Caps on Coasean Transfers

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Abstract

We investigate the efficiency of Coasean bargaining when transfers between agents are capped. We model a two-stage Coasean environment where, in the first stage, property rights are costly to attribute. After the attribution stage agents voluntarily exchange over the level of harm. If property rights are attributed via an all-pay auction, then the introduction of a cap is Pareto improving. Using a Tullock contest we find a cap is Pareto inferior, but may increase Kaldor-Hicks efficiency. Applications include the analysis of tort law.

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1 Introduction

Given zero transaction costs, the Coase ‘theorem’ (1959; 1960) implies that efficiency is independent of initial property rights attribution. In many applications, however, full gains from exchange are not realized, which may result in allocative inefficiency. Although this is often due to the well-established case of high transaction costs, friction in Coasean bargaining may occur for a variety of reasons. The gains from exchange may be restricted or ‘capped’ due to judicial or regulatory influence, for example, by prescriptive laws and regulations.\(^1\) Alternatively, agents may experience a \textit{de facto} cap on transfers simply by being budget constrained. Given limitations to the gains from exchange, \textit{prima facie}, it seems that caps on Coasean transfers may preclude the efficient Coasean equilibrium. Yet are there any rational economic justifications for allowing ‘caps’ on Coasean rent transfers? In general, how do caps on rent transfers alter the overall efficiency of Coasean bargaining?

To answer these questions we model a two-stage Coasean environment. The first stage consists of a contest over property rights attribution, which is endogenously determined by agents’ costly efforts. Introducing such a stage contrasts with the standard Coasean paradigm, but is particularly plausible in many real-world situations. Property rights may be either initially non-existent, such as rights for common-pool resources, or insecure (perhaps ambiguously defined and/or costly to enforce).\(^2\) After property rights attribution, we introduce—in the second stage—a Coasean bargaining game where agents exchange compensation against a reallocation of the permissible level of the harmful activity. We find that the introduction of a cap on transfers may be Pareto and welfare improving. In particular, it turns out that the contest architecture—the process of liability attribution—is fundamental in determining Pareto and welfare improvements.

To model the endogenous attribution of property rights we develop two commonly used contest types with complete information: a first-price all-pay auction and a ‘Tullock contest.’ In both contests, agents choose costly effort to determine the initial endowment of property rights (rents). The distinction between both types of contest rests on the probability of success. In the all-pay auction, the agent with the highest effort wins with certainty, whereas in a Tullock contest the agent with the highest effort has a higher, but proportional, probability of success.

After property rights attribution, agents proceed with Coasean bargaining, but the transfer between agents may be capped, for example, by legislation or budget constraint. The establishment of a cap on negotiated transfers has two main effects. On the one hand, achieving the allocative optimum of the Coasean equilibrium may be precluded. On the other hand, the efficient Coasean equilibrium may exist but agents’ effort levels are reduced due to the cap on (overcompensatory) transfers. The respective trade-off between both effects determines overall efficiency. In principle, there are three different cases possible. First, the cap may be set so high that it is non-binding. In this case, transfers are unrestricted and the efficient Coasean equilibrium is unaffected by the cap. Second, the cap may be set so that overcompensatory rents are restricted but still existent: an efficient Coasean equilibrium is still feasible but rents (and therefore efforts) are now smaller. Lastly, a cap may be set at a sufficiently low level so that compensatory costs to reach an efficient Coasean equilibrium are not met. In this case, an allocatively inefficient equilibrium ex-

\(^1\)Close to the arguments within Coase (1960)—post-trial settlement within tort law—a common observation is the creation of laws that limit transfers between defendant and plaintiff (so-called “caps” on tort damages) (e.g., Dobbs, 2000; Kessler, 2011).

\(^2\)Although Coase (1960) abstracts from all types of transaction costs including socially wasteful appropriation efforts, Coase was well aware of the importance of the social costs of litigation. As explained in Coase (1959, p.27, footnote 54), “a waste of resources may occur when the criteria used by the courts to delimit rights result in resources being employed solely to establish a claim.”
ists. The overall efficiency is hence determined by both countervailing effects: the potential for an allocatively inefficient outcome versus the reduction in socially wasteful effort in appropriating the property right.

We find the establishment of a cap on negotiated transfers is usually Kaldor-Hicks efficient. Furthermore, we provide contrasting results for an all-pay auction and Tullock contest; a cap yields Pareto improvements for the former but not the latter. Interestingly, in the all-pay auction, the agent that experiences a cap on their gain from transfer will be \textit{ex ante} indifferent to its implementation. The intuition is as follows. A cap reduces the agent’s gross valuation of the property rights which, in turn, makes them a “weaker” player in the all-pay auction. Having the lower valuation in the contest means the agent’s expected utility will be equal to his loss from the expected transfer to the rival. Note that this is also the case with unrestricted transfers, where gross valuations of both players are identical. The agent’s expected utility is hence unaltered by the introduction of a cap. Yet, for the rival, introducing a cap implies that any transfers payable in the event of losing the contest are reduced. Even in an extreme case—where a cap precludes efficient Coasean bargaining—the reduction of expected rents transferred (and associated equilibrium efforts) more than offsets any loss from allocative inefficiency for both players. This is not the case within a Tullock contest, where a higher effort only implies an increase in the probability of winning, which will however always be lower than 1. As this contest structure implies a lower level of expected rent dissipation, capping an agent’s transfer gains will result in expected utility being lower under a Tullock contest.

Our findings provide additional insights to a number of applications, including—but not limited to—tort law reform. As within Coase (1960), property rights are mainly established through the stipulation of liability via legal court rulings (declaratory judgment (Bray, 2010)), an intuitive application of our framework is litigation. Interpreting the game as representing litigation with post-trial settlement yields insights on the efficiency of caps on tort law damages.\footnote{Post-trial activities are common and do have an influence on awarded damages. For 880 jury trials in Illinois and California, Shanley (1991) found that in around 25 per cent of all plaintiff verdicts, post-trial activities reduced payments. Cohen (2011) notes that in 2005 for 75 US counties, 28 per cent of civil trials experienced post-trial motions. For additional empirical analysis of motions and settlement see Boyd and Hoffman (2012).}

If litigation is adequately described by a contest with a high level of rent dissipation, any level of cap on damages will result in Pareto improvements. However, when litigation success is rather reflected by a probabilistic approach, where costs of litigation are \textit{ceteris paribus} lower, caps on damages are Pareto inferior. Yet, even in these cases, the use of caps can still be Kaldor-Hicks efficient.

Although a voluminous literature exists on the study of contests with respect to the attribution of property rights (Konrad, 2009), only a limited number of studies have investigated contests with “caps”. Che and Gale (1998) show that, in a first-price all-pay auction representing a lobbying game, setting an exogenous cap on efforts may result in an increase in aggregate expenditure and reduction in welfare.\footnote{For extensions and alternative (budget-constrained) measures see Baye et al. (1993), Laffont and Robert (1996), Gavious et al. (2002), Fang (2002), Kaplan and Wettstein (2006), and Faravelli and Stanca (2012).} Intuitively, capping agents’ efforts increases bidding competition within the all-pay auction; accordingly, this increases overall expenditure (and therefore reduces welfare). Yet, Che and Gale (1998) do not consider the potential exchange of rents (property rights) after the contest. The potential for voluntary exchange provides an alternative institutional setting in which a cap can be implemented, notably, on the negotiated transfers. We show such a cap reduces overall expenditure and thereby increases total surplus.

Only relatively recently have attempts been made to incorporate Coasean bargaining with endogenously determined property rights attribution via a contest. Robson and
Skaperdas (2008) include costly appropriation activity with Coasean bargaining. They show conditions for which a voluntary settlement is to be preferred before or after a potential court ruling as well as pre-commitment conditions for bargaining not to occur. However, they do not consider impediments to the Coasean bargaining game—the main objective of this article. MacKenzie and Ohndorf (2013) consider costly enforcement with Coasean bargaining and focus on the effects of a priori restrictions to the level of harm, such as constitutional laws, and show that it may be Pareto improving. Yet, these restrictions are delimitations of the action space, which only provide maximum and minimum permitted levels of a harmful activity. In contrast, we provide the first investigation into the capping of transfers between two agents when property rights attribution is endogenous and the subsequent rights are voluntarily exchanged.

