Understanding the causes of income inequality in complex economic systems

Brendan Markey-Towler[1] & John Foster[2]

Abstract: We suggest in this paper that inequality in economic systems can be profitably analysed using complex systems analysis. We explain how we can capture, analytically, complexity in an economic system by applying graph theory in networks. We then develop a highly stylised theoretical model of how income inequality arises naturally due to the fact that a skewed income distribution necessarily arises from "preferential attachment" in a complex economic system. We characterise this process, both in the market system broadly defined and, specifically, within a firm. It is argued that such a complex systems approach (despite being vastly simplified here) provides a superior basis for understanding income inequality compared to standard economic analysis.

Keywords: Inequality; theory; complex systems; model

JEL: B00, B41, B59, C02, P40

1. Introduction

It has been widely observed that income inequality has increased steadily in the United States and other countries since about 1980. However, although this has been well reported and reflected upon from a range of perspectives, analyses of why it has occurred using the standard tools of economic theory have been rare. Indeed, some economists feel that it is not the business of economics to deal with the causes and consequences of income inequality:

> "It has been remarked that if one tells an economist that inequality has increased the doctrinaire response is 'so what'"
>
> (Salverda, Nolan, & Smeeding, 2009)

[1] PhD candidate and corresponding author, School of Economics, The University of Queensland, Brisbane, QLD 4072, Australia; email: brendan.markeytowler@uqconnect.edu.au
[2] Professor of Economics, School of Economics, The University of Queensland, Brisbane, QLD 4072, Australia; email: j.foster@economics.uq.edu.au

It is understandable that some economists hold this view because standard economic theory severely constrains our ability to deal with distributional issues. This stems from adherence to a strongly held ontological assumption: that the economy is a fully connected system, not unlike an electromagnetic field, where every individual is directly connected to all the others in a market "space" (Mirowski, 1991). Many are unaware of these ontological foundations and are satisfied with the way that strong and unrealistic assumptions enable the development of calculus-based mathematical models that have clearly defined deductive solutions.

As soon as we move to a more realistic network representation of an economy, where individuals are not directly connected to *every* other individual and, therefore, do not interact with them, the mathematics of an electromagnetic field in a state of general equilibrium no longer applies. When networks are incompletely connected, we can apply modern complex systems science where, for example, graph theory becomes useful in that it can be readily applied in the context of mapping a complex economic system, following the seminal contribution of Potts (2000).

An individual within a complex economic system can be thought of as a node on a graph which is connected to other nodes in the system[3]. These "connections" represent economic interactions such as purchases, employment, and a myriad other transactions (Schweitzer, Fagiolio, Sornette, Vega-Redondo, & White, 2009). At the fundamental level, these connections are created by the decisions of individuals who are pursuing payoffs, and in this way their motivation differs little when compared with standard economic theory. However, in the context of the vastly complex and adaptive system that is the economy, economic agents can only employ traditional constrained optimisation techniques in a very limited sense due to the fact that information is often uncertain to such a degree that event spaces are often not fully known, let alone probabilities defined over this space. Also relevant is the fact that the computational capability of economic agents is self-evidently limited so that it is necessarily the case that they are, at most boundedly rational. Instead of employing constrained optimisation programs they adopt routines (rules) discovered through "trial and error" that enable complex tasks to be completed in an orderly and effective way (Nelson & Winter, 2002).

[3] Mathematical graphs are simply abstract expressions of networks.

It is an established empirical fact that, in the “scale-free” networks that we commonly observe, there is “preferential attachment” that leads some nodes to become “hubs”. Since in an economy it is these connections that generate economic value (being mainly trade and contracting relationships), their pattern tells us how the resources of an economy are allocated and how income is distributed. Foster (2005a) points out that, with this in mind, preferential attachment can be seen as a process by which income inequality emerges because of the concentration of connections with particular nodes. Moreover, the distribution of connections that emerges from preferential attachment matches the commonly observed “Pareto law” in the tail of the distribution of income, a fit noted long ago by Simon (1955b).

Our goal here is to develop this line of argument by using graph theory to construct a model of income inequality in a scale free economic network. This is not unheard of, but unlike much of the modelling done in this area which takes preferential attachment a given we provide a behavioural explanation of what causes the preferential attachment in incomplete economic networks that gives rise to income inequality. We show that this provides us a much more robust understanding of income inequality as would exist in a “healthy” economic system than that which can be obtained using standard economic theory.

2. Inequality as a complex economic system phenomenon

We have suggested elsewhere (Markey-Towler & Foster, 2013) that few of the previous attempts at understanding inequality and its effects do so adequately. In understanding where economic inequality comes from, both neoclassical and post-Keynesian models have serious flaws. What the neoclassical models gain by looking at the micro-foundations of economic phenomena, they also lose insofar as patently false assumptions must be made in order to make the models workable. Moreover, when we look at the causes of inequality, neoclassical economists have to break from the supposed “underlying” model in order to make it fit empirical regularities. Alternative Post-Keynesian distributional models of economic growth that were popular over three decades ago did not explain inequality but simply assumed that income and wealth is unequally split between ‘workers’ and ‘capitalists’. Their differential rates of saving then determined what the rate of growth would be, again making very strong assumptions and using mathematical forms that ensured the existence of point equilibrium.

Despite the obvious flaws in these models they were, in their time, serious attempts to understand underlying truths and indeed shed light on aspects of reality. Their problem is that they are simplistic representations of this reality because they do not account for the complex nature of all economic systems (Foster, 2005a). While there has been a long-standing debate in the philosophy of science about such matters, economists by and large brushed aside the problem some time ago. Friedman (1953) famously argued that, in any meritorious science, it is quite legitimate to make unrealistic and outright wrong assumptions in the quest for the best predictions using the simplest possible model. Why would we go and employ complex systems theory (with the emphasis on 'complex') when we can derive and test simple hypotheses drawn from the Walrasian general equilibrium model, with its "straightforward" neoclassical microfoundations?

Firstly, Hausman (1989) pithily points out that following Friedman's exhortations makes it difficult to revisit the theory and remodel it so that it produces consistent predictions, since we can't know which of our unrealistic or outright wrong assumptions is causing the inconsistency. But at a more fundamental level, Friedman read Occam's instructions incorrectly because these assert that the simplest (and plausible) explanation is usually the correct one. It is an accepted view outside of economics that scientific inquiry aims to understand observable phenomena, rather than merely predict them (Simon, 1998). Moreover Lawson (2003) has suggested that economics has drifted into a crisis of relevance in part due to the position that assumptions are irrelevant to the validity of a model. He argues that economists must match their economic ontology with their heuristics, which really seems to be a statement of common sense: we should know what we are studying and study it *as it exists* if we want to understand existing phenomena.

