Emissions abatement R&D
Dynamic Competition in Supply Schedules

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Abstract

This paper investigates the optimal environmental policy (the mix of emissions tax and R&D subsidy) when two firms, producing differentiated products, compete in the output market over time. Firms compete over supply schedules, which encompasses a continuum of market structures from Bertrand to Cournot. While production generates environmentally damaging emissions, firms can undertake R&D, which has the sole purpose of reducing emissions. In addition to characterising the optimal policy, we examine how the optimal tax and subsidy and the optimal level of abatement change as competition intensifies, as the dynamic parameters change and as the investment in abatement technology changes. In this setting, increased competition no longer necessarily leads to an increase in welfare. Instead, there are two forces. Competition increases welfare through its impact on the final goods price. However, lower prices result in larger quantities and more pollution. Our contribution is to show that the impact depends on the extent of the market, and the nature of preferences and technology.

*Keywords:* Technology; R&D; Environment; Policy; Emission tax; Subsidy

*JEL Classification:* H23; O32; O38; Q55; Q58
1 Introduction

This paper investigates the design of an optimal environmental policy when the output market is a differentiated duopoly, production generates environmentally damaging emissions and firms can pursue R&D to reduce such emissions. By introducing imperfect competition in output markets, we add another market failure to the twin failures associated with environmental pollution and innovation and diffusion of new technologies.

Economic theory suggests that when the production (or consumption) of certain goods generates negative externalities in the form of environmentally damaging pollution, and these costs are not internalised by the producers, there will be excessive production and pollution from a social viewpoint [12]. It is also well understood that when R&D generates positive externalities in the form of positive spillovers for firms who free ride on other firms’ investment, there will be under investment in R&D [10].

Moreover, imperfect competition allows firms to charge prices above competitive (cost-reflective) prices, which leads to less production and consumption than socially optimal. As it is well-understood, under imperfect competition, the second best optimal tax may be less than marginal damages ([6]; [2]) and by a similar second-best logic (a la Lipsey and Lancaster (1956), [17]), the optimal subsidy may be different from the marginal benefit from R&D.

In our setting, Buchanan’s (1969) statement that “imposing a marginal private cost equal to marginal social cost does not provide the “Aladdin’s Lamp” for the applied welfare theorist” holds true and our interest is in the nature of the trade-offs between market failures and policy interventions, controlling for the intensity of competition in the output market.

The literature has typically focused on two of the three market failures. For example, the interaction between environmental policies that use market-based instruments to induce firms to reduce pollution and the development and diffusion of new environmentally beneficial technologies has been studied by [12]. These authors point out that in the absence of environmental policies, there will be under-investment in environmentally friendly technology especially in the presence of adoption spillovers. They further arrive at the sensible conclusion that the complexity of the interaction between the two market failures mentioned above suggests a strategy of experimenting with policy approaches and systematically evaluating their success.

In our model, firms only invest in R&D in the presence of an emissions tax. However, in the absence of a subsidy, firms under-invest in R&D because of adoption spillovers. While this allows us to investigate, in the context of a formal model, the various trade-off discussed by [12], our contribution goes further. By adding imperfect competition, we introduce a further adjustment – more competition leads to lower prices, higher quantities and, therefore, higher emissions. While lower prices and higher quantities are good for consumers, producer surplus is reduced and pollution also increases. Therefore, the final impact on social welfare from an increase in the intensity with which firms compete is a priori ambiguous. Our contribution is to determine the optimal
environmental policy (emissions tax and R&D subsidies) explicitly as a function of the intensity of competition.

The model consists of three building blocks. First, emission abatement R&D is considered with the assumption that R&D investment at the firms' level has the sole purpose of reducing emissions. The model is a dynamic extension of the seminal work of D’Aspremont and Jacquemin (1988), [1] and, more recently, Poyago-Theotoky (2007), [25]. In addition, R&D effective spillovers ( [5]) are considered where the effectiveness of involuntary leakage of innovative information is directly linked to how closely related the two products are. Second, we model a duopoly, producing differentiated goods, and competing over time in the output market. Each firm decides how much to invest in R&D and chooses a supply schedule. This approach allows us to parameterise the degree of competition. We refer to this parameter as the intensity of competition, and it varies continuously from Cournot to Bertrand behaviour. Finally, a social welfare maximising regulator determines a tax per unit of pollution in order to induce the optimal social level of production and pollution and a R&D abatement subsidy to induce the optimal social level of abatement research.

There are several key insights. First, the impact of an increase in the intensity of competition on equilibrium quantities and consumer surplus is, as expected, positive. However, as more production involves more pollution, the impact of more intense competition on welfare depends, in a complex, but quantifiable way, on the environmental policy (emissions tax and abatement R&D subsidies).

Second, as products become less differentiated, the less firms behave as local monopolists and, therefore, quantities and consumer surplus increase, as long as the fixed intensity of competition is sufficiently large. This is a generalisation of a standard result in industrial organisation comparing Bertrand and Cournot outcomes when products are differentiated. See, for example, [3].

Third, as in the case of an increase in the intensity of competition, a reduction in product differentiation that leads to greater equilibrium quantities has, ceteris paribus, an ambiguous impact on welfare as total emissions increase. The ambiguity of the impact on welfare extends the point made by [12] that environmental technology policy is plagued by complex trade-offs not only between market failures associated with pollution and with the diffusion of new technologies but also their interaction in an environment where competition in output markets is imperfect.

Fourth, the optimal (welfare-maximising) emissions tax and the R&D subsidies vary in opposite directions with the intensity of competition. The optimal tax needs to rise as a result of an increase in the intensity of competition to compensate for increased production. The opposite, however, is true for the optimal R&D subsidy. As competition intensifies, and the optimal tax is adjusted accordingly, the benefits of R&D subsidies diminish.

Finally, as the intensity of competition increases, the impact of a tax on emissions is stronger (but diminishing) than the impact of an R&D subsidy. The reason for such asymmetry is that the subsidy is applied to total cost, which is a convex function of R&D output a la D’Aspremont and Jacquemin.
We also show that increased competition can reduce welfare when the emissions tax is set too low and cannot change as a response to changes in the market structure in the final goods market.

This paper is organised as follows. Section 2 describes in detail the various building blocks of the model whereas Section 3 defines the dynamic differential game. Section 4 characterises the optimal emissions tax and R&D subsidy. Sections 5 and 6 analyse how the intensity of competition and other general parameters (such as the discount rate) impact on the optimal environmental policy. Section 7 concludes.

2 The Model

Two firms compete over time in the output market by producing and selling differentiated goods. Time is continuous. Production entails the emission of a pollutant, which is taxed. Firms can respond to the tax by decreasing their production or engaging in R&D, which is assumed to be for the sole purpose of reducing emissions. R&D expenditures are subsidised. Investment in R&D generates spillovers. Below we expound the different building blocks of our model, namely, demand, market (imperfect) competition, and the investment in the production of R&D. For simplicity, we omit the time dimension in most of this section but the reader should keep in mind that firms make decisions at each point in time. As fully explained in Section 3, we will focus on the steady state equilibrium.

2.1 Demand

We follow [28] and assume that the representative consumer has the following quadratic (strictly concave) utility function:

\[ U(q_i, q_j, m) = \alpha(q_i + q_j) - \left(\frac{q_i^2 + q_j^2}{2}\right) - \eta q_i q_j + m; \quad i, j = 1, 2, i \neq j, \]

where \( \alpha > 0 \) is a constant representing the market size, reservation price or maximum willingness to pay, and the parameter \( \eta \in [0, 1] \) represents the degree of product differentiation with \( \eta = 0 \) implying that firms are independent monopolists and \( \eta = 1 \) the case when products are perfect substitutes. In this setting, \( m \) is the numeraire good capturing the consumer’s expenditure on outside goods. The inverse demand function is determined by maximising the utility function:

\[ \max_{(q_i, q_j)} [U(q_i, q_j, m)], \]

with the associated first order conditions:

\[ \frac{\partial U(q_i, q_j, m)}{\partial q_i} = 0 \quad \text{or} \quad P_i = a - q_i - \eta q_j. \]
\[ q_i = \frac{(a - P_i) - \eta(a - P_j)}{1 - \eta^2}. \]  

### 2.2 Competition in Supply Schedules

We use the notion of competition in supply schedules to model competition in the output market. (See, for example, [11], [27], [29], [16], [9], [30], [19] and [13]). By considering families of more or less elastic supply schedules, it is possible to generate spaces of oligopoly games of which Bertrand and Cournot are polar cases. This approach allows us to parameterise the nature of competition in the output market. Firms face a strictly decreasing demand function for each product \( Q_i = D(P_i) \), and produce with constant marginal cost \( c < a \). Firms compete in supply functions \( q_i(\beta_i, \theta_i) \), which are continuously differentiable, convex, and increasing in both variables. We restrict our attention to linear supply schedules of the form

\[ q_i = \beta_i P_i + \theta_i, \]  

but we note that [14] show that under some conditions such a restriction involves no loss of generality.