The article is organized as follows. In Section 2 we model our game with uncapped transfers. Section 3 introduces a capped transfer and provides comparisons with the uncapped regime. In Section 4, the basic model is extended to include an alternative contest structure and Section 5 provides an application of our results. Section 6 provides some concluding remarks.

2 The model

Consider a situation where agent X imposes harm on agent Y. Agent X can reduce harm by choosing an avoidance level \( a \in [0, \bar{a}] \), where \( \bar{a} < \infty \) is a maximum level of avoidance at which agent Y’s costs from harm are reduced to 0. For any \( a \in [0, \bar{a}] \), agent Y suffers harm \( D : [0, \bar{a}] \rightarrow \mathbb{R}_+ \) with \( \frac{\partial D}{\partial a} < 0, \frac{\partial^2 D}{\partial a^2} \geq 0 \), and \( D(\bar{a}) = \frac{\partial D}{\partial a} \bigg|_{a=\bar{a}} = 0 \). We allow uncertainty over agent X’s private costs of avoidance. These costs are initially unknown to a third-party authority, but realized and known by agents X and Y prior to any decisions on the part of both players. The third-party authority’s uncertainty over agent X’s future cost of avoidance is reflected via a random variable \( \theta \) on the support \([\underline{\theta}, \bar{\theta}]\), with distribution function \( F(\theta) \), and corresponding density function \( f(\theta) \). Agent X’s private costs are \( C : [0, \bar{a}] \times [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}_+ \) with the following properties: \( \frac{\partial C}{\partial a} > 0, \frac{\partial^2 C}{\partial a^2} \geq 0, \frac{\partial C}{\partial \theta} > 0 \), and \( C(0, \theta) = \frac{\partial C}{\partial a} \bigg|_{a=0} = 0 \) for all \( \theta \in [\underline{\theta}, \bar{\theta}] \). We denote the maximum cost to X as \( \tilde{C}(\theta) = C(\bar{a}, \theta) \) for \( \theta \in [\underline{\theta}, \bar{\theta}] \) and for agent Y, \( \tilde{D} = D(0) \).

Similar to the logic of Coase (1960), we allow for the establishment of property rights, which are divisible over the entire range of harmful activity \([0, \bar{a}]\). Instead of assuming the initial property rights attribution to be exogenously determined and costless, we provide a contest mechanism that endogenously attributes the initial endowment of property rights entirely either to agent X or Y based on effort. Hence, by expending effort \( x \) and \( y \) respectively, agent \( j \in \{X, Y\} \) can influence the probability \( p_j(x, y) \) of winning the property rights. Without Coasean bargaining, costs to X would correspond to \( \tilde{C}(\theta) \) for a realized \( \theta \in [\underline{\theta}, \bar{\theta}] \) if Y holds the property right. Conversely, costs to Y are \( \tilde{D} \) if agent X owns the property right for \( \theta \in [\underline{\theta}, \bar{\theta}] \). In the main part of the article, we assume the contest to be represented by a first-price all-pay auction with complete information, but extend our analysis to a Tullock contest in Section 4.

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5See also Dixit and Olson (2000) and Anderlini and Felli (2001, 2006) with respect to costly ex ante choices to participate in voluntary exchange. They do not, however, analyze the enforcement of property rights nor the implementation of a cap on transfers.

6Allowing for uncertainty over agent Y’s costs (either separately or combined with uncertainty over agent X’s costs) provides identical results, but is omitted here for the sake of simplicity.
Assumption 1 The contest success function is given by \( p_X(x, y) \) where

\[
p_X(x, y) = \begin{cases} 
1 & \text{if } x > y \\
\frac{1}{2} & \text{if } x = y \\
0 & \text{if } x < y 
\end{cases}
\]  

\((1)\)

and \( p_Y(x, y) = 1 - p_X(x, y) \) for all \( x, y \in [0, \infty] \).

From Assumption 1 the initial endowment of property rights are won by the agent with the largest effort with probability 1. The contest structure can be interpreted in many ways. First, it can be considered as an allocation mechanism for existing property rights, such as a court or political lobbying. Second, the contest can be thought of as a mechanism for agents to firmly establish ownership of a property right, which is either insecure or currently non-existent, such as competition for control of common-pool resources. Thus harm is not initially contractible. The contest, therefore, provides the elimination of this uncertainty via stipulation of liability, and with it, future exchange is now feasible.

The attribution of property rights thus allows both agents the option to enter into a bargaining game as depicted by Coase (1960). In particular, as the property right is divisible, the bargaining game is over the level \( a \in [0, \bar{a}] \). As illustrated by Coase (1960), this is likely in many tortuous situations including nuisance torts where the abatement of harm is feasible. Bargaining, therefore, occurs over the level of harm and its converse—harm avoidance. We define the Coasean bargaining outcome as follows:

Definition 1 Let \( a^*(\theta) = \arg \min_{a \in [0, \bar{a}]} \{ C(a, \theta) + D(a) \} \) be the (ex post) efficient Coasean bargaining equilibrium for some realized \( \theta \in [\underline{\theta}, \bar{\theta}] \).

The above-described bargaining outcome is general enough to allow the analysis of a host of alternative bargaining procedures, such as Nash and alternating-offers equilibria. The existence of an interior solution \( a^*(\theta) \) is provided due to the above-made assumptions on agents’ costs.

In the bargaining process, it is generally assumed that there is no impediment to the transfer of rents. However, as argued in the introduction, it is often the case that transfers between agents are capped, either as an institutional restriction (e.g., law) or as an exogenous restriction on the individual agent (e.g., budget constraint). For sake of clarity, we focus on the former and investigate the potential for Pareto and welfare-improvements. Such improvements are not guaranteed: although a capping of transfers—and hence of achievable rents—will ultimately reduce socially wasteful appropriation efforts, transfers might also be restricted to undercompensatory levels, which precludes attaining the efficient Coasean equilibrium. Before providing the specific details regarding the cap on transfers, it is useful to note that caps are usually set before both parties enter into the contest for property rights (see Section 5 for an extensive discussion of relevant applications). In such a setup, it is plausible that the cap-setting authority has initial uncertainty over the costs and, hence, the ultimate level \( a^*(\theta) \).

We can now detail the timeline of our game. From Figure 1, at \( t = -1 \) a transfer cap is implemented, for example, as a new rule of law or amendments within a constitution. We treat this as exogenous. At \( t = 0 \) the uncertainty over costs is realized and this information becomes common knowledge. At \( t = 1 \), agents invest effort to obtain the initial endowment of property rights and at \( t = 2 \), with property rights attributed, enter into Coasean bargaining. Our two-stage game thus takes place in periods \( t = 1, 2 \). Before initiating our discussion on capped transfers, we first begin with the simpler case where no cap exists.
Hence, the internalization rent is calculated by subtracting \( C \) from the loser’s costs after bargaining has occurred. The loser’s post-bargaining costs consist of his costs without Coasean bargaining (i.e., their expected reservation payoffs \( \bar{\mu} \)). To allow for a wide range of bargaining setups, we denote winner \( j \)'s relative share of rent \( I_j^u \), where superscript “\( u \)” denotes the uncapped regime. This is defined as:

\[
I_j^u(\theta) \equiv \bar{D} - (C(a^*(\theta), \theta) + D(a^*(\theta))) \quad \text{if agent X wins the contest,} \tag{2}
\]

\[
I_j^u(\theta) \equiv \bar{C}(\theta) - (C(a^*(\theta), \theta) + D(a^*(\theta))) \quad \text{if agent Y wins the contest.} \tag{3}
\]

The overall internalization rent (bargaining space) is the difference between the loser’s costs without Coasean bargaining (i.e., their expected reservation payoffs \( \bar{D}, \bar{C}(\theta) \)) and the loser’s costs after bargaining has occurred. The loser’s post-bargaining costs consist of his own costs at \( a^* \), as well as the winner’s compensation for the costs incurred at \( a^* \). The latter is a necessary precondition for successful bargaining, as the owner of the property right would refrain from exchange if he is not fully compensated for the corresponding costs. Hence, the internalization rent is calculated by subtracting \( C(a^*(\theta), \theta) + D(a^*(\theta)) \) from the loser’s costs without bargaining.