2.1 Complex systems science

Complex systems theory finds some of its earliest roots in the work of von Bertalanffy (1950) who observed that the same equations appeared in many different disciplines. He proposed a logico-mathematical discipline, "general systems theory" to understand the laws governing a "system" at a general level. In reality though, a theory has little value outside of the context of study and von Bertalanffy took his level of theorising one step too far in the abstract direction towards studying Platonic "spheres" rather than Aristotelian reality. It is not until Simon (1962) that we find a workable definition of the notion of a "complex system" as made up of parts which interact in a non-simple manner. Complex systems science therefore, offers explanations of the patterns of systemic interactions at any given time and how these evolve between different states of the system.

In modern times, systems science has been invigorated by the application of network theory and has become prevalent in such diverse fields as physics, mathematics, biology and sociology *inter alia* (Watts, 2004). Scientists in this new field started with thinkers such as von Bertalanffy and Simon and expanded upon them hugely using graph theory to characterise and understand the *topology* of the system and its evolution, something earlier thinkers had struggled with. In this style, a complex system can be described as a set of nodes and a set of connections between them which facilitate interactions (Newman, 2003). In this graph theoretical view the "system" becomes a more tractable idea and can be described in terms of its topology and its structural transition can be studied.

2.2 Fundamental mathematics of a complex economic system

Following Potts (2000), the economy can be viewed as consisting of a set of elements, which we know as economic agents, with a set of connections between these agents, generating economic value. Market trading and contractual agreements constitute the 'commercial' part of the system where consumers and firms can conduct transactions, facilitated by an over-arching institutional structure, which is itself constantly changing as new rules supersede old ones (Earl & Wakeley, 2010). For the purposes of the present work though we focus on the commercial aspect of the system alone[4] to characterise its allocation of income, and following Jackson (2010), an economic system can be represented by a graph

$$E_S = \{N, g(N)\}$$

with a set of agents[5] $N = \{n_i\}_{i=1}^{n}$ and a set of connections between them, summarised within an $n \times n$ matrix $g(N) = \{g_{ij}\}$ where, if $n_i, n_j \in E_s$ denotes the existence of an interaction (connection) between agents i and j,then

$$g_{ij} = \begin{cases} 0 \Rightarrow n_i n_j \notin E_s \\ 1 \Rightarrow n_i n_j \in E_s \end{cases}$$

[4] We do this for simplicity of exposition at this early stage of research, and believe that the qualitative results will not greatly differ by the inclusion of an *explicit* institutional structure, though future research must address this gaping lacuna in the realism of our model.

[5] We will eventually split the set of agents into two subsets $\{n_i\} = \{a_i, f_i\}$ where p_i denotes individual consumer i and k_i denotes firm i

When $g_{ij} = g_{ji} \forall\, n_i, n_j \in N$, the graph is undirected so that the connection between the agents is mutual. Insofar as economic behaviour is concerned, this type of graph will be sufficient for our purposes, as economic interactions and transactions usually require the agreement of both parties.0

Nearly all of modern economic theory relies on the assumption that $g_{ij} = 1 \;\forall\; n_i n_j \in N$, or that the economy is a complete network (a field) where all nodes are directly connected to every other[6]. In this sense, if we define the system of interest to be an micro-economy over a *very* short interval, neoclassical economics can be applicable as an approximation, and why Alfred Marshall in setting down his neoclassical theory of marginalism saw it as most useful in short periods in the presence of a high level of knowledge (Foster, 1993).

The beauty of the complex systems representation is that it lends itself to pictorial representation so that we can actually "see" the economy at any point in time, though most systems consist of so many parts that this is unlikely to give us much insight. This is why much of the modern network theory surveyed by, for instance, Newman (2003) consists of attempting to develop measurements of the system which can describe its topology. In particular, if we consider the characteristics of a single node, we can say that the neighbourhood of n_i is given by the set of nodes that an agent is directly connected to

$$N_i(g) = \{n_j \in N : g_{ij} = 1\}$$

In a market context, we can think of this as representative of the *structure* of the budget set, so that the basket of goods any agent buys is facilitated by their interactions with particular firms.[7] Similarly, we can think of this as representing organisation within a firm. In this sense we can take the "degree" of a node to indicate the economic value that is concentrated around that node, the degree of a node being the size of its neighbourhood:

$$d_i(g) = |N_i(g)| = \left|\{n_j : g_{ij} = 1\}\right|$$

What determines this metric is crucial to understanding the inequality of earnings since the connections in an economic system represent value-generating interactions facilitating both production and trade (Earl & Wakeley, 2010). We can then collect the degrees of all the

[6] $g_{ij} = 1 \;\forall\; n_i n_j \in N \Rightarrow E_S = \{N\}$, if we accept that $\{N\} = [\succcurlyeq_i, X_i]_{i=1}^{I}$ then $E_S = E_S^D$

[7] For the purposes of the current work and for the sake of brevity we abstract from one relationship being stronger than another. Again, this is a gaping hole in our realism to be addressed in any future work.

nodes and plot them in a degree distribution $p(d)$ to gain an insight into the structure of the system. Specifically, we gain an understanding of how concentrated the structure is around certain elements. If connections in the economic system represent the earnings capacity and importance of an agent to production within in a firm, then a skewed degree distribution indicates that the structure of the economic system is underlying earnings disparities.

2.3 The scale-free distribution as an empirical regularity

The best studied feature of empirical networks is the almost axiomatically regular scale-free degree distribution (Newman, 2003). A scale-free distribution is characterised by

$$p(d) = cd^{-\gamma}, \qquad c, \gamma > 0$$

indicating that a large number of nodes have a small degree and a much smaller number of nodes have comparatively larger degree, typical of a network/system topology which is concentrated around a few central nodes.

The pioneering study of social networks (Milgram, 1967) showed that it took only five to six steps on average to connect any two people in the USA, suggesting that social networks tend to have a high degree of "clustering" (that is, a degree of connectivity in particular localities) and also a low "diameter" (the largest number of links between two nodes in the network). This property manifests itself in a skewed degree distribution, with a small number of nodes having many links and a large number with few. Such a feature has been observed in many different networks, from de Solla Price (1965), a famous paper which pointed out that certain scholarly articles form the core of networks of citations to Alghmandi and McDonald (2012) which studies the clustering of R&D firms together. In their seminal paper, Barabasi and Albert (1999) noted that degree distributions are common in mappings of the world wide web of hyperlinks. Similarly, Jackson and Rogers (2007) built a probabilistic model of the formation of a network which tends only to fit the data on social networks when random effects parameters are calibrated to be outweighed by parameters which nudge the structure towards concentration.