The market clearing price for each good satisfies:

\[ D_i(P_i(\beta_i, \theta_i, \beta_j, \theta_j)) = q_i(\beta_i, \theta_i, \beta_j, \theta_j). \]  

Firm \( i \)'s profit function can be written as follows:

\[ \pi_i(\beta_i, \theta_i, \beta_j, \theta_j) = [P_i(\beta_i, \theta_i, \beta_j, \theta_j) - c] q_i(\beta_i, \theta_i). \]  

Following [14], we assume that \( \beta_i \) is fixed at \( \beta \) and firms choose \( \theta_i \). \( \beta \) can be interpreted as a measure of market competitiveness. This representation allows us to cover a continuum of market structures from Cournot \( (\beta = 0) \) to Bertrand \( (\beta \to \infty) \).

Given the linear supply schedules, and a linear demand function, \( P_i = a - (q_i + \eta q_j) \), the market clearing prices and quantities for the differentiated goods are obtained by substituting (4) into the demand function, yielding:

\[ P_i(\beta, \theta_i, \theta_j) = \frac{a[1 + \beta(1 - \eta)] - (1 + \beta - \beta \eta^2)\theta_i - \eta \theta_j}{(1 + \beta)^2 - \beta^2 \eta^2} \]  

and

\[ q_i(\beta, \theta_i, \theta_j) = \frac{a \beta[1 + \beta(1 - \eta)] + (1 + \beta)\theta_i - \beta \eta \theta_j}{(1 + \beta)^2 - \beta^2 \eta^2}. \]
2.3 Emissions and Abatement R&D

Firm \(i, i = 1, 2\), emits \(e_0q_i\) units of some pollutant when it produces quantity \(q_i\). The pollutant is subject to a per unit tax \(t_e\). As a response to the tax, firms can either decrease their output or invest in abatement technology ([24]).

Investment in R&D earns a subsidy \(\sigma\), \(0 \leq \sigma < 1\), on a per unit basis. As in [1], we assume that, in the absence of subsidies, if firm \(i\) invests \(\frac{2}{\sigma}I_i^2\) in abatement R&D, then \(i\) reduces its gross emissions \(e_0q_i\) by \((x_i + \eta \gamma x_j)\), where \(x_i\) represents the ability of firm \(i\) to reduce emissions with a similar definition for firm \(j\), and \(\gamma, 0 \leq \gamma \leq 1\), captures the technological spillover. In particular, the effective R&D spillover is represented by \(\eta\gamma\) ([5], [7]), \(0 \leq \eta \leq 1\), which requires that the products must be related for the spillover to be effective. Finally, \(\nu\) relates to the efficiency or productivity of the R&D activities, where a higher value means lower efficiency. We also assume that Firm \(i\)'s R&D emission reduction capital stock evolves over time according to the following standard capital stock accumulation process:

\[
\dot{x}_i = I_i - \delta x_i, x_i(0) = x_{i0}
\]  

(9)

where \(\delta, 0 \leq \delta \leq 1\), is a constant depreciation rate, which measures the instantaneous decrease in emissions reduction effectiveness due to the ageing of technology, and \(x_{i0}\) is a positive constant. While there is a direct relationship between \(x_i\) and \(I_i\), and it should be clear that firm \(i\) chooses \(I_i\), for convenience, we will not make this relationship explicit in some of the definitions that follow. For example, we will write firm \(i\)'s net emissions are given as:

\[
E_i(x_i, x_j, q_i) = e_0q_i - (x_i + \eta \gamma x_j),
\]  

(10)

which are required to be nonnegative.

The environmental damage cost of emissions is assumed to take a linear form:

\[
D = \alpha(E_1 + E_2),
\]

where \(E_i\) is defined in equation (10) and \(\alpha > 0\) is the marginal disutility of pollution. Firm \(i\)'s emissions tax cost function is given by:

\[
T_i(x_i, x_j, q_i) = t_eE_i = t_e[e_0q_i - (x_i + \eta \gamma x_j)].
\]  

(11)

The choice of an emissions tax rate \(t_e\) is not entirely unconstrained. In order to ensure an interior solution we need to impose the following condition:

**Assumption A0** \(a - c - e_0t_e \geq 0\).

This assumption states that the maximum willingness to pay net of the marginal cost of production and net of the marginal cost of the emissions tax must be nonnegative.

When a per unit subsidy \(s\) is applied to R&D investment, the firm's total cost function can be expressed as:
\[ C_i(q_i, x_i, x_j, I_i) = c q_i + t_e[e_0 q_i - (x_i + \eta \gamma x_j)] + (1 - s) \frac{\nu}{2} I_i^2. \] (12)

Note that the Porter Hypothesis (\[23\]) does not hold in our setting as the investment in R&D has no impact on production costs.

We can write firm \(i\)'s profit function as:

\[ \pi_i(q_i, q_j, x_i, x_j, I_i) = (a - q_i - \eta q_j - c)q_i - t_e[e_0 q_i - (x_i + \eta \gamma x_j)] - (1 - s) \frac{\nu}{2} I_i^2. \] (13)

Note that as degree of differentiation increases – that is, \(\eta\) decreases – there is a positive impact via demand and a negative impact as the ability of the firms to free ride on their opponent’s R&D decreases.

### 2.4 Social Welfare

The regulator sets its emissions tax and R&D subsidies to maximise the discounted stream of social welfare over an infinite horizon. Consumer’s surplus is defined as:

\[ CS = U(q_i, q_j) - \sum_{i=1}^{2} p_i q_i = a \sum_{i=1}^{2} q_i - \frac{1}{2} \sum_{i=1}^{2} q_i^2 - \eta q_i q_j - \sum_{i=1}^{2} p_i q_i. \] (14)

Firm \(i\)'s profits, excluding emissions taxation cost and R&D subsidy, are given by

\[ \pi_i = (a - c)q_i - q_i^2 - q_i q_j \eta - \frac{\nu}{2} I_i^2. \] (15)

Social welfare is defined by consumer’s surplus plus the profits of both firms (excluding taxes and subsidies) minus the environmental damage:

\[ W = CS + \pi_1 + \pi_2 - D. \] (16)

### 3 The Differential Game

We analyse a non-cooperative differential game where firms 1 and 2 each choose quantity \((q_i, i = 1, 2)\) and investment in R&D \((I_i, i = 1, 2)\) to maximise the following discounted stream of profits over an infinite horizon:

\[ \pi_i = \int_{0}^{\infty} e^{-rt} \{ (a - q_i - \eta q_j - c)q_i - t_e[e_0 q_i - (x_i + \eta \gamma x_j)] - (1 - s) \frac{\nu}{2} I_i^2 \} dt \] (17)

where \(r (0 \leq r \leq 1)\) is the discount rate. The functional objective (profit function) above can also be expressed in terms of the strategic variables \(\theta_i, \theta_j\).
In differential game terminology, $x_i$ are the state variables and $I_i$ and $\theta_i$ are the control variables, where $x_i$ and $I_i$ are related through (9) and $\theta_i$ and $q_i$ are linked through (4).

The differential game is linear both in state and control variables and, therefore, it follows that the value functions and strategies are also linear. As it is standard in the analysis of infinite-horizon differential games, the analysis is confined to stationary Markovian feedback strategies. Applying a standard sufficient condition for a stationary feedback equilibrium, the objective is to find bounded and continuously differentiable value functions $V(x_i, x_j)$, satisfying the Hamilton-Jacobi-Bellman equations (8):

$$rV_i(x_i, x_j) = \max_{I_i, \theta_i} \{(a - q_i - \eta q_j - c)q_i - t_e[e_0q_i - (x_i + \eta \gamma x_j)] - (1-s)\frac{\nu}{2} I_i^2 +$$

$$+ \frac{\partial V_i(x_i, x_j)}{\partial x_i}(I_i - \delta x_i) + \frac{\partial V_i(x_i, x_j)}{\partial x_j}(I_j - \delta x_j)\}.$$  \hspace{1cm} (18)

The above equation can be expressed in terms of the strategic variables by replacing the quantities using equation (8).