The sharing of the internalization rent is dependent on the underlying bargaining game. To allow for a wide range of bargaining setups, we denote winner \( j \)'s relative share of rent as \( \mu_j \in [0,1] \). The value of this sharing parameter is then dependent on both players’ relative bargaining power. We can now define the winners’ rent \( R_j^u \), and the corresponding loser’s rent \( z_j^u \), as follows:

\[
R_j^u(\theta) \equiv \mu_X \cdot I_j^u(\theta) \geq 0, \quad z_j^u(\theta) = (1 - \mu_X) \cdot I_j^u(\theta) \geq 0, \tag{4}
\]

\[
R_j^u(\theta) \equiv \mu_Y \cdot I_j^u(\theta) \geq 0, \quad z_j^u(\theta) = (1 - \mu_Y) \cdot I_j^u(\theta) \geq 0. \tag{5}
\]

where (4) and (5) represent the net rents obtained by both agents when agents X and Y win the contest, respectively. When \( \mu_j = 1 \) (\( \mu_j = 0 \)) the winner (loser) of the contest captures the entire internalization rent.

By use of backward induction, we can now determine the agents’ expected payoffs before entering into the contest over property rights. For any realization of \( \theta \), the expected

\[7\] A declaratory judgment, therefore, provides liability determination without awarding damages (Bray, 2010). Such judgments are common in tort law. The court has the ability to rule—in terms of a preventative adjudication—if a potential action (e.g. potential trespass, patent infringement, noise nuisance, and so on) is tortuous or not. Once liability has been determined, bargaining is frequently observed. For example, when a patent has been infringed, in many cases, the defendant and plaintiff are able to come to a mutually beneficial agreement over patent use with a negotiated license (Chiang, 2013). For rent seeking or conflict (war of attrition), the appropriation of resources to the winner may, over time, provide potential benefits of future exchange (e.g. the sale of land or resources).
payoffs are:

\[
U_X^i(x,y;\theta) = p_X(x,y)R_X^i(\theta) - (1 - p_X(x,y))(C(a^*(\theta),\theta) + D(a^*(\theta)) + R_X^i(\theta)) - x, \quad (6)
\]

\[
U_Y^i(x,y;\theta) = p_Y(x,y)R_Y^i(\theta) - (1 - p_Y(x,y))(C(a^*(\theta),\theta) + D(a^*(\theta)) + R_Y^i(\theta)) - y. \quad (7)
\]

The accumulated costs under bargaining—even for the loser of the property right in the contest—are always smaller than for the no-bargaining case. Thus, in our analysis, there always exists an incentive to enter into a bargaining process.

For the contest success function specified by (1), agents’ payoff functions (6) and (7) are of the general form \( p_j(x,y)W_j(\theta) + (1 - p_j(x,y)L_j(\theta) \), with \( j \in \{X,Y\} \) where \( W_j(\theta) \) and \( L_j(\theta) \) are the respective awards and losses, which can be re-arranged to:

\[
p_X(x,y)v_X(\theta) - L_X(\theta) - x, \quad (8)
\]

\[
p_Y(x,y)v_Y(\theta) - L_Y(\theta) - y, \quad (9)
\]

where \( v_j(\theta) = W_j(\theta) + L_j(\theta) \) denotes the gross value to agent \( j \) of winning relative to not participating in the contest. For this formulation, the solution to the all-pay auction is well known (Hillman and Riley, 1989; Baye et al., 1996). As shown in the appendix, the corresponding Nash equilibrium is in mixed strategies and expected utilities of agent \( X \) and \( Y \) are, respectively:

\[
U_X^i(\theta) = E\left[-\bar{C}(\theta) + (1 - \mu_Y)I_X^i(\theta)\right], \quad (10)
\]

\[
U_Y^i(\theta) = E\left[-\bar{D} + (1 - \mu_X)I_Y^i(\theta)\right]. \quad (11)
\]

## 3 Capping Transfers within a Coasean bargaining game

We now investigate the effects of a cap on transfers from agent \( X \) to agent \( Y \) that is effectively enforced by some authority. In tort law, for example, such caps on plaintiff-awarded damages are common (Dobbs, 2000). Stipulating maximum levels of transferable damages via court ruling, will, in our framework, restrict the feasible rents of bargaining via altered reservation utilities. For simplicity, we assume that agents’ relative bargaining powers are unaltered by the introduction of a cap. Hence, we continue to represent the sharing of rents by the same parameter \( \mu_j \) for \( j \in \{X,Y\} \).

The cap on transfers is assumed to be stipulated \emph{a priori} to the two-stage game, i.e., at time 0 of the timeline presented in Figure 1. At this point in time the uncertainty over the actual cost, and hence over \( a^* \), is still not resolved. As a consequence, the cap can only be formulated as a restriction on the overall gross transfer from \( X \) to \( Y \). We hence define this cap as follows: let \( K \) be an absolute cap stipulated \emph{a priori} on the total transfers that are obtained by agent \( Y \) if he wins the contest.

As \( K \) is a restriction on the total transfer from \( X \) to \( Y \), there exist three possible cases if agent \( Y \) wins the contest. First, the cap might be large enough not to affect transfers in the Coasean bargaining stage at all. In this case the corresponding restriction is simply non-binding, i.e., if \( K > R_Y^i(\theta) + D(a^*(\theta)) \). Second, there is the possibility that the cap

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8For an additional cap on the transfer from agent \( Y \) to agent \( X \), the same logic applies. Identical incentives are present when caps on transfers for both agents are introduced: one must consider the binding and non-binding nature of both caps as well as their potential changes to effort levels.

9The results presented below also hold in principle for changes in parameter \( \mu_j \) in that our findings are now additionally dependent on the changes in \( \mu_j \) before and after the introduction of a cap. By comparing expected utilities before and after the introduction of a cap, a range of post-cap \( \mu_j \) levels can be calculated (with respect to the pre-cap \( \mu_j \)) that satisfy our results.
restricts agent Y’s share of the residual rent, while still being large enough to compensate him for costs $D(a^*(\theta))$ that arise at the Coasean equilibrium. Clearly, this situation arises if $D(a^*(\theta)) < K < R_Y^u(\theta) + D(a^*(\theta))$. This case is, in fact, the one intended by a benevolent authority, as such a reduction in transferable rent will ultimately reduce socially wasteful appropriation efforts. There exists, however, a third possibility: if $K < D(a^*(\theta))$, the cap restricts transfers to undercompensatory levels. In this case the efficient Coasean equilibrium is effectively precluded, as Y will not agree to avoidance level $a^*$ without being compensated for the associated costs. Instead, both agents will settle on an allocatively inefficient level of $a$.

**Definition 2** For a given cap $K$, define the following ex post threshold levels:

- Denote with $\theta_K$ the level of $\theta$ for which $D(a^*(\theta_K)) = K$. Then, transfer $K$ is defined as “undercompensatory” if $\theta \in [\theta_K, \bar{\theta}]$.
- Denote with $\theta_{R_Y}$ the level of $\theta$ for which $\tilde{K} = R_Y^u(\theta_{R_Y})$, where
  
  $$\tilde{K}(\theta) \equiv K - D(a^*(\theta)).$$

Then, transfer $K$ is defined as “overcompensatory with restricted rent” if $\theta \in [\theta_{R_Y}, \theta_K]$.

Hence, for any cap $K$, there can exist a maximum of three different regions of $\theta$ that are to be considered: $[\theta, \theta_{R_Y}]$, $[\theta_{R_Y}, \theta_K]$, and $[\theta_K, \bar{\theta}]$. For a range of $\theta$ where all of these regions exist, expected payoffs (denoted with a superscript “$c$”) are generally formulated as:

$$U^c_X(\theta) = \int_{\theta}^{\theta_{R_Y}} U^c_X(\theta) f(\theta) \, d\theta + \int_{\theta_{R_Y}}^{\theta_K} U^c_X(\theta) f(\theta) \, d\theta + \int_{\theta_K}^{\bar{\theta}} U^c_X(\theta) f(\theta) \, d\theta,$$

$$U^c_Y(\theta) = \int_{\theta}^{\theta_{R_Y}} U^c_Y(\theta) f(\theta) \, d\theta + \int_{\theta_{R_Y}}^{\theta_K} U^c_Y(\theta) f(\theta) \, d\theta + \int_{\theta_K}^{\bar{\theta}} U^c_Y(\theta) f(\theta) \, d\theta,$$

where superscripts “$l$,” “$m$,” and “$w$” denote utilities within each region of $\theta$, respectively.