In terms of the processes which underlie the emergence of such degree distributions, network theorists (including the relatively new grouping of econophysicists) have derived, many times, over degree distributions such as these indicating a high level of concentration in networks using multiplicative stochastic processes which as Markose (2005) points out are largely special cases of Simon's ground-breaking *Biometrika* paper (Simon, 1955b). He showed that from two simple assumptions about what would come to be known as "preferential attachment," one can derive a skewed stationary distribution. Importantly, both Simon (1955b) and von Bertalanffy (1950), as well as many modern authors including Markose (2005), Newman (2003) and (albeit in a different theoretical paradigm) von Weizsacker (1993), note that such a skewed distribution is very similar to the tail of the income distribution.

The most elegant modern model of a skewed degree distribution through preferential attachment can be found in the famous Barabasi and Albert (1999) incarnation of the Simonian model, a very simple version of which is presented in Jackson (2010). First, assume a new node entering the system will tend to attach itself to a higher degree node rather than a lower degree node. Then take a series of nodes being born sequentially and indexed by $i \in \{1, \dots, t, \dots\}$ and suppose that the new node attaches to m nodes. So, using the common mean-field approximation technique,[8] we can express the growth rate of the degree of node i as dependent on the degree of the node relative to the total degree of the system

$$\frac{dd_i(t)}{dt} = m\frac{d_i(t)}{\sum_{j=1}^{t} d_j(t)} \tag{1}$$

Now, noting that the total number of links in the system at this point in time is tm, we can say that

$$\dot{d}_i(t) = \frac{d_i(t)}{2t} \Rightarrow \frac{\dot{d}_i(t)}{d_i(t)} = \frac{1}{2t}$$

Setting $d_i(i) = i$ we can solve this differential equation to yield

$$\int_i^t \frac{\dot{d}_i(t)}{d_i(t)} dt = \frac{1}{2}\int_i^t \frac{1}{t} dt \Rightarrow d_i(t) = m\sqrt{\frac{t}{i}}$$

which can be rearranged to identify the birth date of a node with degree $d\colon i = t(m/d)^2$. This is important as it allows us to identify the nodes which will have a degree lower than d,

[8] This is commonly used in the physics literature on complex systems and which takes probabilistic outcomes and takes the expected outcome as deterministic in order to visualise the expected situation (Jackson, 2010).

which will be the nodes born after date i. Hence, since the number of nodes in the system at any point is t, the cumulative degree distribution will be defined as the proportion of nodes with a degree less than or equal to d so $P(d) = 1 - i/t$. By taking its derivative, we find that preferential attachment yields a power law in the degree distribution (where $c = 2m^2$ and $\gamma = 3$)

$$P(d) = 1 - (m/d)^2$$

$$\Rightarrow p(d) = cd^{-\gamma} \tag{2}$$

As noted above, such a distribution is very similar to those which describe the "Pareto law" skew in income distribution. And, as we have argued in the economic sphere, these connections create value. In market systems, the degree of a firm is associated with its earnings capacity: the connections it maintains being connections to customers who purchase goods from it. Within the firm, the connections between the elements economists refer to as factor inputs are indicative of the centrality of that input to production. Accordingly, the degree of an individual within a firm indicates that individual's contribution to the production process and hence will likely determine remuneration.

However, while such a skewed distribution has been well modelled in the econophysics literature[9], these models lack economic content. Indeed as Markose (2005) notes, the econophysics literature betrays its intellectual heritage in physics, as its models conceive of the economy as physicists do of a natural system, a network of interacting particles. This literature is largely devoid of any behavioural content that could explain why economic activities commonly display preferential attachment, or why even economic activity occurs at all. But, if we can understand why a skewed degree distribution emerges in an economic system, we can understand what processes underlie the distribution of income within such a system.

3. A model of inequality

The highly stylised model of the distribution of income that we develop here is an extension of the Bianconi and Barabasi (2001) which incorporates a "fitness" parameter η_i into equation (1) so that the likelihood that node i gets a new link in any given period is

[9] See Corso, Lucena, and Thome (2003) and Dahui, Li, and Zengru (2008)

$$\Pi_i = \frac{\eta_i d_i}{\sum_\ell \eta_\ell d_\ell}, \qquad \eta_i \sim \rho(\eta) \tag{3}$$

They show that, if $\rho(\eta)$ is a uniform distribution, then this model will generate a degree distribution which is scale-free, while if $\rho(\eta)$ is normally distributed then the resulting degree for each node will display a power law such that the fitter nodes attract more connections. Again, there is an obvious lack of economic content in such models. So as economists, we wish to find some behavioural cause for this form of preferential attachment to exist.

3.1 A model of market-level inequality with behavioural micro-foundations

Herbert Simon (Simon, 1959) argued that there was a simple yet incredibly important aspect of human decision making missed in neoclassical economics. *Every* human being has limits on their cognitive capabilities in that they can only perceive and consider limited amounts of information at any point in time. So, even though individuals make considered decisions, they are simply not capable of meeting the definition of "rationality" required by neoclassical economics. Instead of weighing every possible alternative carefully by solving a constrained maximisation problem that would take an absurd amount of time to solve (*if* all the information required was available), people simply make decisions on the basis of what information they have and adapt in a pragmatic way if that information turns out to be misleading and loss-making. Thus, choices are outcomes of a process of competitive selection amongst alternatives. We suggest that a view of decision making informed by Simon's ideas will, in a complex economic system lead to preferential attachment[10].