Differentiating the right-hand side w.r.t. $I_i$ and $\theta_i$ and equating it to zero yields:

$$\theta_i^* = \frac{a[1 - \beta^2(1 - \eta^2)] - (c + e_0t_e)(1 + \beta)[1 + \beta(1 + \eta)]}{2 + \beta(2 - \eta)(1 + \eta)}. \hspace{1cm} (19)$$

Therefore, the equilibrium quantities can be determined as follows:

$$q_i^* = \frac{(a - c - e_0t_e)(1 + \beta)}{2 + \beta(2 - \eta)(1 + \eta)}. \hspace{1cm} (20)$$

**Remark 1** From (20), setting $\beta = 0$ we obtain the Cournot equilibrium quantity $\frac{(a-c-e_0t_e)}{2+\eta}$, where $e_0t_e$ represents the incremental cost associated with the emissions tax. Similarly, we obtain the Bertrand equilibrium quantity when $\beta \rightarrow \infty$.

The equilibrium investment in R&D is determined by:

$$I_i^* = \frac{1}{(1-s)\nu} \frac{\partial V_i(x_i, x_j)}{\partial x_i}. \hspace{1cm} (21)$$

Replacing (21) in (18) yields:

$$rV_i(x_i, x_j) = \{(a - q_i^* - \eta q_j^* - c)q_i^* - t_e[e_0q_i^* - (x_i + \eta \gamma x_j)] -$$

$$-(1-s)\frac{\nu}{2} \left\{ \frac{1}{(1-s)\nu} \frac{\partial V_i(x_i, x_j)}{\partial x_i} \right\}^2 + \frac{\partial V_i(x_i, x_j)}{\partial x_i} \left\{ \frac{1}{(1-s)\nu} \frac{\partial V_i(x_i, x_j)}{\partial x_i} - \delta x_i \right\} +$$
Replacing (20) in the above equation, we can show that the following linear value functions are solutions to the above system of partial differential equations:

\[
V^*_i(x_i, x_j) = \frac{1}{2r} \left( \frac{2(a - c - e_0 t_e)^2(1 + \beta)[1 + \beta(1 - \eta^2)]}{[2 + \eta + \beta(2 - \eta)(1 + \eta)]^2} + \frac{t_e^2(1 + 2\eta)}{nu(1 - s)(r + \delta)^2} \right) + \\
\frac{t_e}{(r + \delta)} x_i + \frac{t_e \eta}{(r + \delta)} x_j.
\]  

(23)

We are now ready to state our first result characterising the steady state equilibrium.

**Proposition 1:** Suppose assumption A0 holds and that

\[
\nu \geq \frac{t_e(1 + \gamma \eta)[2 + \eta + \beta(2 - \eta)(1 + \eta)]}{e_0(1 - s)(a - c - e_0 t_e)(1 + \beta) \delta (r + \delta)}.
\]  

(24)

Then the general feedback equilibrium steady state is given by:

\[
x^*_i = \frac{t_e}{(r + \delta)(1 - s) \delta \nu},
\]  

(25)

the steady state value for the control variable is expressed by:

\[
I^*_t = \frac{t_e}{(r + \delta)(1 - s) \nu},
\]  

(26)

and the dynamic emissions reduction function is given by:

\[
x_i(t) = \frac{e^{-\delta t}[(r + \delta)(1 - s) \delta \nu x_{i0} - (1 - e^{-\delta t}) t_e]}{(r + \delta)(1 - s) \delta \nu}.
\]  

(27)

**Proof:** Equations (25) and (26) are obtained by substituting the first derivative of the value function (equation (23)) with respect to \( x_i \) in equation (21) and then use equation (9). Condition (24) implies that \( e_0 q^*_0 \geq x^*_i (1 + \gamma \eta) \), which ensures that costs remain non-negative post innovation and that firms’ emissions are nonnegative.

The following corollary summarises how equilibrium steady state quantities (20), investment in abatement R\&D (26), consumer surplus (CS), producer surplus (PS), environmental damage cost of emissions (D) and welfare (W) vary with the emissions tax \( t_e \), R\&D subsidy \( s \), intensity of competition \( \delta \), discount rate \( r \), technology depreciation rate \( \delta \) and product differentiation \( \eta \):

8
Corollary 2  The table below summarises the comparative statics.  \(^1\).

<table>
<thead>
<tr>
<th>( q_i )</th>
<th>( \frac{\partial q_i}{\partial \tau} )</th>
<th>( \frac{\partial q_i}{\partial \varepsilon} )</th>
<th>( \frac{\partial q_i}{\partial \sigma} )</th>
<th>( \frac{\partial q_i}{\partial \rho} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_i )</td>
<td>(&lt; 0)</td>
<td>( n/a )</td>
<td>( &gt; 0 )</td>
<td>( n/a )</td>
</tr>
</tbody>
</table>

if \( 1/2 < \eta \leq 1 \) and \( \beta > \frac{1}{2n-1} \)

<table>
<thead>
<tr>
<th>( J_i^* )</th>
<th>( &gt; 0 )</th>
<th>( &gt; 0 )</th>
<th>( n/a )</th>
<th>( &lt; 0 )</th>
<th>( &lt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CS )</td>
<td>( &lt; 0 )</td>
<td>( n/a )</td>
<td>( &gt; 0 )</td>
<td>( n/a )</td>
<td>( n/a )</td>
</tr>
</tbody>
</table>

if \( \beta > \frac{1}{n(1+\eta)} \)

<table>
<thead>
<tr>
<th>( PS )</th>
<th>( i/t )</th>
<th>( i/t )</th>
<th>( &lt; 0 )</th>
<th>( &gt; 0 )</th>
<th>( &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>( &lt; 0 )</td>
<td>( i/t )</td>
<td>( i/t )</td>
<td>( i/t )</td>
<td>( i/t )</td>
</tr>
</tbody>
</table>

| \( W \) | \( i/t \) | \( i/t \) | \( i/t \) | \( i/t \) | \( i/t \) |

The proof is straightforward but provided in the appendix for completeness. There are several noteworthy features in the Table above. First, the impact of an increase in the intensity of competition on equilibrium quantities and consumer surplus is, as expected, positive. However, as more production involves more pollution, for a fixed emission tax rate, the impact of more intense competition on welfare is ambiguous. This is a new insight, which will be explored further in Section 5 below. Second, as products become less differentiated (that is, \( \eta \) increases), the firms behave less as local monopolists and, therefore, quantities and consumer surplus increase, as long as the fixed intensity of competition is sufficiently large. This is a generalisation of the standard result in industrial organisation that, in the absence of pollution taxes, quantities increase with \( \eta \) under Bertrand Competition only if \( \eta > 0.5 \). \([3]\) \(^2\). As in the case of an increase in the intensity of competition, here too a reduction in product differentiation that leads to greater equilibrium quantities has, ceteris paribus, an ambiguous impact on welfare as total emissions increase. Finally, we note that the marginal impact of all the variables of interest on welfare is ambiguous. This extends the point made by \([12]\) that environmental technology policy is plagued by complex trade-offs not only between market failures associated with pollution and with the diffusion of new technologies but also their interaction when competition is imperfect in output markets. In the next section we study the combination of emissions tax and R&D subsidies that can be designed to overcome these market failures and the trade-offs that they entail.