Depending on the relative stringency of cap $K$, there might also be cases where $\theta_{R_Y}$ and/or $\theta_K$ lie beyond the support of $\theta$. In this case the expected utilities are reduced to the respective intervals that are still relevant. Let us now derive all potential components of $U^c_X(\theta)$ and $U^c_Y(\theta)$ by considering each region of $\theta$ while temporarily assuming that both thresholds lie within the distribution’s support.

In the region $[\theta, \theta_{R_Y}]$, the cap is larger than the transfer under the unconstrained case, i.e., $K > R_Y^u(\theta) + D(a^*(\theta))$, such that $K$ does not influence the agents’ payoffs. As bargaining powers are constant, the values of the $R_j^u$ remain the same. Hence, both agents’ payoffs are analogous to (6) and (7) for any realization of $\theta$ in this range.

$$U^c_X(x,y;\theta) = p_X(x,y)(R^u_X(\theta)) - (1 - p_X(x,y))(R^u_X(\theta) + C(a^*(\theta),\theta) + D(a^*(\theta))) - x,$$

$$U^c_Y(x,y;\theta) = p_Y(x,y)(R^u_Y(\theta)) - (1 - p_Y(x,y))(R^u_Y(\theta) + C(a^*(\theta),\theta) + D(a^*(\theta))) - y.$$

More interesting cases arise in those areas where the constraint on transfers is binding. The authority’s intention when introducing such a cap is probably the reduction of overall effort spent by reducing agent Y’s share of the internalization rent. More precisely, the regulator intends to induce the above-mentioned case of restricted overcompensation to reduce socially wasteful appropriation costs. As shown above, this case arises for realizations
of \( \theta \) in the interval \( ]\theta_{R^Y}, \theta_K) \), where \( K \) restricts agent Y’s residual rent in case of winning, while the efficient Coasean equilibrium remains feasible. The corresponding payoffs in this range of \( \theta \) are:

\[
\begin{align*}
U^w_X(x, y; \theta) &= p_X(x, y)(R^w_X(\theta)) - (1 - p_X(x, y))(K + C(a^*(\theta), \theta)) - x, \\
U^w_Y(x, y; \theta) &= p_Y(x, y)(\bar{K}(\theta)) - (1 - p_Y(x, y))(R^w_X(\theta) + C(a^*(\theta), \theta) + D(a^*(\theta))) - y.
\end{align*}
\]  

(17)  

(18)

Hence, agent Y’s net gains from winning are effectively restricted to the level of \( K \), although the cap does not affect the payoffs in case agent X wins the property right.

In the last interval, i.e., \( \theta \in ]\theta_K, \overline{\theta} \), the introduction of a sufficiently restrictive cap results in a reduction in allocative efficiency. Within this range of \( \theta \), when agent X loses (Y wins), obtaining the efficient Coasean equilibrium is precluded due to undercompensation. As in these cases \( D(a^*(\theta)) > K \), player Y cannot be compensated for his costs incurred at the equilibrium level. As a consequence, agent Y would simply not agree to a reduction of \( a \) to its efficient level. Still, there is some internalization rent to be gained. As a consequence, there will be bargaining over some reduced (but suboptimal) level of avoidance \( a_L \).

Successful bargaining then means that agent Y agrees to incur the cost \( D(a_L) \) in exchange for the allowed compensation \( K \), transferred by agent X. The expected payoffs in the interval \( \theta \in ]\theta_K, \overline{\theta}) \) are:

\[
\begin{align*}
U^w_X(x, y; \theta) &= p_X(x, y)(R^w_X(\theta)) - (1 - p_X(x, y))(K + C(a_L, \theta)) - x, \\
U^w_Y(x, y; \theta) &= p_Y(x, y)(K - D(a_L)) - (1 - p_Y(x, y))(R^w_X(\theta) + C(a^*(\theta), \theta) + D(a^*(\theta))) - y.
\end{align*}
\]  

(19)  

(20)

The efficient Coasean solution is only precluded if agent X loses. If agent X wins, bargaining will yield the efficient level \( a^* \), as only X’s transfer is capped by \( K \).

Note further that \( a_L \) is the outcome of bargaining between both agents. Hence, the domain of \( a_L \) is limited by the reservation points of both agents. Agent X would only accept to transfer \( K \) if the corresponding reduction in costs are at least as large. Conversely, agent Y will only agree to an \( a_L \) if his costs are covered by the transfer of \( K \). Hence, we can establish the following definition of reservation points.

**Definition 3** For \( \theta \in ]\theta_K, \overline{\theta} \), the bargaining reservation points \( \{a_L, \pi_L\} \), are implicitly defined by \( D(a_L) = K \) and \( \bar{C}(\theta) - C(\pi_L, \theta) = K \), which are the lower bounds of the participation constraints of agents Y and X, respectively.

The overall residual rent in the region \( \theta \in ]\theta_K, \overline{\theta} \] is \( \bar{C}(\theta) - C(a_L, \theta) - D(a_L) \). Hence, interestingly, the size of the rent itself is dependent on the sharing of the internalization rent resulting from bargaining over \( a_L \). The maximum size of the rent can be achieved if X holds all the bargaining power. The corresponding level of avoidance is then \( \overline{a_L} \). The rent is smallest if Y holds all the bargaining power and X’s participation constraint holds with equality.

Generally, X’s share \( z^X_L \) and agent Y’s share \( R^Y_L \) of internalization rent are:

\[
z^X_L = \bar{C}(\theta) - C(a_L, \theta) - K, \quad R^Y_L = K - D(a_L), \quad \text{with } a_L \in [\overline{a_L}, \pi_L].
\]  

(21)

In this case the sharing of the internalization rent is independent of the monetary transfer, which always amounts to \( K \). Instead, distribution of rent is solely determined by the agreed level of \( a_L \). Hence, using sharing parameter \( \mu_Y \), we can rewrite the shares of rent.
defined in (21) as follows:

\[ R_Y^c = \mu_Y \cdot [(C(\bar{a}_L(\theta), \theta) - D(\bar{a}_L(\theta))) - (C(a_L, \theta) - D(a_L))] , \quad (22) \]
\[ z_X^c = (1 - \mu_Y) \cdot [(C(\bar{a}_L(\theta), \theta) - D(\bar{a}_L(\theta))) - (C(a_L, \theta) - D(a_L))] . \quad (23) \]

After having established the different payoff components, we will now proceed with comparing agent’s payoffs under a cap with the unconstrained case.

3.1 Comparison: Pareto improvements

We now turn our attention to the Pareto- and welfare-improving aspects of a cap on transfers. First, let us define the gross valuation of agent \( X \) and \( Y \) in the contest by \( v_X^c, v_Y^c \), respectively. The gross valuations of each agent are the sum of possible benefits and losses avoided from winning the contest. We then have the following lemma.

**Lemma 1** For a cap \( K \) and gross contest valuations \( v_X^c(\theta), v_Y^c(\theta) \) for agents \( X \) and \( Y \), respectively, if \( \theta \in [\theta, \theta_K] \) then \( v_X^c(\theta) = v_Y^c(\theta) \), otherwise, if \( \theta \in [\theta_K, \bar{\theta}] \) then \( v_X^c(\theta) > v_Y^c(\theta) \). Also, the ranking of agents’ gross valuations of the Coasean game is independent of agents’ (relative) costs.

**Proof.** See Appendix. ■

When a cap does not preclude efficient Coasean bargaining, i.e., for \( \theta \in [\theta, \theta_K] \), Lemma 1 shows that agents’ gross valuations are identical. When a cap is sufficiently strict to preclude full (efficient) Coasean bargaining, we observe asymmetry in agents’ gross valuations. In particular, the agent which experiences the cap on transfers (here agent \( Y \)) has a lower valuation than the remaining agent. This is independent of bargaining powers: even if agent \( Y \) held all bargaining power, their gross valuation would continue to be smallest.

Gross valuations are, in fact, independent of agents’ costs. When \( \theta \in [\theta, \theta_K] \) agents’ valuations are identical and any change in costs (for either agent) will result in an equivalent shift in both gross valuations. For \( \theta \in [\theta_K, \bar{\theta}] \), although gross valuations are asymmetric, any change in the composition and level of costs will maintain the ranking of gross valuations. This follows from \( v_X^c(\theta) > v_Y^c(\theta) \iff C(a_L, \theta) + D(a_L) > C(a^*(\theta), \theta) + D(a^*(\theta)) \), where the latter inequality will always remain strict given our assumptions on the cost functions.