Specifically, let us consider a market system where individuals choose to purchase goods and services from firms, thus forming relationships with those firms. At any point in time t, this market system can be represented by a graph of a set individuals and firms N_t and the relationships between them represented in $g_t(N_t)$

$$E_s^t = \{N_t \; g_t(N_t)\} \tag{4}$$

In this system, there are two subsets of agents, individual consumers, $A_t = \{a_i\}_{i=1}^{|A|_t}$, and firms, $F_t = \{f_i\}_{i=1}^{|F|_t}$. Suppose that in every period each individual must make a purchase of an

[10] It is in fact possible to derive preferential attachment as the equilibrium of a game of rational agents (see (Kim & Jo, 2010)), though we believe that the boundedly-rational approach is more generally representative of reality while giving the same results.

item[11] from one firm, which is represented as a link $g_t(N_t)$. For simplicity, because we are concerned with the allocation of resources in the system, let us impose $a_i a_j \notin g_t(N_t)\ \forall\ a_i, a_j \in N$ as well as $f_i f_j \notin g_t(N_t)\ \forall\ f_i f_j \in N$, thus omitting social and strategic relationships from our study of the market system[12]. So we can think of the system graph here as a representation of "who's buying from whom". Clearly, $A_t \cup F_t = N_t$, but, in an evolving system, the makeup of this partition changes as we have new individuals and firms entering and exiting the system, i.e.,

$$\frac{d|F|_t}{dt} \neq 0, \qquad \frac{d|A|_t}{dt} \neq 0$$

In this market system the earnings of the firm immediately follow from the firm's degree at any given time, since any link represents a purchase in that time period, so earnings of firm i in the market system are determined largely by the degree of the firm in that time period

$$d_i(t) = \left|\left\{a_i \in N : a_j f_i \in g_t(N_t)\right\}\right|$$

So when we see a non-degenerate degree distribution, we are effectively observing inequalities of earnings and hence income. Clearly, the degree of each firm is determined by the choices of each of the individuals in the market. Now, following the framework of Simon (1955a), we can explain the process by which these choices are made under bounded rationality.

The first element of the decision making process is to perceive and interpret the information presented in any situation. As argued by Simon (1959), human beings simply cannot perceive every possible alternative and consider each carefully against the others, since they have a limited cognitive capacity. This early idea was extensively tested and demonstrated to hold empirically by Tversky and Kahneman (1974) who found that many biases and illusions can be introduced into perception due to the limited computing power of the human brain. Indeed, as emphasised in a later Nobel prize acceptance lecture, Kahneman (2003) noted that the perception of stimuli largely depends on the "availability" of certain bits of information within a vastly complex situation. In the context of the market model here, this means that

---

[11] As noted above (note 7), this is quite unrealistic and future work should try to relax this assumption.

[12] Or, equivalently, $a_i a_j = 0\ \forall\ i \neq j$, of course, this is patently false and indeed social networks and firm strategy are highly important, though we believe that the model of preferential attachment here would not display hugely different results in terms of network topology merely strengthen them. The effects of social networks and industrial organisation on the income distribution are an important topic for future research.

the set of alternatives which a_i considers, B_i^t, is a subset of the overall set of firms in the market. So we can define the set of *considered* alternatives for agent a_i as

$$B_i^t = \{f_j \in F_t | b_i^t(f_j) = 1\} \subset F_t \tag{6}$$

where b_i^t is a mapping which assigns elements in the overall set of alternatives F_t to either the set of considered alternatives B_i^t or not, so $b_i^t: F_t \to \{0,1\}$, such that $b_i^t(f_j) = 0$ indicates f_j is perceived or considered. However, this mapping is not arbitrary - more "prominent" firms are more likely to be in the considered set of alternatives. In this case we can say that

$$p\left(b_i^t(f_j) = 1\right) = \psi_i\left(d_t(t)\right), \qquad \psi'(\cdot) > 0$$

This captures the "availability effect,"[13] insofar as a firm with more "prominence" in the system, in the sense of having a larger customer base, will be more easily perceived than a firm which the agent must search for. For instance, a firm with greater degree will also be likely to have more outlets, and hence will be geographically closer to the individual in question. Also, a firm with a larger customer base may have a firmly established reputation and brand recognition with individuals, as well as a greater ability to spend on advertising, making it more noticeable.

Now that we have the set B_i^t of alternatives that each individual considers, we wish to understand the choice function which operates on this set. Traditionally, the assumed heuristic is a maximisation rule over some utility function $u: B_i^t \to \mathbb{R}$ defined over the set of alternatives, so that

$$C_i^t(B_i^t) = \arg\max_{f_j \in B_i^t} u\left(f_j\right) \tag{7}$$

However, this sort of rule is inconsistent with extensive evidence presented in Kahneman (2003) and Rabin (1998) that individuals typically follow heuristics such as "routines" when making decisions. Indeed, traditional theory in (7) implies that choice is far less stable than the evidence suggests. Why constrained optimisation of a utility correspondence defined over a set of alternatives is not followed in reality was understood as early as 1955 by Herbert Simon. Most human beings simply cannot hold a high-dimensional object such as u in their

[13] One of the key results of the behavioural economics literature for which Daniel Kahneman was awarded the Nobel Prize surveyed by (see his acceptance speech in Kahneman (2003)).

mind much less if the object must be continuous and differentiable over its domain space. If individuals *do* make comparisons of the utility derived from various alternatives, Simon (1955a) suggested that they will consider only approximations of u at various points in B_i^t without interpolating the rest of the surface between them. While this still permits an optimising operator to exist, the choice made does not necessarily correspond to one that maximises utility since the domain of (7) is a finite set of points on the object u.

On a more technical note, utility functions which are differentiable (the easiest to handle) cannot exist if the data on preferences are to be given any weight. As Earl (1990) pointed out, most preference systems studied by decision psychologists involve evaluating alternatives in a number of distinct categories ranked in terms of importance i.e. preferences seem to be lexicographic in nature. It is a fact that such preferences are not continuous, which breaks a fundamental assumption in Debreu's theorem of the existence of a "well-behaved" utility function (Mas-Colell, Whinston, & Green, 1995). Moreover, in a complex market system, most alternatives are not even sampled, prohibiting the existence of the data which a constrained optimiser requires if we do not make the strong assumption agents know precisely the distribution of possible payoffs. Thus, specifying a utility function seems to add little to our understanding of choice, since most positive theories of decision-making prohibit its existence in any meaningful sense and preferences are unlikely to be known completely.

Kahneman (2003) explains that choice is born of the interaction between two "systems of thought" in the brain which operate on the information provided by external perception. The first of these systems is the "reasoning" mind, which monitors the activity of the rest of the mind and which has a slow, methodical and effortful decision process. The reasoning mind takes more time to interpret perception, to weigh up alternatives on various categories and then make a considered decision. The second system, the "intuitive" mind is fast, associative and characterised by the employing of simple heuristics in decision making which take a fair degree of the effort out of decision making. This intuitive decision-making process leads to *almost* automatic choice processes, which are highly efficient in terms of cognitive effort, and is monitored by the reasoning mind to mitigate the effects of its various biases (catalogued extensively by Tversky and Kahneman (1974)).