4 Optimal Emissions Tax and R&D Subsidies

A regulator, who can fully commit to policy, sets its emissions tax and R&D subsidies to maximise the discounted stream of social welfare over an infinite horizon. This dynamic control problem can be defined as maximising:

\[ \max L(t) = \sum_{t=0}^{\infty} \frac{1}{(1+\delta)^t} \left( \sum_{i=1}^{n} w_i q_i(t) - \tau q_i(t) - \frac{1}{2} (q_i(t) - q_j(t))^2 \right) \]

\(^1n/a: \text{not applicable. } i/t: \text{indeterminate or determinate only under complex conditions.} \)

\(^2\text{Equilibrium quantities on a Bertrand duopoly with differentiated products and symmetric/constant marginal cost: } q_B = \frac{\sigma - \eta}{2(\sigma - \eta)(1+\eta)} \text{ (output falls if } 0 \leq \eta < 0.5, \text{ reaches a minimum at } \eta = 0.5 \text{ and rises for } \eta > 0.5)\)
where \( r (0 \leq r \leq 1) \) is the discount rate and \( \rho \) is an environmental purification factor \((0 \leq \rho \leq 1)\). The environmental damage is assumed to be subject to exponential decay. The constraint (29) determines how emissions generated today adds to the current stock of environmental damages \( d(t) \). The initial stock is \( d(0) = d_0 \). The term \( \rho d \) is included in the above equation as it is assumed that the natural rate of purification is proportional to the existing stock. The state variable is \( d(t) \) and the control variables are the tax on emissions \( (t_e) \) and R&D subsidies \( (s) \). The tax revenue and the subsidy expenditures are simply transfers and, therefore, can be omitted from the welfare function. The government in this setting is not budget constrained.

### 4.1 Current Value Hamiltonian

Given the equilibrium steady state, the emissions reduction, and equilibrium quantities given by, respectively, (25), (26), and (20), we can write the current value Hamiltonian function as:

\[
H_c(d, t_e, s) = (1 + \eta)q^* + 2q^*[\alpha - c - q^*(1 + \eta)] - \nu I^* - \rho d + \\
+ \lambda \{2[\epsilon_0 q^* - (1 + \eta)\bar{x}] - \rho d\}.
\]

Costate variable \( \lambda \) is a measure of the sensitivity of the optimal total welfare to the given initial level of environmental damage. In general, \( \lambda(t) \) is the shadow price of environmental damage at a given point in time. From an economics perspective, this problem can be viewed as an autonomous problem as the \( t \) argument only appears explicitly via the discount factor.

The first four terms of the Hamiltonian function represent the welfare value at time \( t \), based on the current environmental damage and the current policy decisions in terms of emissions taxes and R&D subsidies, taken at this point in time. The last component of this function represents the rate of change of environmental damages corresponding to tax and subsidy policy. When multiplied by the shadow price \( \lambda(t) \), it is converted to a monetary value. Unlike the first four terms, which relate to current welfare effect of tax/subsidy policy, the last term can be viewed as the future welfare impact of the policy. The current environmental damage accumulation leads to future deterioration of general welfare.
Applying the [22] maximum principle conditions, the problem can now be solved for \( t_e(t), s(t), d(t) \) and \( \lambda(t) \), satisfying the following conditions:

\[
\frac{\partial H_e}{\partial t_e} = 0; \quad \frac{\partial H_e}{\partial s} = 0; \quad \frac{\partial H_e}{\partial d} + r\lambda = \dot{\lambda};
\]

(31)

\[
\dot{d} = \frac{\partial H}{\partial \lambda} = 2[e_0q^* - (1 + \gamma\eta)x^*] - \rho d =
\]

(32)

\[
= \frac{2e_0(a - c - e_0t_e)(1 + \beta)}{2 + \eta + \beta(2 - \eta)(1 + \eta)} - \frac{2t_e(1 + \gamma\eta)}{(1 - s)\delta(r + \delta)\nu} - \rho d;
\]

(33)

\[
d(0) = d_0; \quad \text{and}
\]

(34)

\[
\lim_{t \to \infty} e^{-rt}\lambda(t)d(t) = 0 \quad \text{(transversality condition)}.
\]

(35)

The conditions for the maximum principle are sufficient for the global maximisation in the infinite-horizon problem provided the current value Hamiltonian is concave in the state and control variables (second derivatives of the Hamiltonian with respect to those variables are negative) for all \( t \) and that the transversality condition is met.

**Assumption A1** \( s > 1 - \frac{3t_e\delta(r + \rho)}{2\alpha(1 + \beta)(1 + \gamma\eta)} \).

This assumption ensures that the current value Hamiltonian is concave in \( t_e \), (second derivative of the Hamiltonian with respect to \( t_e \) is negative).

The next proposition characterises the optimal solutions.

**Proposition 2:** Suppose that assumption A1 holds. The maximisation of (28) subject to (29) yields the optimal tax

\[
t_e^* = \frac{\alpha[2 + \eta + \beta(2 - \eta)(1 + \eta)]}{(1 + \beta)(1 + \eta)(r + \rho)} - \frac{(a - c)(1 + \beta(1 - \eta^2))}{e_0(1 + \beta)(1 + \eta)},
\]

(36)

and the optimal abatement R&D subsidy

\[
s^* = \frac{(a - c)\delta[1 + \beta(1 - \eta^2)](r + \rho)}{e_0\alpha(1 + \beta)(r + \delta)(1 + \eta)(1 + \gamma\eta)} - \frac{\delta[1 + (1 + \eta)[\beta(1 - \eta) - \gamma\eta(1 + \beta)]]}{(1 + \beta)(r + \delta)(1 + \eta)(1 + \gamma\eta)} + \frac{r}{(r + \delta)}.
\]

(37)
Proof: Conditions (31) to (35) yield:

\[
\frac{\partial H_c}{\partial \epsilon_c} = -\frac{2\epsilon_0(1 + \beta)}{[\epsilon + \beta(2 - \eta)(1 + \eta)]^2} \left\{ [a - c + e_0 t_c(1 + \eta)](1 + \beta) + (a - c)\beta \eta^2 \right\} - \frac{2t_c}{(1 - s)^2(r + \delta)^2 \nu} - \frac{2\epsilon_0(1 + \beta)}{2 + \eta + \beta(2 - \eta)(1 + \eta)} + \frac{2(1 + \gamma \eta)}{(1 - s)\delta(r + \delta) \nu} = 0; \quad \text{and}
\]

\[
\frac{\partial H_c}{\partial s} = \frac{2t_c[-t_c\delta - (1 - s)(r + \delta)(1 + \gamma \eta)\lambda]}{(1 - s)^3 \delta(r + \delta)^2 \nu} = 0; \quad \text{and}
\]

\[
\frac{\partial H_c}{\partial \delta} = -\alpha - \lambda \rho; \alpha + \lambda(\rho + r) = \dot{\lambda}.
\]

At steady state we have \( \dot{\lambda} = 0; \dot{d} = 0 \). We can then use the three equations to solve for \( \lambda, t_c \) and \( s \), yielding the optimal emissions tax (36) and the optimal abatement R&D subsidy (37).

From inspection of (36), we note that the optimal emissions tax increases as the disutility of pollution (\( \alpha \)) increases but it does not depend on the level of industry spillover (\( \gamma \)) nor does it depend on the level of technology depreciation (\( \delta \)). The emissions tax is introduced to correct the negative externality of firms producing more than what it is socially desirable, due to the accompanying generation of harmful emissions. The first term of the optimal emissions tax (36) represents the net present value of the disutility of emissions adjusted by product differentiation and intensity of competition. The second term represents the maximum willingness to pay net of marginal cost, per unit of emissions, also adjusted by product differentiation and intensity of competition. This term implies a decrease in the tax due to the benefit of consuming the product.

Similarly, inspection of (37) reveals that the optimal subsidy decreases as \( \alpha \) increases. The social returns of R&D may often exceed firms’ returns due to the presence of spillovers. The abatement R&D subsidy addresses this externality. The technology depreciation is compensated through the subsidy, as indicated in (37).

4.2 Characterising Equilibrium Outcomes

This subsection computes the equilibrium values of the various variables of interest. The next two sections then investigate how the maximised values of these variables change with exogenous variables such as the discount rate, the nature of the abatement technology and the intensity of competition.