Having established \( v_X^c(\theta) \geq v_Y^c(\theta) \) in Lemma 1, we can calculate each agent’s expected payoffs for capped transfers for each component of (13) and (14). It follows that in an all-pay auction with a cap \( K \) on rent transfers, agents’ expected utilities are given by:

\[ U_X^c(\theta) = \int_{\theta}^{\theta_K} (-\bar{C}(\theta) + (1 - \mu_Y)I_X^c(\theta)) f(\theta) \, d\theta + \int_{\theta_K}^{\bar{\theta}} (-K - C(a^*(\theta), \theta)) f(\theta) \, d\theta + \int_{\theta}^{\theta_K} (K + D(a_L)) f(\theta) \, d\theta, \]

\[ U_Y^c(\theta) = E_{\bar{D}}([-\bar{D} + (1 - \mu_X)I_X^c(\theta)]. \quad (24) \]

The first integral in (24) reflects the case where the transfer from \( X \) to \( Y \) is unconstrained by the cap. The second integral reflects the intermediate case where this transfer is capped but the cap remains large enough to be overcompensatory. If transfers are restricted to undercompensatory levels, then this is reflected in the third integral of (24). Comparison of (11) and (25) provides an immediate proposition.

**Proposition 1** In an all-pay auction with a cap \( K \) on rent transfers, \( U_X^c(\theta) = U_Y^c(\theta) \) for \( \theta \in [\theta, \bar{\theta}] \).
Proposition 1 provides a counter-intuitive result: if an agent has his potential transfer capped—in this case agent $Y$—then his *ex ante* expected utility level will be identical to that experienced when no cap on transfers is implemented. The logic behind this result is as follows. If agent $Y$’s potential transferable rent is capped, then from Lemma 1 agent $Y$ has a (weakly) lower gross valuation than without a cap. From the mixed strategies associated with the all-pay auction, his expected utility loss will be equal to the transfer loss to agent $X$. As agent $X$’s transfer is not capped—i.e., identical to the uncapped case—it follows that agent $Y$’s expected payoff will be identical.

From (10) and (24) we define

$$
\Phi \equiv U_c^X(\theta) - U_u^X(\theta)
$$

(26)

as the relative expected benefit of a cap to agent $X$. Using $\Phi$ we can show the following proposition.

**Proposition 2** In an all-pay auction with a cap $K$ on rent transfers, $\Phi > 0$, which yields (weak) Pareto improvements.

**Proof.** See Appendix.

Pareto improvements can be observed by comparing the relevant expected utility functions for agent $X$ in (10) and (24). When the cap is non-binding, agent $X$’s expected utility is identical under both regimes. For $\theta \in [\theta_K, \theta_L]$, although Coasean compensation is fulfilled, the total rent transfer is now bounded by the cap. In this case, agent $X$’s expected utility is unambiguously larger if transfers are restricted. Intuitively, the cap reduces the transfer from agent $X$ to agent $Y$, which clearly benefits agent $X$. Surprisingly, for the case where there exists undercompensation, agent $X$’s expected utility is larger under a cap—even though an efficient Coasean equilibrium is precluded. As shown in the proof of Proposition 2, the transferable rent from agent $X$ to agent $Y$ is lower under a capped regime, which results in agent $X$’s expected utility being higher under a capped regime.

We, therefore, have shown that if property rights are attributed via an all-pay auction and there exists potential Coasean bargaining then it is Pareto-improving to cap an agent’s Coasean rent transfer. In fact, Proposition 2 suggests that even if a cap is established that precludes Coasean bargaining *ex post* (i.e., the realized value of $\theta \in [\theta_K, \theta_L]$), it is always Pareto improving to maintain the cap. This means that the reduction in contest effort outweighs the gains from trade achievable within the Coasean bargaining game. Although the absolute Pareto improvements are largest for the case of restricted overcompensation (i.e., the second integral in (24)), a cap remains (weakly) Pareto-improving independent of the level of $\theta$ that is ultimately realized.

It is important, however, to discuss the caveats of this result. First, a sufficient condition for this result to hold is that relative rent shares between different regimes must be constant. Intuitively it makes sense to assume some form of consistency, as the relative sharing in rent is likely to be independent of the game itself. Yet, even when relative sharing consistency does not hold, Pareto improvements still may exist. Comparing (10) and (24) provides a
range of parameters where Pareto improvements exist. Notably, the key factors are the relative size of \((1 - \mu)^{I_u} I_u \geq 0\) compared to a different \(z^c_{X}\) in the first integral as well as \(K - D(a(\theta))\) in the third integral of (24). Low enough values of \((1 - \mu^{I_u}) I_u\) and \(K - D(a(\theta))\) would result in unambiguous Pareto improvements. Furthermore, it is quite obvious that if both integrals do not exist or are associated with a low enough probability weighting, a cap would also lead to (quite significant) Pareto improvements. In other cases, however, Pareto improvements might not be achieved.

A second caveat might be associated with the fact that the all-pay auction mechanism distributes property rights with probability 1 to the agent with the highest effort level. Under an all-pay auction the contestable rents are often significantly dissipated by the aggregate effort invested. Clearly, the larger efforts are, the larger improvements can exist for the implementation of a cap. To show this precisely, we now extend our model to an alternative contest mechanism, which uses a probability of winning the property right as a proportional share of effort.

4 An alternative contest structure

We have previously assumed that the mechanism that allocated property rights was a first-price all-pay auction. In this section, we relax this assumption by introducing another frequently used contest, namely, a Tullock contest (Tullock, 1975, 1980).

Assumption 2 The contest success function is given by \(p_X(x, y)\) where

\[
p_X(x, y) = \begin{cases} 
\frac{x^r}{x^r + y^r} & \text{if } \max\{x, y\} > 0, \\
\frac{1}{2} & \text{otherwise,} 
\end{cases}
\]

(27)

with \(r \leq 1 + \left(\frac{v_Y}{v_X}\right)^r\) where \(v_X, v_Y\) are the gross valuations for agents X and Y respectively, and \(p_Y = 1 - p_X(x, y)\).

The probability of winning is now proportional to effort. In contrast to a setup with an all-pay auction, assuming a Tullock contest implies that the agent with the highest effort no longer wins with certainty. Assumption 2 limits the Tullock contest to relatively small marginal returns from effort, i.e., relatively small \(r\). Note, however, that if this assumption is relaxed so that \(r > 2\), then the use of a Tullock contest produces expected equilibrium efforts and payoffs identical to the results of the all-pay auction as shown in Sections 2 and 3 (see Alcalde and Dahm, 2010). Given Assumption 2, we obtain the following proposition regarding Pareto improvements.

Proposition 3 Under a Tullock contest with a cap \(K\) on rent transfers, the implementation of a cap is not Pareto-improving.

Proof. See Appendix. ■

In contrast to a setup with an all-pay auction, a cap is Pareto inferior when the allocation of property rights is modeled as a Tullock contest. In the Tullock contest, expected aggregate efforts are less than what would be experienced in an all-pay auction. A cap, therefore, has less scope to reduce efforts and, consequently, a loss of expected utility for agent Y is encountered. Although Pareto improvements have been shown not to exist this does not preclude the possibility of welfare improvements along the lines of the Kaldor-Hicks criterion.
Proposition 4 Under a Tullock contest, the relative welfare (Kaldor-Hicks) improvement $\Delta_T$ of implementing a cap is

$$
\Delta_T = \frac{1}{2} \int_{\theta_K}^{\theta_Y} \left( R^u_Y(\theta) - \tilde{R}(\theta) \right) f(\theta) \, d\theta + \int_{\theta_K}^{\theta_Y} \left( \frac{v^c_X(\theta) + v^c_Y(\theta) }{ (1+\gamma)^2 } - v^c_X(\theta) + \frac{v^u(\theta) }{2} (2-r) \right) f(\theta) \, d\theta
$$

(28)

where $\gamma = \frac{\tilde{v}^c}{\tilde{v}^c}, \, v^c_X(\theta) = R^u_X(\theta) + K + C(a_L), \, v^c_Y(\theta) = R^u_X(\theta) + K - D(a_L) + C^*(a^*(\theta),\theta) + D(a^*(\theta)), \, \text{and} \, v^u(\theta) = R^u_X(\theta) + R^u_Y(\theta) + C^*(a^*(\theta),\theta) + D(a^*(\theta)).$

Proof. See Appendix. ■

Proposition 4 shows that Kaldor-Hicks improvements may exist under a Tullock contest. The first term in Proposition 4 is unambiguously positive, whereas the second term is ambiguous. It follows that welfare improvements exist under a number of circumstances. When $K$ is set such that an allocatively inefficient outcome is unlikely or non-existent, then $\Delta > 0$. Essentially, this requires that the probability distribution of $\theta$ to have “small tails.” Further, the asymmetric valuations of agents, represented by parameter $\gamma$, have an important role. For example, one can see as asymmetry increases, $\gamma \rightarrow 0$, then $\Delta$ increases.