We can deduce a common thread from these theories: that individuals form routines for making decisions in complex situations and adapt when a "threshold" of acceptability is reached. In the context of our model, individuals in a complex market system will, for example, tend to continue their relationship with a particular firm until the characteristics of that relationship breach some "acceptability" constraint. The most reasonable trigger event in our model is the existence of an element in the considered alternative set with a relative price that is below some tolerance level ε_i. If this event occurs then the agent will move from a routine choice to a considered assessment of the set of alternatives. Denoting p_c as the price of the previous choice and p_j as the prices of the various other alternatives

$$C_i^t = \begin{cases} C_i^{t-1}(B_i^{t-1}) \; if \; p_c - p_j \leq \varepsilon_i \; \forall \; f_j \in B_i^t \\ f_j^t \; if \; \exists \; f_j \in B_i^t \; s.t. \; p_c - p_j > \varepsilon_i \end{cases} \tag{8}$$

So the routine is revisited if there are sufficient changes in the prices of alternatives in the considered set B_i^t or when this set expands to include firms whose prices make the current relative price paid unacceptable. Of course, this sort of choice rule only has significant meaning if the tolerance level is not trivial. Why would individuals not revisit their routines as soon as there was some difference in price relatives, as in the neoclassical conception of the perfect market? Firstly, $\varepsilon_i > 0$ can be a justifiable assumption, even in neoclassical theory, since (7) is not necessarily the same as $C_i^t(B_i^t) = \arg\min_{f_j \in B_i^t} p_j$. Consumers have preferences over even slightly different types of products, due to personal preference and considerations such as the quality of the product and its brand reputation, so they will tolerate a degree of difference in prices. In essence, if they are "satisfied" with the particular business relationship they are maintaining they will not change their routine until there is some significant disconnect in prices.

We can also find various behavioural reasons why routines are maintained even when "cheaper" alternatives are perceived. The most famous of these reasons is perhaps the "loss aversion" hypothesis of Kahneman and Tversky (1979). Related is the hypothesis of Schwartz (2005) stating that more alternatives on offer can lead to less choice actually occurring. A more "irrational" perspective on this aversion to changing routine can be found in "hypothesis filtering" and "confirmation bias" perspectives. In developing heuristics to deal with complexity, individuals will often interpret new information through the prism of choices previously made in order to make those choices seem more appropriate, which is complemented by the tendency to attach a larger ex-post likelihood to events that have occurred (Rabin, 1998). So boundedly rational individuals believe new alternatives are less attractive than their current routine choice, which they believe is more likely to be the best available to them. This "conservative" bias has also been termed "default option" bias by Kahneman (2003) and Ariely (2008). When presented with a bewildering array of alternatives, individuals will typically stick with a routinized choice rather than take a risk and expend effort. Frydman and Goldberg (2011) have argued that this "conservative bias" can be an entirely rational (albeit under a different definition of that term) updating rule for expectations and actions. Rather than change rules every time there is new evidence at hand, they suggest that it is rational to maintain the same rule until there is sufficient evidence that the chosen outcome is inefficient due to more than just a stochastic shock.

We can conclude then that the threshold ε is not trivial and that therefore we are justified in setting down a choice rule for individuals in our model in which they will follow a routine until that routine becomes unsatisfactory. But we are yet to understand what occurs in scenarios of non-routine choice, when the individual re-engages their "reasoning" mind to consider fully the alternatives set and change habits. Since we have rejected utility maximisation as an appropriate general rule, we are left with the suggestion of Simon (1955a) that individuals will aim to find an alternative which gives them a "satisfactory" payoff. However, Simon noted that this process of search does not necessarily give a unique solution to the problem of choice without imposing some iterative presentation of the set of alternatives to an individual. This non-uniqueness is much more likely when an individual doesn't know his or her preferences over alternatives *ex-ante*. So it is not possible to work out which alternatives will even be considered "satisfactory" once sampled. So we can say that, if choice is more or less a stochastic process, the scientist and even the individual, to some extent, will not be able to know the final choice until it is made. However, we can still impose some structure on the likelihood of certain alternatives being selected.

Two reasonable determinants of this likelihood would seem to be price and the quality of the product on offer, which may be taken as an indication of the payoff it would yield to the individual. However, most products in a complex system will not have been sampled by the individual, so unbiased information on relative quality will be almost non-existant. So how do we account for the effect of increased quality if individuals cannot observe it directly? We would suggest that again, the degree of the firm can be taken as the relevant determinant. We can take the degree of the firm as a proxy for quality, at least in the mind of the individual. Degree's role as a proxy for quality rests on the idea that the customer base which it represents is one signal of quality and this allows brand reputation to be built and this can be spread throughout social networks. But also, what applies in the construction of the perceived set of alternatives will apply to the choice function as well, insofar as greater "availability" in the cognitive and physical sense will increase the likelihood of the firm in question being selected. So we can say that

$$C_i^t(B_i^t) \sim c\left(\left\{p_j\, d_j\right\}_{f_j \in B_i^t}\right)$$

where $c: \mathbb{R}_+^2 \to [0,1]$ & $c'_{p_i} < 0, c'_{d_i} > 0$. Now, to analyse system dynamics, and the income dynamics which follow from it, we need to set down a functional form for this relationship. To maintain the direction of our intuition, let us assume the simplest relationships: linear, but bounded between zero and one. If we assume that *differences* in prices have an impact on the likelihood of selection (as would seem reasonable) and that this is amplified by size ($d_j(t)$) relative to the size of the market (indicating how much the firm "stands out" from the other firms in the market), we can express the likelihood of a particular firm gaining a business relationship from individual $i \in A_t$ as

$$p_i\big(C_i^t(B_i^t) = f_j\big) = I_i(p_c)\left\{\alpha_i \frac{d_j(t)}{\sum_{f_\ell \in F_t} d_\ell(t)} \sum_{f_\ell \in B_i^t} \delta_{j\ell}\big(p_\ell - p_j\big)\right\}, \qquad p_i(\cdot) \in [0,1]$$

where α_i is an individual-specific constant (low α_i *possibly* being a "novelty-seeker"), $\delta_{j\ell}$ is the ability of firm j to attract customers from firm ℓ and $I_i(p_c)$ is an indicator function[14] assigning us to the two cases of (8)

$$I_i(p_c) = \begin{cases} \left.\begin{matrix} 0\ if\ j \neq c \\ \dfrac{1}{\varphi_j}\ if\ j = c \end{matrix}\right\} if\ p_c - p_j \leq \varepsilon_i\ \forall\ f_j \in B_i^t \\ \quad 1\ if\ \exists\ f_j \in B_i^t\ s.t. p_c - p_j > \varepsilon_i \end{cases}$$

so that, if the price of the routine choice is within the bounds of acceptability, it collapses the distribution so that the routine choice is made with a likelihood one, while if the routine becomes unacceptable, the distribution becomes non-degenerate. Now that we have a plausible relationship between the characteristics of the alternative set B_i^t and the choice of