The steady state welfare for given values of emissions tax and abatement R&D subsidy can be calculated by replacing the steady state quantity and investment values into (16), yielding:

\[
W^{**} = \frac{(a - c - e_0 t_c)(1 + \beta)[3a - 3c + e_0 t_c](1 + \beta) + (a - c + e_0 t_c)(1 + \beta)\eta - 2(a - c)\beta \eta^2}{2 + \eta + \beta(2 - \eta)(1 + \eta)^2}.
\]
\[- \frac{t_e^2}{(r + \delta)^2(1 - s)^2 \nu} - \frac{2 \alpha \gamma \varepsilon}{\rho} \left[2 + \eta + \beta(2 - \eta)(1 + \eta) \right] - \frac{t_e(1 + \gamma \eta)}{(1 - s) \delta (r + \delta) \nu}. \] (38)

Steady state damage accumulation for given values of emissions tax and abatement R&D subsidy is similarly computed as:

\[ d_{ss} = \frac{2}{\rho} \left[ \varepsilon_0 (a - c - \varepsilon_0 t_e)(1 + \beta) \right] - \frac{t_e(1 + \gamma \eta)}{(1 - s) \delta (r + \delta) \nu} = \frac{2}{\rho} \varepsilon_0 \eta - x_i(1 + \gamma \eta). \] (39)

We can then compute the steady state welfare and damage accumulation function for the optimal tax and subsidy by simply replacing (36) and (37) into the equations above, obtaining:

\[ W_{(s = s^*, t_e = t_e^*)} = \frac{a[a \rho - 2(\varepsilon_0 \alpha + c \rho)]}{(1 + \eta) \rho} + \frac{\alpha^2(1 + \gamma \eta)^2(2r + \rho)}{\delta^2 \nu \rho(r + \rho)^2} + \frac{[\varepsilon_0 \alpha + c(r + \rho)] [c \rho(r + \rho) + \varepsilon_0 \alpha(2r + \rho)]}{(1 + \eta) \rho (r + \rho)^2} \] (40)

and

\[ d_{(s = s^*, t_e = t_e^*)} = \frac{2 \varepsilon_0 [(a - c)(r + \rho) - \varepsilon_0 \alpha]}{(1 + \eta) \rho (r + \rho)} - \frac{2 \alpha(1 + \gamma \eta)^2}{\delta^2 \nu \rho(r + \rho)}. \] (41)

In order to ensure that the steady state emissions damage accumulation is non-negative: \((a - c)(r + \rho) - \varepsilon_0 \alpha \geq \frac{\alpha(1 + \eta)(1 + \gamma \eta)^2}{\varepsilon_0 \alpha^* \beta_0} \).

The steady state quantities and abatement R&D outcomes, at the optimum, are given by:

\[ q_i^{ss} = \frac{(a - c)(r + \rho) - \varepsilon_0 \alpha}{(1 + \eta)(r + \rho)} = \frac{(a - c)}{(1 + \eta)(r + \rho)} - \frac{\varepsilon_0 \alpha}{(1 + \eta)(r + \rho)} \] (42)

and

\[ x_i^{ss} = \frac{\alpha(1 + \gamma \eta)}{\delta^2 \nu (r + \rho)}. \] (43)

Note that the optimised values expressed in equations (40) to (43) do not depend on the market competitiveness parameter \( \beta \). The dynamic optimisation process adjusts the emissions tax and R&D subsidy in an optimal way to account for the different levels of competitive intensiveness. This is a major departure from the static case where more competition may have a negative impact on welfare as more quantity implies more pollution. This effect is again present.
when taxes and/or subsidies are not set at optimal level as explored in the next section.

In our framework, it is possible to reproduce the Cournot and Bertrand cases, which are dealt with in the literature, as special cases:

**Remark:** From the linear supply schedules the **Cournot** duopoly game or corresponding market structure can be analysed by setting the competitiveness parameter $\beta = 0$. The **Bertrand** duopoly game or corresponding market structure can be analysed by setting the competitiveness parameter $\beta \to \infty$. The optimal pollution or emissions tax in a Cournot setting is

$$ t^C_e = \frac{\alpha(2 + \eta)}{(1 + \eta)(r + \rho)} - \frac{(a - c) e_0}{e_0(1 + \eta)}. $$

(44)

The optimal pollution or emissions tax in a Bertrand setting is

$$ t^B_e = \frac{\alpha(2 - \eta)}{r + \rho} - \frac{(a - c)(1 - \eta)}{e_0}. $$

(45)

The optimal subsidy in a Cournot setting is:

$$ s^C = \frac{(a - c)\delta(r + \rho)}{e_0\alpha(r + \delta)(1 + \eta)(1 + \gamma\eta)} - \frac{\delta[1 - \gamma\eta(1 + \eta)]}{(r + \delta)(1 + \eta)(1 + \gamma\eta)} + \frac{r}{(r + \delta)}. $$

(46)

The optimal subsidy in a Bertrand setting is:

$$ s^B = \frac{(a - c)\delta(r + \rho)(1 - \eta)}{e_0\alpha(r + \delta)(1 + \gamma\eta)} - \frac{\delta[1 - \eta(1 + \gamma)]}{(r + \delta)(1 + \gamma\eta)} + \frac{r}{(r + \delta)}. $$

(47)

As $t^B_e - t^C_e = \frac{\eta^2((a-c)(r+\rho)-e_0\alpha)}{e_0(1+\eta)(r+\rho)} > 0$ for all admissible levels of the several variables, the optimal tax under Bertrand competition is always larger than under Cournot competition but below marginal damage of emissions. Katsoulacos and Xepapadeas (1995) also found that under imperfect or oligopolistic competition, the optimal emission tax should be smaller than the marginal damage of emissions. Moreover, our model shows that, under a static setting ( $r = 0$, $\delta = 1$ and $\rho = 1$), when the products are near perfect substitutes (that is, as $\eta$ increases and approaches 1) the optimal emission tax under Bertrand competition is equal to the marginal damage of emissions ($\alpha$). These results are similar to Poyago-Theotoky (2003) and in line with Pigou (1932), which established that, under perfect competition the optimal tax on emissions should equal the marginal damage
of emissions. Our approach is more general as it covers a continuum of strategy behaviour, yielding Cournot and Bertrand as special polar cases. Moreover, as $s^B - s^C = -\frac{\delta q^2[(a-c)(r+p)-c_0\alpha]}{c_0\alpha(r+p)(1+\gamma)(1+\eta)} < 0$ for all admissible levels of the several variables, the optimal abatement R&D subsidy under Bertrand competition is always smaller than under Cournot competition. The intuition behind this result is that, as the optimal tax increases with the level of competition, and the R&D activities are undertaken for the sole purpose of decreasing emissions (lowering the tax burden), the increase in the emissions tax lowers the need for a subsidy to foster R&D.

5 The Impact of the Intensity of Competition

As discussed above, when both tax and subsidy are set optimally, they change in a way to perfectly offset any changes in the intensity of competition. That is, setting the tax/subsidy combination optimally addresses not only the market failures associated with emissions and R&D production but also those associated with the existence of imperfect competition in the output market.

The next proposition establishes how the change in intensity of competition affects the optimal tax and subsidy. It also establishes how the steady state damage accumulation and welfare are impacted by changes in the intensity of competition when the tax and subsidy cannot be instantaneously adjusted.

**Proposition 3:** The impact of changes in the intensity of competition is summarised in the following table:

<table>
<thead>
<tr>
<th>$W^{ss}$</th>
<th>$t^*$</th>
<th>$s^*$</th>
<th>$\frac{\partial}{\partial \beta}$</th>
<th>$\frac{\partial^2}{\partial \beta^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t^*$</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>$s^*$</td>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>$W^{ss}$</td>
<td>&gt; 0 if $t_e &gt; \frac{\alpha(2-c)}{r+p} - \frac{(a-c)(1-\eta)}{\gamma} &gt; 0$</td>
<td>[ \frac{\alpha(2-c)}{r+p} - \frac{(a-c)(1-\eta)}{\gamma} ]</td>
<td>&lt; 0 if $t_e &lt; \frac{\alpha(2-c)}{r+p} - \frac{(a-c)(1-\eta)}{\gamma}$</td>
<td></td>
</tr>
</tbody>
</table>

The proof of Proposition 3 is straightforward and it is included in the appendix. This proposition establishes that the optimal tax needs to rise as a result of an increase in the intensity of competition. This arises as more competition implies more output and, consequently, more pollution. Of course, more competition also implies higher consumer surplus (gross of pollution). From a welfare perspective, these two effects (increase in both consumer surplus and pollution) work in opposite directions. As $\beta$ approaches infinity (Bertrand case), the required increase in the optimal tax decreases as the first effect becomes more important than the second effect.

Similarly, the optimal R&D subsidy decreases with the increase in the intensity of competition – more competition implies more pollution, which requires a higher optimal tax to induce firms to invest more in R&D. There is less of a
need for a subsidy when the emission tax increases. Unlike the impact on tax, as $\beta$ approaches infinity, the decrease in optimal R&D subsidy diminishes. From a welfare perspective, as the intensity of competition increase, the impact of a tax on emissions is stronger (but diminishing) than a subsidy on R&D activities. The reason for such asymmetry is that the subsidy is applied to total cost, which is a convex function of R&D output a la D’Aspremont and Jacquemin (1988).