The fact that, given a Tullock contest, a cap might yield Kaldor-Hicks improvements but not Pareto improvements, is worth some discussion. As shown above, the introduction of a cap increases the sum of overall payoffs (Kaldor-Hicks improvement), but agent $Y$ would fare worse than under unrestricted transfers, i.e., a cap would be Pareto inferior. One of the most prominent justifications for using the Kaldor-Hicks criterion as a rule to evaluate policy changes is that the winner might compensate the loser ex post via a mechanism of re-distribution that is exogenous to the policy changes considered. This compensation does not necessarily have to be actually realized for the Kaldor-Hicks criterion to hold. For this reason, the Kaldor-Hicks criterion is also often referred to as ‘potential Pareto-superiority’ (Coleman, 1980). Yet, if there is indeed the possibility to provide for such compensations ex post, Kaldor-Hicks efficiency can be equivalent to Pareto efficiency.

Note, however, that establishing such an equivalence is not possible for the sort of situations modeled here. Any ex post mechanism that redistributes wealth from $X$ to $Y$ subsequent to the game would yield a change in ultimate payoffs, which would effectively offset the effect of capping the transfer in the first place. Hence, it is not possible to conceive of such ex post compensations without increasing the stakes of the property rights attribution game, which will in turn lead to higher socially wasteful attribution efforts. As a consequence, assuming reasonable foresight of actors with clearly assigned roles in the game, the Kaldor-Hicks improvement derived above is never equivalent to a Pareto improvement. Hence, introducing a cap on Coasean transfers will always be associated with issues of distributional justice.

Distributional concerns might be reduced if it is assumed that the game is repeated with changing roles of the agents over an unforeseeable time horizon. That is, if there is a sufficient probability that any individual will benefit in the long run from such a change, i.e., any actor might take the position of agent $X$ at some point. Yet, if the possibility of changing roles is precluded, the introduction of a cap as modeled here corresponds to one of those institutional changes where the overall size of the pie (i.e., the sum of payoffs) is not independent of its distribution (Rawls, 1971).
5 Applications

Our analysis applies to situations with costly property right attribution followed by (capped) Coasean exchange. Such scenarios may exist in litigation, appropriation of natural resources as well as patent races, to name but a few. A clear prediction is that a cap—either in an institutional form or based on an individual’s budget constraint—may be Pareto improving. The key component is the contest architecture. Thus for litigation Pareto improvements will depend on the legal system. For appropriation of natural resources it will depend on the intensity and probability of successful conflict, whereas for patent races it may very well depend on firms’ incumbent technological knowledge. Applications abound, it seems worthwhile to revisit the main application that drives the argument within Coase (1960); namely, tort law and, in particular, the potential capping of tort law damages.

One of the most prominent fields that applies the Coase ‘theorem’ is Law and Economics. In order to showcase the reciprocity of harm and the neutrality of allocative efficiency in ownership of property rights, Coase (1960) cites a number of court cases taken from tort law. The predominantly discussed context within Coase (1960) is the liability attribution for harmful activities, which is, in fact, a specific method for allocating (relative) property rights. Yet, what is usually abstracted from in this context is that the parties have to go to court to establish ownership (or enforcement) of a property right via a ruling on liability. The act of litigation, however, is not costless. To the contrary, with larger values at stake, both parties in litigation usually expend a considerable amount of resources in order to be granted specific rights via adjudication. Hence, our abstract extension of the Coasean framework might also be applied in the concrete context of litigation, i.e., the preferred application in Coase (1960).

In this case, the appropriation stage of our game would represent a lawsuit over a nuisance tort (e.g., noise or air pollution), establishing (relative) property rights, while the bargaining stage would represent some form of post-trial settlement over the level of tort (pollution). Litigation thus is in the form of a declaratory judgment—or more generally—preventative adjudication (Bray, 2010), where a judgment is made over liability without ruling on compensatory damages. In many cases, a jury’s decision is not the end of the lawsuit: litigants often participate in post-trial settlement (Shanley, 1991). For example, in all thirteen US federal courts of appeal, mediation programs exist with the primary objective to facilitate a settlement between parties after a jury has provided a decision (and prior to any formal appeal process) (Niemic, 2006). Usually mediation is free to parties.\footnote{Post-trial settlement over jury-determined damages may also exist in order for the plaintiff to secure the defendant’s agreement that no appeal will be sought (Shanley, 1991). This reduces expected future costs of appeals and (potential) future trials. Another possible interpretation is that the trial outcome becomes clear to both parties during the trial (and before any judgments have been delivered). This can occur, for example, when “indisputable” evidence is presented that favors one party. In this case, parties would find it beneficial to participate in an out-of-court settlement in order to agree on a mutually beneficial damage settlement (relative to a jury’s decision).}

From the 1970s onwards, a contentious debate has occurred over the need to reform US tort law. One of the major tort reforms over this period has been the establishment of statutory limitations to damages—or ‘caps’ on tort damages—in lawsuits (Dobbs, 2000).\footnote{Other tort reforms have also been considered including, for example, the collateral source rule (Flemming, 1966), compensation of litigation fees, either an “American rule” or “English rule” (Baye et al., 2005), and the rule of joint and several liability (Carvell et al., 2012).} In our model, a cap on tort damages will result in restrictions to the level of post-trial settlement, that is, a damage cap will alter agents’ outside options in the bargaining process. Hence, as it seems, when introducing caps on tort damages, the main trade-off to be considered is between the reduction in socially wasteful costs of (over)-litigation and the
negative distributional effects for the plaintiffs.\textsuperscript{12}

In the interpretation of the above-derived game as a model of declaratory judgments with post-trial settlement, part of the transfer from \(X\) (the defendant) to \(Y\) (the plaintiff) would, in the unrestricted case, represent compensatory damages. Obviously, this share of the transfer would correspond to \(Y\)'s costs at equilibrium, \(D(a^*(\theta))\). As outcome \(a^*\) is the outcome of an efficient bargaining process, this compensation would have to be interpreted as including both, pecuniary and non-pecuniary damages.\textsuperscript{13} In contrast, the transfer \(R_Y(\theta)\), i.e., the share of internalization rent of the injured party, is by definition overcompensatory. In the legal context, such overcompensatory transfers are often referred to as 'punitive' damages. In our analysis, where we assume a restriction on total damages, the cap \(K\) can also be interpreted as a cap on punitive damages given a specific expectation by the lawmaker over the transfers necessary for compensation. To see this, note that the level of actual non-pecuniary damages is uncertain due to non-observability by the legislator as well as the court. Hence, underestimating the level of non-monetary losses could well lead to a situation where the cap, meant to only restrict overcompensatory damages, actually restricts the bargaining outcome itself. Indeed, in reality, such caps have been implemented in one way or another within US States; at least half of all states have some type of cap (Dobbs, 2000). For example, a cap on total damages of $2.10 million is present in Virginia (among others) while caps on non-pecuniary damages are common, such as in California where non-economic damages are limited to $250,000 (Virginia, 2013; California, 2013).

Given unobservability over the actual level of non-pecuniary damages, our analysis shows that capping the transfers in a Coasean setup are often welfare improving in the sense of the Kaldor-Hicks-criterion. Furthermore, when litigation is modeled as an all-pay auction any form of cap would also yield genuine Pareto-improvements, i.e. restrictions on transfers are preferable to both parties. As caps decrease the transferrable value of the property right, (legal) efforts to appropriate this right are reduced, leaving both parties better off. At the extreme, precluding any transfer is always Pareto superior to a settlement. Such Pareto improvements do not exist if the contest takes the form of a Tullock-game, where rent dissipation is lower. Hence, in those cases where a Tullock contest provides Kaldor-Hicks improvements, caps would only be preferable to the defendant.