[14] setting $\varphi_j = \alpha_i \frac{d_j(t)}{\sum_{f_\ell \in F_t} d_\ell(t)} \sum_{f_\ell \in B_i^t} \delta_{j\ell}\big(p_\ell - p_j\big)$

the agents, we can treat these probabilities as deterministic rates of change in the degree of the firm when summed across all individuals[15]

$$
\begin{aligned}
\dot{d_j}(t) = \frac{dd_j(t)}{dt} &= \sum_{i \in A_t} p_i\big(C_i^t(B_i^t) = f_j\big) \\
&= \sum_{i \in A_t} I_i(p_c)\left\{\alpha_i \frac{d_j(t)}{\sum_{f_\ell \in F_t} d_\ell(t)} \sum_{f_\ell \in B_i^t} \delta_{j\ell}\big(p_\ell - p_j\big)\right\}
\end{aligned}
\tag{10}
$$

3.2 An approximation of (10)

Clearly, it is difficult to do much with (10) other than "black box" simulation without imposing some regularity on its various objects. To begin with, let introduce one firm and one individual in each time period so that

$$\frac{d|F|_t}{dt} = \frac{d|A|_t}{dt} = 1 \Rightarrow \sum_{f_j \in F_t} d_j(t) = t$$

and we index the set N_t by birth date. Also, set the rate of transformation of the degree and relative price variables on average to be $\bar{\alpha}$, so

$$\dot{d_j}(t) = \sum_{i \in A_t} I_i(p_c)\left\{\bar{\alpha}\frac{d_j(t)}{t} \sum_{f_\ell \in B_i^t} \delta_{j\ell}\big(p_\ell - p_j\big)\right\}$$

Now, we still have two objects with unspecified relationships to time in the indicator function $I_i(p_c)$ and in the considered alternatives set, B_i^t. First, we can define a scalar to be the fraction of consumers in whose considered alternatives set, f_j, exists

$$\kappa_j(t) = \frac{|\{a_i \in A_t : I_i(p_c) = 1\}|}{|A_t|}$$

We assume that this scalar is relatively constant in time, so $\bar{\kappa}_j \approx \kappa_j(t)\ \forall\ t$, which can be justified by the tendency for there to be a continuous increase in the number of consumers in the economy, somewhat counteracting the likelihood of being in B_i^t. Also, the effects of

[15] See Jackson (2010) for a justification of this common approach to analysing system dynamics using "mean-field approximations"

increases in "size" on considered alternatives sets are already captured in the linearity of $\dot{d_j}(t)$ in $d_j(t)$. Now we can write our equation for the firms' degree as

$$\dot{d_j}(t) = \sum_{i \in A_t} \bar{\kappa}_j \bar{\alpha} \frac{d_j(t)}{2t} \sum_{f_\ell \in F_t} \delta_{j\ell}(p_\ell - p_j)$$

$$\Rightarrow \dot{d_j}(t) = \bar{\kappa}_j \frac{\bar{\alpha} d_j(t)}{2} \sum_{f_\ell \in F_t} \delta_{j\ell}(p_\ell - p_j)$$

This differential equation has a solution, using $d_j(j) = \nu_j$ as a starting point, perhaps a set of "adventurous" consumers with a preference for novelty and, assuming for simplicity, that prices are relatively constant for each particular firm[16]

$$\int_j^t \frac{\dot{d_j}(s)}{d_j(s)} ds = \frac{\bar{\kappa}_j \bar{\alpha}}{2} \int_j^t \sum_{f_\ell \in F_t} \delta_{j\ell}(p_\ell - p_j)\, ds$$

$$d_j(t) = \nu_j \exp\left[\frac{\bar{\kappa}_j \bar{\alpha}}{2} \sum_{f_\ell \in F_t} (t - \max\{j, \ell\}) \delta_{j\ell}(p_\ell - p_j)\right] \tag{11}$$

which is clearly a power law for degree. However, due to the complex nature of this relationship we cannot easily derive a closed form for the degree distribution. The profit function for any firm in this industry can be represented by a function $\pi: \mathbb{R}^2 \to \mathbb{R}$ which maps prices and degree (or quantity) to profits

$$\pi_{f_j}(t) = p_j d_j(t) - c_j(d_j) \tag{12}$$

where $c_j: \mathbb{R} \to \mathbb{R}$ is an operating cost function (i.e. not including wages) that, for our purposes, we will assume the agent is endowed with. Notice from this that the distribution of earnings has the potential to have a high skew especially given the results of Bianconi and Barabasi (2001). A firm that is "fit" in the sense of having a higher degree will continue to gain "fitness" provided that, on average, it is more fit in the price set than the other firms, so much so that the distribution of income may not be proportional to the distribution of prices

[16] This is of course a very strong assumption to make, though the direction of results still stands if we include the whole history of relative prices. However, we would do well to keep in mind that in reality, price history is vitally important.

around the mean. So, while (11) does seem rather complex, it still has a simple narrative to tell. In (10) we essentially have the preferential attachment law in (3) where the rate of degree accumulation is increasing in the current degree.

Where this model departs from most network models is that it tells us what is *causing* preferential attachment. In this economic system, preferential attachment arises out of the behaviour of each of the agents, whose choices are driving the system equation above. Specifically, as our defence of these equations' assumptions illustrates, preferential attachment arises out of the bounded rationality of individuals within this market system. Since individuals have limited cognitive ability, their set of considered alternatives is made up of firms that *ceteris paribus* have a larger customer base and then, when making decisions about which firm to purchase from, they use *inter alia* "size" as a proxy for quality, or resort to choosing the most "available" option, in the proximity, or reputational sense.

Another point of interest is that the structure of the system emerges completely through the actions of the individuals within it and is in this sense a "self-organisational system" of the sort described by Foster (2005b). But also present in this model are terms bearing a close resemblance to the replicator dynamic models introduced to economics to model market share promoted, most notably, by Metcalfe (1998), and indeed the relative price term in this equation was inspired by that work. The preferential attachment process here, like a replicator dynamic, assigns more and more market share to "successful" or "fit" firms in the sense that firm's with cheaper products on average attract more business, while firms with a larger customer base appear to have a more satisfactory product, causing the distribution of income to skew toward these dominant firms.