As mentioned before, when the intensity of competition increases quantities produced also increase. For given values of emissions tax and R&D subsidy, damage accumulation increases when the intensity of competition increases since emissions are proportional to output. Quantities produced show a diminishing increase with the intensity of competition, therefore, damage accumulation behaves in a similar manner (concave shape).

As discussed above, an increase in the intensity of competition generates two opposing effects on welfare. More competition results in lower prices and higher quantities increasing consumer’s surplus (gross of pollution). When the tax on emissions is lower than the optimal Cournot tax (equation (44)) the welfare always decreases with the intensity of competition. On the other hand, when the tax on emissions is higher than the optimal Bertrand tax (equation (45)) the welfare always increases with the intensity of competition (Appendix, Proof of Proposition 3). An emissions tax lower than the optimal Cournot emissions tax (the lowest optimal tax) will result in emissions damage being underpriced. An increase in competition will increase quantities produced and, therefore, an increase in emissions that are underpriced - welfare will decrease. Conversely, an emissions tax higher than the optimal Bertrand emissions tax (the highest optimal tax) will result in emissions damage being overpriced. An increase in quantities produced due to an increase in competition will increase emissions that are overpriced. The additional consumer surplus will imply an increase in welfare.

The relationship between intensity of competition and welfare is an important and novel finding. Only when the emissions tax is sufficiently high (above its optimum for a given level of intensity of competition) will it be socially beneficial for policy makers to foster market competition. As in the case of a natural monopoly where competition inefficiently duplicates fixed costs, here competition can be detrimental when the emissions tax is set too low and cannot change as a response to changes in the market structure in the final goods market.

### 6 Some General Comparative Statics

The next proposition shows how the optimal tax, the optimal subsidy, and the steady state values of the damage accumulation function and welfare change with the discount rate ($r$), the R&D emission reduction capital stock depreciation ($\delta$), and the environmental purification factor ($\rho$).

**Proposition 4:** Key comparative statics results are summarised in the following table:
Moreover, \( t^*_e \) does not depend on \( \delta \).

The optimal value of the tax on emissions (\( t^*_e \)) and emissions abatement R&D subsidy (\( s^* \)) (per unit of R&D investment, therefore varying from 0 to 1) were determined simultaneously in order to maximise the discounted stream of social welfare over an infinite time horizon (at a steady state). The cumulative emissions damage increases when future generations are less important (meaning an increase in the discount rate). The optimal tax maximises welfare through the reduction in cumulative damage. It follows general intuition that, as the discount rate increases (so individuals value the present increasingly more than the future), the optimal emissions tax decreases (in a decreasing way). The optimal tax on emissions decreases when the environmental purification factor increases. It stands to reason that, as the natural decay of emissions increases, lower taxes can deliver the same social welfare, which means that the optimal emissions tax decreases.

Both producer surplus and damage accumulation increase when either (or both) discount rate or technology depreciation increase. These have opposite effects in the total welfare. Technology depreciation has a stronger effect on damage accumulation (square term in the denominator). This eliminates the effect of the increase in decay in pollution so the optimal subsidy increases with the discount rate as long as the technology depreciation is greater than pollution decay. Investment in R&D decreases when either (or both) discount rate or technology depreciation increase. By definition, the optimal R&D subsidy varies between 0 and 1, therefore, in order to compensate for the decrease in R&D investment we find a diminishing increase in optimal subsidy or a concave function of \( r \), if \( \delta > \rho \) and a diminishing decrease or a convex function of \( r \), if \( \delta < \rho \).

Our results show a linear increase in optimal R&D subsidy when the environmental purification factor increases. Since we consider a linear environmental damage function, at steady state, the optimal subsidy is a linear function of the environmental purification factor but a more complex function of \( r \) and \( \delta \). The optimal R&D subsidy increases (in a diminishing way) when the depreciation rate of technology increases only if \( a - c - \frac{e^{\omega(2-\eta)}}{r+\eta} > 0 \) (the maximum willingness to pay, net of the marginal cost of production and net of the present value of the marginal cost of the emissions tax, adjusted for product differentiation, must be positive). When the products are close to perfect substitutes
(\(\eta\) approaching 1) the optimal subsidy always decreases when the depreciation rate of technology increases.

Damage accumulation increases when investment in R&D decreases. Since investment in R&D decreases when the discount rate or the technology depreciation increases, the steady state damage accumulation increases (in a diminishing way) when the discount rate increases and when the depreciation of technology increases. The steady state damage accumulation decreases when the environmental purification factor increases. Taking into account that, when emissions decay at a higher rate, damage accumulation will be reduced, these results are unsurprising.

It also follows general intuition that steady state social welfare increases (in a diminishing way, as the decay factor is just multiplied by the value of damage accumulation) when the environmental purification factor increases. A higher factor is better for welfare as the emissions damage dissipates quicker. The welfare decreases in a straightforward manner when the discount rate increases and when depreciation decreases.

The model presented in this paper enables a novel understanding on how the value of the dynamic parameters (discount rate, technology depreciation and pollution decay) affect the optimal emission tax, optimal abatement R&D, damage accumulation and welfare. From an environmental policy perspective, a lower discount rate, lower technology depreciation and a high decay of pollution are preferable. However, a tax on emissions and an abatement R&D subsidy can be adjusted to optimally manage a variation in the above dynamic parameters.

7 Concluding Remarks

This paper studies environmental policy design when there are three sources of market failure (imperfect competition, R&D spillovers and pollution). Understanding the impact of imperfect competition on environmental policy is important as existing research focuses by and large on competitive markets. See, for example, [26].

Our contribution is to determine the optimal environmental policy (emissions tax and R&D subsidies) explicitly as a function of the intensity of competition in a dynamic setting. We show that the interaction between increased competition and the optimal environmental policy is complex. Competition interacts in a non-trivial way with the emission of pollution and the incentives to undertake R&D. The key message is that the optimal emissions tax and R&D subsidy need to be calibrated to account for the impact of imperfect competition.

Further research is needed, however, to explore the links between the nature of competition and the firms’s choices about abatement technology, investment in R&D and the environmental policy environment in general. While this paper takes the intensity of competition as exogenous, it is very likely that firms (and governments) may attempt to influence it through their actions.
Appendix

Table 1: Steady state quantities are given by (20) and investment in abatement R&D are given by (26). As per section 2.3, consumer surplus (CS), producer surplus (PS), environmental damage cost of emissions (D) and welfare (W) are defined as follows:

\[ CS = \frac{(a - c - e_0 t_e)^2 (1 + \beta)^2 (1 + \eta)}{[2 + \eta + \beta(2 - \eta)(1 + \eta)]^2}, \]

\[ PS = \frac{2(a - c - e_0 t_e) (1 + \beta)[(a - c - e_0 t_e)(1 + \beta) + e_0 t_e (1 + \beta)\eta - (a - c)\beta \eta^2]}{[2 + \eta + \beta(2 - \eta)(1 + \eta)]^2} \]

\[ D = \frac{2\alpha_2 e_0(a - c - e_0 t_e)(1 + \beta)}{\rho [(2 + \eta + \beta(2 - \eta)(1 + \eta)] - \frac{t_e(1 + \gamma\eta)}{(1 - s)\delta(r + \delta)\nu}] \text{ and} \]

\[ W = CS + PS - D. \]

The signs of the first derivatives with respect to the parameters of interest are as follows.