Interpreting a litigation as all-pay auctions may be appropriate when the facts of the case are difficult to present and where there is factual disputes for the jury to decide.\textsuperscript{14} In this case, as Baye et al. (2005) suggest, it may appear intuitive to model litigation as a “best case wins” scenario. Additionally, if one considers litigation costs to be significantly large—in that the value at stake is mostly dissipated through litigation costs—then this outcome is similar to the prediction of an all-pay auction. Recall that if one were to model litigation as a standard Tullock contest, litigation costs would have to be substantially smaller.\textsuperscript{15}

A lawmaker’s decision to implement a cap on damages should, therefore, be based on their expectation of the litigation process (relative size of litigation costs). The provision

\textsuperscript{12}Note, in many US states, caps on tort damages have also been challenged as unconstitutional, which include infringements on the rights of trial by jury, equal protection, and due process (Dobbs, 2000, Chapter 27).

\textsuperscript{13}If compensation would only cover monetary damages, additional gains from trade could be achieved by taking non-pecuniary losses into account.

\textsuperscript{14}See Dobbs (2000, p31) for discussion on jury deliberation.

\textsuperscript{15}Interestingly, Tullock (1996, p186) has the opinion “[t]he first thing to be said about in the American procedure is that it is extremely costly. If you take the fees of the attorneys on both sides, together with, in some cases, some very expensive expert witnesses ... and the government’s expenditures in maintaining the court ... the costs of the trial normally are about as large as whatever is at issue.” From this, excluding government expenditure, we extrapolate that the view expressed in Tullock (1996) is closer to full rent dissipation characteristics of an all-pay auction (or a Tullock contest with a sufficiently large parameter value for marginal efforts) rather than rent dissipation under a ‘conventional’ Tullock contest with \(r = 1\).
of “clear evidence” may vary with respect to the type of tort under contention as it may be easier (or harder) to determine liability. Furthermore, our analysis is more suitable to cases where there exists a continuing relationship between plaintiff and defendant. Hence, at least in tendency, our analysis suggests that discriminating among different torts when determining legal restrictions to tort damages, could yield significant Pareto and welfare improvements.

6 Conclusion

The purpose of this article is to investigate the efficiency of Coasean bargaining when transfers between agents are capped. We provide a two-stage framework where, in stage one, property rights for a harmful activity are attributed via a contest, which is followed, in stage two, by Coasean bargaining. We show that a cap on the value of exchange may provide Pareto and welfare improvements. We model the attribution of property rights as a contest and show a Coasean transfer cap may result in Kaldor-Hicks efficiency. Further, we show stark differences occur when applied to an all-pay auction and Tullock contest, where the former unambiguously results in Pareto improvements and the latter is unambiguously Pareto inferior.

For an all-pay auction, an agent who experiences a cap on gains from transfer will have a lower gross valuation of the property rights and become “weaker” in the contest. As a consequence, the agent’s expected utility will be equal to his loss from transferring rent to the rival, which is identical for cases with and without a cap. For the rival, in contrast, expected utility increases with the introduction of a cap, as transfers payable in the event of losing the contest are reduced. Interestingly, even if a cap precludes an efficient Coasean equilibrium it is still Pareto improving, as the reduction in transfers and equilibrium efforts offsets any allocative inefficiency. For a Tullock contest, however, where expected aggregated efforts are lower, a cap will reduce the agent’s expected utility.

Aside from litigation, our framework can be applied to any context in which property rights attribution is costly and endogenously determined with the potential for future bargaining. Such examples may include rent seeking, patent races, and resource conflicts.
Appendix

Determination of expected utilities under an all-pay auction

For expected payoffs:

\[ p_X(x, y)v_X(\theta) - L_X(\theta) - x, \]
\[ p_Y(x, y)v_Y(\theta) - L_Y(\theta) - y, \]

If \( v_X(\theta) \geq v_Y(\theta) \), then the equilibrium mixed strategies are generally described by

\[ G_X = \begin{cases} 
\frac{x}{v_Y(\theta)} & \text{for } x \in [0, v_Y(\theta)] \\
1 & \text{for } x > v_Y(\theta) 
\end{cases} \]

and

\[ G_Y = \begin{cases} 
\left[ 1 - \frac{v_Y(\theta)}{v_X(\theta)} \right] + \frac{y}{v_X(\theta)} & \text{for } y \in [0, v_Y(\theta)] \\
1 & \text{for } y > v_Y(\theta). 
\end{cases} \]

The expected payoffs of both agents are hence:

\[ U_X^\mu(\theta) = -L_X(\theta) + G_Yv_X(\theta) - \mu Y \varepsilon = -L_X(\theta) + v_X(\theta) - v_Y(\theta), \]
\[ U_Y^\mu(\theta) = -L_Y(\theta) + G_Xv_Y(\theta) - \mu Y \varepsilon = -L_Y(\theta). \]

Note, from agents’ payoff functions (6) and (7), that for the here-assumed setup both agents’ net valuations \( v_j \) are identical, as we have

\[ v_X^\mu(\theta) = v_Y^\mu(\theta) = E \left[ D(a^*(\theta)) + C(a^*(\theta), \theta) + R_X^\mu(\theta) + R_Y^\mu(\theta) \right]. \]

Hence expected utilities of agent X and Y are, respectively:

\[ U_X^\mu(\theta) = E \left[ -C(\theta) + (1 - \mu Y) I_Y^\mu(\theta) \right], \]
\[ U_Y^\mu(\theta) = E \left[ -D + (1 - \mu X) I_X^\mu(\theta) \right]. \]

Proof. Lemma 1:

As \( v_j(\theta) = W_j(\theta) + L_j(\theta) \) for \( j \in \{X, Y\} \), it is obvious from pairwise comparison of (6) and (7), (15) and (16), and (17) and (18) that for \( \theta \in [\theta, \theta_K] \) both agents have identical gross valuations.

For \( \theta \in [\theta_K, \theta] \), gross valuations for agents X and Y are:

\[ v_X(\theta) = R_X^\mu(\theta) + K + C(a_L, \theta) , \quad (A.1) \]
\[ v_Y(\theta) = K - D(a_L) + R_X^\mu(\theta) + C(a^*(\theta), \theta) + D(a^*(\theta)) . \quad (A.2) \]

Hence, for \( \theta \in [\theta_K, \theta] \), \( v_X(\theta) > v_Y(\theta) \) iff

\[ C(a_L, \theta) + D(a_L) > C(a^*(\theta), \theta) + D(a^*(\theta)) . \quad (A.3) \]

This condition always holds under the initial assumptions for both cost functions, as for \( C(a^*(\theta), \theta) + D(a^*(\theta)) \) aggregate costs are minimized and \( a_L > a^*(\theta) \).

Further, note from (15) and (16), and (17) and (18) that under the initial assumptions for both cost functions, valuations \( v_X, v_Y \) remain identical for any change in the agents’ costs.
For the case \( \theta \in [\theta_K, \overline{\theta}] \), condition (A.3) continues to hold for any such change in agents’ costs. It follows that the ranking of agents’ valuations for \( \theta \in [\theta, \overline{\theta}] \) is independent of the size of agents’ costs. 

**Proof. Proposition 2:** Given the equivalence result of agent Y’s expected utility in Proposition 1, it remains to show that agent X’s expected utility is higher under a capped regime. For \( \theta \in [\theta, \theta_{\nu_Y}] \), comparison of the first term in (24) and (10) results in both terms canceling each other. Comparison of the remaining terms in (24) with (10) yields the following condition:

For \( \theta \in [\theta_{\nu_Y}, \theta_K] \) expected utility is larger under a cap than under an unconstrained regime iff

\[
\int_{\theta_{\nu_Y}}^{\theta_K} C(\theta) - z_X^u(\theta) \, d\theta > \int_{\theta_{\nu_Y}}^{\theta_K} K + C(a^*(\theta), \theta) \, d\theta
\]

\[
\Leftrightarrow \int_{\theta_{\nu_Y}}^{\theta_K} C(a^*(\theta), \theta) + D(a^*(\theta)) + R_Y^u(\theta) \, d\theta > \int_{\theta_{\nu_Y}}^{\theta_K} \tilde{K}(\theta) + C(a^*(\theta), \theta) + D(a^*(\theta)) \, d\theta
\]

\[
\Leftrightarrow \int_{\theta_{\nu_Y}}^{\theta_K} R_Y^u(\theta) \, d\theta > \int_{\theta_{\nu_Y}}^{\theta_K} \tilde{K}(\theta) \, d\theta.
\]

We know from Definition 2 that at the realization \( \theta = \theta_{\nu_Y} \), we have \( \tilde{K} = R_Y^u \). Given \( C(\cdot) \) is convex, for the region \( \theta \in [\theta_{\nu_Y}, \theta_K] \), we know that \( \tilde{K} \) decreases and \( R_Y^u \) increases as \( \theta \to \theta_K \). It follows that within this region \( \int_{\theta_{\nu_Y}}^{\theta_K} R_Y^u(\theta) \, d\theta \geq \int_{\theta_{\nu_Y}}^{\theta_K} \tilde{K}(\theta) \, d\theta \).