3.3 A model of firm level inequality

This model can explain why we see discrepancies between the incomes of small business owners, medium enterprise owners and entrepreneurs of all stripes. But the income of most individuals is derived from working within productive organizations. How do we explain wage differentials? Traditional economic theories of the firm rely on a production function and explain how the derivatives of this production function determine the productivities of various inputs and hence their returns. This is not entirely fallacious, but to define a production function requires an integral space in the domain of the function across which it is defined, a complete network. Such a "black box" view of the firm is prevalent in many of the canonical texts of microeconomics but has been criticised in particular by Williamson (2002) for leaving unstudied a crucial part of economic activity.

Instead of an input-output table we can think of the firm as a production *graph* so that the firm itself is a subsystem of the broader market. That is, firm f_j at any given time can be characterised as a graph

$$f_j = \{N_f^t \; g_f^t(N_f^t)\}$$

where N_t is the set of workers used in producing the firm's output indexed by their date of entry and $g_f^t(N_f^t)$ is a set of connections between them representing their interaction in the production process.

A similar story of concentration of activity around certain agents within the firm as in the market is, we would suggest, what is driving wage differentials. It seems obvious from looking at any organisation chart that firms are inherently hierarchical structures, and tend to be concentrated around particular central co-ordinating jobs such as managers. These jobs are essentially natural monopolies, not easily divisible but much more crucial to the firm's continued operation than other more divisible jobs, and hence they attract more value. The firm system however, is slightly different to the market system in that the firm is not a "self-organised" system as the market is. Instead, those at the top of the hierarchy try to allocate resources to where they are best used. As per Williamson (2002) and Simon (1957) we can see our firms as hierarchical systems of individuals within a governance structure, or organisation, bound by a contract to engage in certain production activities in return for a wage. In particular, it seems reasonable to assume that remuneration out of firm earnings is proportional to the "criticality" of the individual to the production system. So wages would be proportional to the degree $d_{n_i}(t)$ as a proportion of the total degree of the production system and then adjusted by some scaling factor $\zeta_{n_i} \in (0,1]$ reflecting the bargaining power of the individual n_i in wage negotiations so that

$$w_{n_i} = \zeta_{n_i} \left[\frac{d_{n_i}(t)}{\sum_{n_k \in f_k} d_{n_k}(t)} \pi_{f_j}(t) \right] \tag{13}$$

In this particular case, suppose that the firm's management allocate degree to particular agents within the system according to the skill level ϕ_i of agent relative to some required skill level $\bar{\phi}$ within the firm. An agent with a higher degree can be interpreted as a higher degree of centrality within the firm's production system, as the worker will be interacting with more of their colleagues to produce output. Clearly, it would be more efficient for managers to promote higher skilled individuals to more central to the production process as managers themselves, co-ordinators of shopfloors, senior staff, etc. So we could say that without any assumptions on the growth in the size of the firm through time, the change in the degree of the agent (a quantification of "promotion") depends on their current degree and their relative skills

$$\dot{d_{n_i}}(t) = \delta_{n_i}(\phi_i - \bar{\phi}) d_{n_i}(t)$$

where δ_{n_i} is an individual specific constant capturing the ability of the individual to make their skills and potential known. Again, we have a preferential attachment phenomenon within this system, though this time it is due to the necessities of the firm existing as a command economy in some guise. This differential equation has an elegant solution, setting $d_{n_i}(i) = \nu_{n_i}$

$$d_{n_i}(t) = \nu_{n_i} \exp\{(t-j)\delta_{n_i}(\phi_i - \bar{\phi})\} \tag{14}$$

We can then easily determine the dynamics of wages for each worker in a given firm $f_j \in F_t$ by substitution of (14) into (13). A point of interest to note before turning to an analysis of earnings is that a firm being more profitable than another would indicate that there are potentially more profits to be paid to the factors of production, and if the labour market itself is a non-complete network with only localised competition (which we observe in reality), this would allow wage differentials for similar positions.

3.4 Earnings

To summarise our model, we can characterise earnings from the two systems under analysis. Substituting (5.11) into (5.12) the profits of a firm in the market system are

$$\pi_{f_j}(t) = p_j \nu_j \exp\left[\frac{\bar{\kappa}_j \bar{\alpha}}{2} \sum_{f_\ell \in F_t} (t - \max\{j, \ell\}) \delta_{j\ell}\left(p_\ell - p_j\right)\right] - c_j\left(d_j\right) \tag{15}$$

and similarly, substituting (5.14) into (5.13) we get wages of a worker within the firm

$$w_{n_i} = \zeta_{n_i}\left[\frac{\nu_{n_i}\exp\{(t-j)\delta_{n_i}(\phi_i-\bar{\phi})\}}{\sum_{n_k\in f_k} d_{n_k}(t)}\pi_{f_j}(t)\right] \tag{16}$$

Clearly, these equations are complicated and cannot be easily "solved" for their distributions analytically, although it would be possible to simulated distribution in such a system. However, we can still see there is a high potential for skew, even if the "fitness" parameters p_j and ϕ_i are even normally distributed (as demonstrated by Bianconi and Barabasi (2001)). As importantly, we can also see what it takes to achieve a higher income and understand how this is not entirely controlled by the choices of an individual, and in this sense, even from this highly stylised model we can understand more fully how skewed distributions of income can arise without having to make *ad hoc* divisions of the labour force into skilled and unskilled workers or capitalists and workers the centrepiece driving inequality.

An individual within the firm needs a high skill level, which can to some degree compensate for a late arrival and a poor "start-point" within the production system ν_{n_i}, both of which largely depend on luck. But moreover, this individual must also have bargaining power within a *profitable* firm. In order to be profitable, the firm will preferably have entered the market relatively early in order to establish and attract a base level of business. But also, a necessary condition for success is that it be in the considered sets of a large number of consumers, which requires it to be sizeable in the first place. As time passes in this system preferential attachment becomes stronger and the effect of price differentials will have a compounding effect on profitability, as will the firm's ability to attract customers from its rivals. However, as with the individual within the firm, not all of success can be put down to choice. The firm needs to be lucky enough to enter the market early and establish the basis for preferential attachment, and will also need there to be an environment such that $\delta_{j\ell}$ reflects the "true" ability of the business to compete, i.e. rent-seeking by established firms must not be too pervasive.

4. Conclusions

We have attempted to lay a foundation for a theory of income distribution based on complex systems science. Moving away from the view of the economy as akin to an electromagnetic field to a network with incomplete connections associated with trade and contracting, we have argued that boundedly rational behaviour can result in concentrated power nodes in the network structure of an economic system. This phenomenon has been widely studied in the complex systems literature as “preferential attachment” and known to concentrate structure around a few central elements of a system. Since, in an economic system, the relevant connections are within markets and within firms, in the form of trading and contracting, complex system analysis allows us to understand how income is distributed. Space has prevented us from offering more than a basic version of this kind of theory of income inequality so there remains considerable scope for further work.