Quantities:

\[ \frac{\partial q^*_i}{\partial t_e} = -\frac{e_0(1 + \beta)}{2 + \eta + \beta(2 - \eta)(1 + \eta)} < 0 \]

\[ \frac{\partial q^*_i}{\partial \beta} = \frac{(a - c - e_0 t_e)\eta^2}{[2 + \eta + \beta(2 - \eta)(1 + \eta)]^2} > 0 \]

\[ \frac{\partial q^*_i}{\partial \eta} = \frac{(a - c - e_0 t_e)(1 + \beta)[\beta(2\eta - 1) - 1]}{[2 + \eta + \beta(2 - \eta)(1 + \eta)]^2} > 0 \text{ if } 1/2 < \eta \leq 1 \text{ and } \beta > \frac{1}{2\eta - 1} \]

Abatement R&D investment:

\[ \frac{\partial I^*_i}{\partial t_e} = \frac{1}{(r + \delta)(1 - s)\nu} > 0 \]

\[ \frac{\partial I^*_i}{\partial s} = \frac{t_e}{(r + \delta)(1 - s)\nu} > 0 \]

\[ \frac{\partial I^*_i}{\partial r} = -\frac{t_e}{(r + \delta)^2(1 - s)\nu} < 0 \]

\[ \frac{\partial I^*_i}{\partial \delta} = -\frac{t_e}{(r + \delta)^2(1 - s)\nu} < 0 \]
Consumer surplus:

\[
\frac{\partial CS}{\partial t_c} < 0
\]

\[
\frac{\partial CS}{\partial \beta} = \frac{2(a - c - e_0 t_c)^2(1 + \beta)^2(1 + \eta)}{[2 + \eta + \beta(2 - \eta)(1 + \eta)]^2} > 0
\]

\[
\frac{\partial CS}{\partial \eta} = \frac{(a - c - e_0 t_c)^2(1 + \beta)^2 \eta^3[3\beta(1 + \eta) - 1]}{[2 + \eta + \beta(2 - \eta)(1 + \eta)]^3} > 0 \text{ if } \beta > \frac{1}{3(1 + \eta)}
\]

Producer surplus:

\[
\frac{\partial PS}{\partial s} = -\frac{2t_c^2}{(r + \delta)^2(1 - s)^{3/2}} < 0
\]

\[
\frac{\partial PS}{\partial r} = \frac{2t_c^2}{(r + \delta)^3(1 - s)^{3/2}} > 0
\]

\[
\frac{\partial PS}{\partial \delta} = \frac{2t_c^2}{(r + \delta)^3(1 - s)^{3/2}} > 0
\]

\[
\frac{\partial PS}{\partial \beta} = -\frac{2(a - c - e_0 t_c)^2[3\beta(1 + \eta)^3 - 2e_0 t_c(1 + \beta)(1 + \eta) + (a - c)\eta(1 + \beta + \beta \eta)]}{[2 + \eta + \beta(2 - \eta)(1 + \eta)]^3}
\]

The producer surplus always decreases with the competition \(\frac{\partial PS}{\partial \beta} < 0\) if

\[
a - c > \frac{2e_0 t_c(1 + \eta)}{\eta}
\]

or if

\[
\frac{(a - c)\eta}{(1 + \eta)} < 2e_0 t_c < (a - c)\eta \text{ and } \beta > \frac{(a - c)\eta - 2e_0 t_c(1 + \eta)}{(1 + \eta)[2e_0 t_c - (a - c)\eta]}
\]

Under the classic duopoly competition model (Cournot setting when \(\beta = 0\) and Bertrand setting when \(\beta \to \infty\), producer surplus always decreases with the intensity of competition. This can be easily understood as the firm’s profit function is concave on quantities and the equilibrium quantities are greater than the quantity corresponding to the maximum profit \(q_i^{\text{Max}} = \frac{a - c}{2(1 + \eta)}\); monopoly quantity: \(q_i^{\text{Mon}} = \frac{a - c}{2}\). In our model, the equilibrium quantities contain an additional term: \(e_0 t_c\). The emissions tax value, \(t_c\), determines whether the equilibrium quantities are greater or smaller than the quantity corresponding to the maximum profit. The above conditions simply state that, if the emissions tax is sufficiently low \((t_c < \frac{(a - c)\eta}{2e_0(1 + \eta)})\), producer surplus will always decreases with the intensity of competition. If the emissions tax is sufficiently high \((t_c > \frac{(a - c)\eta}{2e_0(1 + \eta)})\), producer surplus always increases with the intensity of competition. For values
in-between producer surplus decreases with the intensity of competition only if 
\[ \beta > \frac{(a-c)\eta-2e_0t_c(1+\eta)}{1+\eta} \frac{1}{2e_0t_c(a-c)\eta}. \]

Damage accumulation:

\[ \frac{\partial D}{\partial t_e} < 0 \]
\[ \frac{\partial D}{\partial s} = -\frac{2t_e\alpha(1+\gamma\eta)}{(r+\delta)(1-s)^2}\nu^2 < 0 \]
\[ \frac{\partial D}{\partial \beta} = \frac{2e_0(a-c-e_0t_e)\eta^2\alpha}{\rho(2+\eta+\beta(2-\eta)(1+\eta))^2} > 0 \]
\[ \frac{\partial D}{\partial r} = \frac{2t_e\alpha(1+\gamma\eta)}{(r+\delta)^2(1-s)\rho\nu} > 0 \]
\[ \frac{\partial D}{\partial \delta} = \frac{2t_e\alpha(r+2\delta)(1+\gamma\eta)}{(r+\delta)^2(1-s)\rho\delta^2\nu} > 0 \]

**Proof of Proposition 3:** The first and second derivatives of \( t_e^* \) with respect to \( \beta \) are given by:

\[ \frac{\partial t_e^*}{\partial \beta} = \frac{\eta^2[(a-c)(r+\rho)-e_0\alpha]}{e_0(1+\beta)^2(1+\eta)(r+\rho)} > 0 \]
\[ \frac{\partial^2 t_e^*}{\partial \beta^2} = -\frac{2\eta^2[(a-c)(r+\rho)-e_0\alpha]}{e_0(1+\beta)^3(1+\eta)(r+\rho)} < 0. \] (48)

The first and second derivatives of \( s^* \) with respect to \( \beta \) are given by:

\[ \frac{\partial s^*}{\partial \beta} = -\frac{\delta^2[(a-c)(r+\rho)-e_0\alpha]}{e_0\alpha(1+\beta)^2(1+\eta)(1+\gamma\eta)} < 0 \]
\[ \frac{\partial^2 s^*}{\partial \beta^2} = \frac{2\delta^2[(a-c)(r+\rho)-e_0\alpha]}{e_0\alpha(1+\beta)^3(1+\eta)(1+\gamma\eta)} > 0. \] (49)

The first and second derivative of the steady state accumulation function with respect to \( \beta \) are given by:

\[ \frac{\partial d_{ss}}{\partial \beta} = \frac{2e_0(a-c-e_0t_e)\eta^2}{[2+\eta+\beta(2-\eta)(1+\eta)]^2\rho} > 0 \]
\[ \frac{\partial^2 d_{ss}}{\partial \beta^2} = -\frac{4e_0(a-c-e_0t_e)(2-\eta)\eta^2(1+\eta)}{[2+\eta+\beta(2-\eta)(1+\eta)]^3\rho} < 0 \] (50)

Finally, the derivative of the steady state welfare function for given values of emissions tax and abatement R&D subsidy with respect to \( \beta \) is given by:
\[ \frac{\partial W^{ss}}{\partial \beta} = \frac{2(a-c-e_{0}t_{e})\eta^{2}((a-c)[1+\beta(1-\eta^{2})]\rho+e_{0}t_{e}(1+\beta)(1+\eta)\rho)}{[2+\eta+\beta(2-\eta)(1+\eta)]^{3}\rho} - \frac{2(a-c-e_{0}t_{e})\eta^{2}e_{0}\alpha}{[2+\eta+\beta(2-\eta)(1+\eta)]^{2}\rho} \] (51)

Recall that all parameters are positive, 0 \leq \eta \leq 1, 0 \leq \rho \leq 1, and, by assumption, \( a-c > e_0t_e \) and \((a-c)(r+\rho) > e_0\alpha \). Thus, it follows that \( \frac{\partial W^{ss}}{\partial \beta} > 0 \) when
\[(a-c)[1+\beta(1-\eta^{2})]\rho+e_{0}t_{e}(1+\beta)(1+\eta)\rho - e_{0}\alpha[2+\eta+\beta(2-\eta)(1+\eta)] > 0, \]
or
\[ a - c > \frac{e_{0}}{[1+\beta(1-\eta^{2})]\rho}\{\alpha[2+\eta+\beta(2-\eta)(1+\eta)] - t_e(1+\beta)(1+\eta)\rho\}. \]

Taking the limits on \( \beta \) (zero and infinity) of the above inequality’s right-hand side yields:

\[ \lim_{\beta \to -\infty} \frac{e_{0}}{[1+\beta(1-\eta^{2})]\rho}\{\alpha[2+\eta+\beta(2-\eta)(1+\eta)] - t_e(1+\beta)(1+\eta)\rho\} = \]
\[ = \frac{e_{0}}{(1-\eta)}\frac{\alpha(2-\eta)}{\rho} - t_e \]

\[ \lim_{\beta \to 0} \frac{e_{0}}{[1+\beta(1-\eta^{2})]\rho}\{\alpha[2+\eta+\beta(2-\eta)(1+\eta)] - t_e(1+\beta)(1+\eta)\rho\} = \]
\[ = e_{0}\frac{\alpha(2+\eta)}{\rho} - t_e(1+\eta) \]

Since \( \frac{e_{0}}{(1-\eta)}\frac{\alpha(2-\eta)}{\rho} - t_e > e_{0}\frac{\alpha(2+\eta)}{\rho} - t_e(1+\eta) \) for all admissible values of the variables, it follows that \( \frac{\partial W^{ss}}{\partial \beta} > 0 \) when \( t_e > \frac{\alpha(2-\eta)}{\rho} - \frac{(a-c)(1-\eta)}{e_0} \) (emissions tax greater than the optimal Bertrand emissions tax (44)).