For \( \theta \in [\theta_K, \overline{\theta}] \) expected utility is larger under a cap than under an unconstrained regime iff

\[
\int_{\theta_K}^{\overline{\theta}} C(\theta) - z_X^u(\theta) \, d\theta \geq \int_{\theta_K}^{\overline{\theta}} K - D(a_L) + C(a^*(\theta), \theta) + D(a^*(\theta)) \, d\theta
\]

\[
\Leftrightarrow \int_{\theta_K}^{\overline{\theta}} C(a^*(\theta), \theta) + D(a^*(\theta)) + R_Y^u(\theta) \, d\theta \geq \int_{\theta_K}^{\overline{\theta}} K - D(a_L) + C(a^*(\theta), \theta) + D(a^*(\theta)) \, d\theta
\]

\[
\Leftrightarrow \int_{\theta_K}^{\overline{\theta}} R_Y^u(\theta) \, d\theta \geq \int_{\theta_K}^{\overline{\theta}} K - D(a_L) \, d\theta
\]

\[
\Leftrightarrow \int_{\theta_K}^{\overline{\theta}} \mu_Y \cdot [\tilde{C}(\theta) - (C(a^*(\theta), \theta) + D(a^*(\theta)))] \, d\theta \geq \int_{\theta_K}^{\overline{\theta}} K - D(a_L) \, d\theta. \quad (A.4)
\]

To prove that this inequality always holds, we start out by showing that it holds for the extreme cases \( z_X^u = z_X^c = 0 \) and \( R_Y^u = R^c = 0 \).

If \( z_X^u = z_X^c = 0 \), then \( \mu_Y = 1 \) and \( \tilde{C}(\theta) - C(a_L, \theta) = K \). Hence, in this case (A.4) can be transformed into

\[
\int_{\theta_K}^{\overline{\theta}} C(a_L, \theta) + D(a_L) \, d\theta \geq \int_{\theta_K}^{\overline{\theta}} C(a^*(\theta), \theta) + D(a^*(\theta)) \, d\theta.
\]

As this always holds, a cap is welfare improving for \( z_X^u = 0 \) with constant bargaining power.

If \( R_Y^u = R_Y^c = 0 \), then it follows from (21) that \( K - D(a_L) = 0 \). Hence, both sides of (A.4) are zero, which implies that agent X is in fact indifferent with respect to the introduction
of a cap.

We now proceed to the proof for intermediate cases of $R_Y$ and $z_X$. For $\theta \in [\theta_K, \bar{\theta}]$, and agent $X$ winning the contest, as we assume constant bargaining powers, we can, for $0 < \mu_Y < 1$, substitute (22) into the right-hand side of condition (A.4), which yields

$$\mu_Y \cdot [\tilde{C}(\theta) - (C(a^*(\theta), \theta) + D(a^*(\theta)))] \geq \mu_Y [C(\pi_L(\theta), \theta) - D(\pi_L(\theta))] - (C(\bar{a}_l, \theta) - D(\bar{a}_l)).$$

$$\Leftrightarrow \tilde{C}(\theta) - (C(a^*(\theta), \theta) + D(a^*(\theta))) \geq C(\bar{a}_L(\theta), \theta) - D(\bar{a}_L(\theta)) - (C(\bar{a}_l, \theta) - D(\bar{a}_l)).$$  \hspace{1cm} (A.5)

The right-hand side of (A.5) always represents a subset of the internalization rent expressed on the left-hand side, hence the condition always holds and $\Phi > 0$ where Pareto improvements exist.

**Proof. Proposition 3:** Consider the expected utility of agent $Y$. Substituting (27) into (6) and (7), and solving yields the expected utility without a cap:

$$\Pi_Y = \frac{R_X^u(\theta) + R_Y^u(\theta) + C^*(a^*(\theta), \theta) + D(a^*(\theta))}{4} (2 - r) - (R_X^u(\theta) + C^*(a^*(\theta), \theta) + D(a^*(\theta))).$$  \hspace{1cm} (A.6)

Consider now the implementation of a cap. For $\theta \in [\theta_L, \theta_{R_Y}]$ the cap is non-binding and the resulting expected utility is identical to (A.6). For $\theta \in [\theta_{R_Y}, \theta_K]$ it is easily shown that

$$\frac{R_X^u(\theta) + \tilde{K}(\theta) + C^*(a^*(\theta), \theta) + D(a^*(\theta))}{4} (2 - r) - (R_X^u(\theta) + C^*(a^*(\theta), \theta) + D(a^*(\theta)))< \frac{R_X^u + R_Y^u + C^*(a^*(\theta), \theta) + D(a^*(\theta))}{4} (2 - r) - (R_X^u(\theta) + C^*(a^*(\theta), \theta) + D(a^*(\theta)))$$

as $\tilde{K}(\theta) < R_Y^u(\theta)$, due to $\theta \in [\theta_{R_Y}, \theta_K]$. For $\theta \in [\theta_K, \bar{\theta}]$ expected utility for agent $Y$ is smaller under a cap, if after canceling terms:

$$\frac{(v_X^r)^{r+1} ((v_Y^X)^r + (v_Y^X)^r (1 - r))}{((v_Y^X)^r + (v_Y^X)^r)^2} < \frac{v^u}{4} (2 - r)$$  \hspace{1cm} (A.7)

where $v_X^r = R_X^u(\theta) + K + C(a_L, \theta)$, $v_Y^X = R_Y^u(\theta) + K - D(a_L) + C^*(a^*(\theta), \theta) + D(a^*(\theta))$, and $v^u = R_X^u(\theta) + R_Y^u(\theta) + C^*(a^*(\theta), \theta) + D(a^*(\theta))$. The left-hand side can be rewritten as $v_Y^X \gamma' \left(\frac{1 + \gamma'}{\gamma'}\right)^r$ where, from Lemma 1, $\gamma = \frac{v_Y^X}{v_X^r} < 1$. Note first that $\gamma' \left(\frac{1 + \gamma'}{\gamma'}\right)^r < \frac{(2 - r)}{4}$ for all $\gamma \in [0, 1]$ and $r \leq 1 + \left(\frac{v_X^r}{v_Y^X}\right)^r$. Additionally, note when $v_Y^X < v_Y^u \iff K - D(a_L) < R_Y^u$. As shown in Proposition 2, for $\theta \in [\theta_K, \bar{\theta}]$, $K - D(a_L) < R_Y^u$, therefore $v_Y^X < v_Y^u$. It follows that agent $Y$ has lower utility than under the unconstrained case.

**Proof. Proposition 4** Substituting (27) into (6) and (7), and solving for expected welfare yields:

$$-\frac{R_X^u(\theta) + R_Y^u(\theta) + C(a^*(\theta), \theta) + D(a^*(\theta))}{2} (2 - r) - C(a^*(\theta), \theta) - D(a^*(\theta)).$$  \hspace{1cm} (A.9)

For $\theta \in [\theta_L, \theta_{R_Y}]$, welfare is identical to (A.9). For $\theta \in [\theta_{R_Y}, \theta_K]$, the difference in welfare can easily be calculated as $\frac{1}{2} (R_Y^u(\theta) - \bar{K})$ which is always non-negative.
For $\theta \in [\theta_K, \tilde{\theta}]$, solving for welfare, improvements exist when:

$$\frac{(v_X^c + v_Y^c)r\gamma}{(1 + \gamma r)^2} - v_X^c - C(a^*(\theta), \theta) - D(a^*(\theta)) > -\frac{v^u}{2} (2 - r) - C(a^*(\theta), \theta) - D(a^*(\theta))$$  (A.10)

where $\gamma = \frac{v_Y^c}{v_X^c}$. □

References