University of Queensland, Australia 7 May 2013

References

Alghmandi, M., & McDonald, S. (2012). *The Emergence of a Small World in a Network of Research Joint Ventures*. Paper presented at the 14th International Schumpeterian Society Conference, Brisbane.

Ariely, D. (2008). *Predictably irrational : the hidden forces that shape our decisions* (1st ed.). New York, NY: HarperCollins.

Barabasi, A., & Albert, R. (1999). Emergence of Scaling in Random Networks. *Science, 286*(5439), 509-512.

Bianconi, G., & Barabasi, A. (2001). Bose-Einstein Condensation in Complex Networks. *Physical Review Letters, 86*(24), 5632-5635.

Corso, G., Lucena, L., & Thome, Z. (2003). The small-world of economy: a speculative proposal. *Physica A, 324*, 430-436.

Dahui, W., Li, Z., & Zengru, D. (2008). *Bipartite Producer-Consumer Networks and the Size Distribution of Firms; unpublished mimeo*.

de Solla Price, D. (1965). Networks of Scientific Papers. *Science, 149*(3683), 510-515.

Earl, P. (1990). Economics and psychology: a survey. *The Economic Journal, 100*(402), 718-755.

Earl, P., & Wakeley, T. (2010). Alternative perspectives on connections in economic systems. *Journal of Evolutionary Economics, 20*(2), 163-183.

Foster, J. (1993). Economics and the self-organisation approach: Alfred Marshall revisited? *The Economic Journal, 103*(419), 975-991.

Foster, J. (2005a). From simplistic to complex systems in economics. *Cambridge Journal of Economics, 29*(6), 873-892.

Foster, J. (2005b). The self-organizational perspective on economic evolution: a unifying paradigm. In K. Dopfer (Ed.), *The Evolutionary Foundations of Economics* (pp. 367-390). Cambridge: Cambridge University Press.

Friedman, M. (1953). The Methodology of Positive Economics. In M. Friedman (Ed.), *Essays in Positive Economics* (pp. 3-43). London: University of Chicago Press.

Frydman, R., & Goldberg, M. (2011). *Beyond mechanical markets : asset price swings, risk, and the role of the state*. Princeton: Princeton University Press.

Hausman, D. (1989). Economic Methodology in a Nutshell. *The Journal of Economic Perspectives (1986-1998), 3*(2), 115-115.

Jackson, M. (2010). *Social and Economic Networks*. Princeton: Princeton University Press.

Jackson, M., & Rogers, B. (2007). Meeting Strangers and Friends of Friends: How Random Are Social Networks? *American Economic Review, 97*(3), 890-915.

Kahneman, D. (2003). Maps of Bounded Rationality: Psychology for Behavioral Economics. *American Economic Review, 93*(5), 1449-1475.

Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica, 47*(2), 263-291.

Kim, J., & Jo, H. (2010). A signalling explanation for preferential attachment in the evolution of social networks. *Journal of Evolutionary Economics, 20*, 375-393.

Lawson, T. (2003). *Reorienting economics*. London ; New York: Routledge.

Markey-Towler, B., & Foster, J. (2013). *Why economics can say little about inequality*. School of Economics Discussion Paper No. 476. University of Queensland.

Markose, S. (2005). Computability and Evolutionary Complexity: Markets as Complex Adaptive Systems (CAS). *The Economic Journal, 115*(504), F159-F192.

Mas-Colell, A., Whinston, M., & Green, J. (1995). *Microeconomic Theory*. Oxford: Oxford University Press.

Metcalfe, J. (1998). *Evolutionary Economics and Creative Destruction*. London: Routledge.

Milgram, S. (1967). The Small World Problem. *Psychology Today, 2*, 60-67.

Mirowski, P. (1991). *More Heat Than Light: Economics as Social Physics, Physics as Nature's Economics*: Cambridge University Press.

Nelson, R., & Winter, S. (2002). Evolutionary theorizing in economics. *The Journal of Economic Perspectives, 16*(2), 23-46.

Newman, M. (2003). The Structure and Function of Complex Networks. *SIAM Review, 45*(2), 167-256.

Potts, J. (2000). *The New Evolutionary Microeconomics: Complexity, Competence, and Adaptive Behaviour*. Cheltenham: Edward Elgar.

Rabin, M. (1998). Psychology and Economics. *Journal of Economic Literature, 36*(1), 11-46.

Salverda, W., Nolan, B., & Smeeding, T. (2009). Introduction. In W. Salverda, B. Nolan & T. Smeeding (Eds.), *Oxford Handbook of Inequality*. Oxford: Oxford University Press.

Schwartz, B. (2005). *The Paradox of Choice: Why More Is Less*. New York: HarperCollins.

Schweitzer, F., Fagiolio, G., Sornette, D., Vega-Redondo, F., & White, D. (2009). Economic Networks: What Do We Know and What Do We Need to Know? *Advances in Complex Systems, 12*(4-5), 15.

Simon, H. (1955a). A Behavioral Model of Rational Choice. *The Quarterly Journal of Economics, 69*(1), 99-118.

Simon, H. (1955b). On a Class of Skew Distribution Functions. *Biometrika, 42*(3/4), 425-440.

Simon, H. (1957). The Compensation of Executives. *Sociometry, 20*(1), 32-35.

Simon, H. (1959). Theories of Decision-Making in Economics and Behavioral Science. *The American Economic Review*, *49*(3), 253-283.

Simon, H. (1962). *The architecture of complexity.* Paper presented at the Proceedings of the American Philosophical Society.

Simon, H. (1998). Discovering Explanations. *Minds and Machines, 8*(1), 7-37.

Tversky, A., & Kahneman, D. (1974). Judgment under Uncertainty: Heuristics and Biases. *Science, 185*(4157), 1124-1131.

von Bertalanffy, L. (1950). An Outline of General System Theory. *The British Journal for the Philosophy of Science, I*(2), 134-165. doi: 10.1093/bjps/I.2.134

von Weizsacker, R. (1993). *A Theory of Earnings Distribution*. Cambridge: Cambridge University Press.

Watts, D. J. (2004). The "New" Science of Networks. *Annual Review of Sociology, 30*(1), 243-270.

Williamson, O. (2002). The Theory of the Firm as Governance Structure: From Choice to Contract. *The Journal of Economic Perspectives*, *16*(3), 171-195.