \(X \alpha (\varphi + 1)^{2 \alpha - 1} \varphi^{-\alpha} [2 - \varphi^{-1} (\varphi + 1)] > 0\)
or, \(2 - \left(1 + \frac{1}{\varphi}\right) > 0\)

Substituting \((1 + \mu) = \varphi\)

\[
2 - \left(1 + \frac{1}{\varphi}\right) = 2 - \left(\frac{1 + \mu + 1}{1 + \varphi}\right) = \frac{\varphi}{1 + \varphi} = \frac{1}{1 + 1/\varphi}
\]

Since \(0 < \varphi < 1\), \(\frac{1}{1 + 1/\varphi} > 0\). This implies \(\frac{d [B^2 (1 + \mu)]}{d(1 + \mu)} > 0\).