It also follows that \( \frac{\partial W^{ss}}{\partial \beta} < 0 \) when \( t_e < \frac{\alpha(2-\eta)}{(1+\eta)(r+\rho)} - \frac{(a-c)(1-\eta)}{e_0} \) (emissions tax smaller than the optimal Cournot emissions tax (45)).

If \( \frac{\alpha(2+\eta)}{(1+\eta)(r+\rho)} - \frac{(a-c)(1-\eta)}{e_0} < t_e < \frac{\alpha(2-\eta)}{e_0(1+\eta)} - \frac{(a-c)(1-\eta)}{e_0} \), it follows that \( \frac{\partial W^{ss}}{\partial \beta} > 0 \) when \( \beta > \frac{e_0\alpha(2-\eta)-[a-c+e_0t_e(1+\eta)]\rho}{(1+\eta)(a-c)(1-\eta)^{2}-e_0\alpha(2-\eta)-e_0t_e} \).

**Proof of Proposition 4:** The proof is straightforward but included here for completion:
\[ \frac{\partial t_e^*}{\partial r} = -\frac{\alpha[2+\eta+\beta(2-\eta)(1+\eta)]}{(1+\beta)(1+\eta)(r+\rho)^{2}} < 0 \]
\[
\frac{\partial^2 t_e^*}{\partial r^2} = \frac{2\alpha[2 + \eta + \beta(2 - \eta)(1 + \eta)]}{(1 + \beta)(1 + \eta)(r + \rho)^3} > 0
\]
\[
\frac{\partial t_e}{\partial \rho} = \frac{\alpha[2 + \eta + \beta(2 - \eta)(1 + \eta)]}{(1 + \beta)(1 + \eta)(r + \rho)^2} < 0
\]
\[
\frac{\partial^2 t_e^*}{\partial \rho^2} = \frac{2\alpha[2 + \eta + \beta(2 - \eta)(1 + \eta)]}{(1 + \beta)(1 + \eta)(r + \rho)^3} > 0
\]
\[
\frac{\partial s^*}{\partial r} = \frac{\delta\{e_0\alpha[2 + \eta + \beta(2 - \eta)(1 + \eta)] + (a - c)[1 + \beta(1 - \eta^2)](\delta - \rho)\}}{e_0\alpha(1 + \beta)(r + \delta)^2(1 + \eta)(1 + \gamma\eta)} > 0
\]
if \(\delta > \rho\)
\[
\frac{\partial^2 s^*}{\partial r^2} = -\frac{2\delta\{e_0\alpha[2 + \eta + \beta(2 - \eta)(1 + \eta)] + (a - c)[1 + \beta(1 - \eta^2)](\delta - \rho)\}}{e_0\alpha(1 + \beta)(r + \delta)^3(1 + \eta)(1 + \gamma\eta)} < 0
\]
if \(\delta > \rho\)
\[
\frac{\partial s^*}{\partial \delta} = \frac{r\{(a - c)[1 + \beta(1 - \eta^2)](r + \rho) - e_0\alpha[2 + \eta + \beta(2 - \eta)(1 + \eta)]\}}{e_0\alpha(1 + \beta)(r + \delta)^2(1 + \eta)(1 + \gamma\eta)} > 0
\]
if \((a - c)(1 - \eta)(r + \rho) > e_0\alpha(2 - \eta)\)
\[
\frac{\partial^2 s^*}{\partial \delta^2} = -\frac{2r\{(a - c)[1 + \beta(1 - \eta^2)](r + \rho) - e_0\alpha[2 + \eta + \beta(2 - \eta)(1 + \eta)]\}}{e_0\alpha(1 + \beta)(r + \delta)^3(1 + \eta)(1 + \gamma\eta)} < 0
\]
if \((a - c)(1 - \eta)(r + \rho) > e_0\alpha(2 - \eta)\)
\[
\frac{\partial s^*}{\partial \rho} = \frac{(a - c)\delta[1 + \beta(1 - \eta^2)]}{e_0\alpha(1 + \beta)(r + \delta)(1 + \eta)(1 + \gamma\eta)} > 0
\]
\[
\frac{\partial^2 s^*}{\partial \rho^2} = 0
\]
\[
\frac{\partial d^{ss}}{\partial r} = \frac{2\alpha[e_0^2\delta^2\nu + (1 + \eta)(1 + \gamma\eta)^2]}{\delta^2\nu(1 + \eta)\rho(r + \rho)^2} > 0
\]
\[
\frac{\partial^2 d^{ss}}{\partial r^2} = -\frac{4\alpha[e_0^2\delta^2\nu + (1 + \eta)(1 + \gamma\eta)^2]}{\delta^2\nu(1 + \eta)\rho(r + \rho)^3} < 0
\]
\[
\frac{\partial d^{ss}}{\partial \delta} = \frac{4\alpha(1 + \gamma\eta)^2}{\delta^4\nu\rho(r + \rho)} > 0
\]
\[
\frac{\partial^2 d^{ss}}{\partial \delta^2} = -\frac{12\alpha(1 + \gamma\eta)^2}{\delta^4\nu\rho(r + \rho)} < 0
\]
\[
\frac{\partial d^s}{\partial \rho} = - \frac{2\{e_0 \delta^2 \nu[(a - c)(r + \rho)^2 - e_0 \alpha(r + 2\rho)] - \alpha(1 + \eta)(r + 2\rho)(1 + \gamma \eta)^2\}}{\delta^2 \nu(1 + \eta)\rho^2(r + \rho)^2} < 0
\]

\[
\frac{\partial^2 d^s}{\partial \rho^2} = 4\{(a - c)e_0 \delta^2 \nu(r + \rho)^3 - \alpha(2r^2 + 3r\rho + 3\rho^2)[e_0 \delta^2 \nu + (1 + \eta)(1 + \gamma \eta)^2]\} > 0
\]

\[
\frac{\partial W^s}{\partial r} = - \frac{2\rho^2(2r - \rho)[e_0 \delta^2 \nu + (1 + \eta)(1 + \gamma \eta)^2]}{\delta^2(1 + \eta)\nu\rho(r + \rho)^4} < 0
\]

\[
\frac{\partial^2 W^s}{\partial r^2} = \frac{2\rho^2(2r - \rho)}{\delta^2(1 + \eta)\nu\rho(r + \rho)^4} > 0 \text{ if } r > \frac{\rho}{2}
\]

\[
\frac{\partial W^s}{\partial \delta} = - \frac{2\rho^2(1 + \gamma \eta)^2(2r + \rho)}{\delta^2 \nu(1 + \rho)^2} < 0
\]

\[
\frac{\partial^2 W^s}{\partial \delta^2} = \frac{6\rho^2(1 + \gamma \eta)^2(2r + \rho)}{\delta^4 \nu(1 + \rho)^2} > 0
\]

\[
\frac{\partial W^s}{\partial \rho} = \frac{2\rho(2r - \rho)}{\delta^2(1 + \eta)\nu\rho^2(r + \rho)^3} < 0
\]

\[
\frac{\partial^2 W^s}{\partial \rho^2} = - \frac{2\rho(2r - \rho)}{\delta^2(1 + \eta)\nu\rho^3(r + \rho)^4} < 0
\]

References


