Are Early Educational Choices Affected by Unemployment Benefits? New Theory

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Abstract

In this paper we contribute to the recent literature that has investigated the potential economic advantages of unemployment benefits, by developing a model where young individuals in education have to irreversibly choose a degree of specialisation positively related to future unemployment risk, market wages for different types of specialisation have to compensate risk-averse individuals for these risks. Unemployment benefits affect the incentives for specialisation and thereby the long-run composition of the workforce, the wage structure, and output. I address this issue in OLG search models with risk-aversion, talent heterogeneity, endogenous specialisation distributions, and competitive wage formation.

The main results are that 1) higher unemployment benefits are related to higher mean specialisation and at low levels raise efficiency and welfare; 2) even though wages compensate for risks, unemployment risk and individual wages are generically negatively related when there is talent heterogeneity; 3) the composition of the workforce changes slowly with changing incentives, leading to long lags between welfare system changes and average outcomes (unemployment and output) and lead to longer lasting effects of shocks in regions with higher unemployment benefits.

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1 Introduction and literature

Recent years have seen a considerable growth of interest by economists into the potential advantages of unemployment benefits (UB) on the decision-making and welfare of individuals following their return to work. One particular advantage, empirically assessed by Gruber (1997), is that benefits smooth incomes. A second possible advantage occurs when benefits increase the outside option of unemployed individuals and hence allows them to make better work-related decisions (Mortensen and Pissarides, 1999). For example, Acemoglu and Shimer (1999, 2000) show that unemployment benefits provide an added incentive for the unemployed to hold out for jobs that involve a higher risk of lay-off or that are simply harder to find. In their model, this can raise welfare and output for low levels of benefits. Similarly, Marimon and Zilibotti (1999) extended the basic argument made at least as early as Burdett (1979), that when individuals receive unemployment benefits they have a greater incentive to hold out for 'better matches' i.e. jobs that are higher paying. When waiting for better job matches has an external beneficial effect on the probability that others find their optimal match, unemployment benefits can be welfare and output increasing. Whilst these theoretical arguments are appealing, it is not clear from the empirical evidence if such potential advantages of unemployment benefits are enough to outweigh the decreased incentives to search for jobs in the presence of moral hazard (e.g. Hopenhayn and Nicolini, 1997).

In this paper, we aim to contribute to this literature by investigating the effects that unemployment benefits have on the economy via the irreversible choices by youngsters before they enter the labour market, in the form of education or training. Our main argument runs as follows. Education not only affects the future level of expected wages, but also ties an individual to the unemployment risks of the jobs that the individual is then suited for. Youngsters who choose a very general education can apply to and function effectively in many jobs. Consequently, their job-arrival rate will be high once they enter the labour market and their job destruction rate will be lower.
In contrast, those who opt for more specialist skills will have a far smaller pool of jobs to search over and will be more vulnerable to technology shocks that might diminish or even wipe-out the value of such specialist knowledge. Therefore, the choice of the type of education or training to undertake is also a choice about the level of future employment risk. This choice may change if the risks or rewards to various jobs change. Consequently, the presence (or introduction) of a risk-sharing device like unemployment benefits reduces the importance of these risks and therefore will affect educational choices. This in turn will change unemployment dynamics and average outcomes, such as average output, and relative wages.

To investigate the importance of risk-relevant irreversible early educational choices within a general equilibrium OLG-model. In the simplest version of the model, identical risk-averse individuals who live only two periods choose a degree of specialisation in the first period. Greater specialisation translates to an increased risk of unemployment in the second period. Those who work with a specific specialisation make intermediary goods that are transformed into a homogeneous consumption good by a continuous CES production function. This production function clearly illustrates macroeconomic complementarities between the activities of individuals that so far have always been largely unexplored in the job search literature. The resulting model yields an observed wage distribution that is unique and that can have virtually any shape, depending on the underlying functions.

Introducing unemployment benefits paid for by a single marginal tax rate turns out to make investments into more risky occupations relatively more attractive, which increases production, expected utility and unemployment. In this respect, the insurance provided by the benefits has the same effect in this model as in the models by Acemoglu and Shimer (1999, 2000) and Marrimon and Zilibotti (1999), albeit via a different mechanism. An empirical prediction of these results is that the percentage of individuals taking general education will be lower in an geographical area or country with higher unemployment benefits. As a stylised example, if we view the US as an area with relatively low unemployment benefits and the countries in the EU as an area with high unemployment benefits, this would translate in there being more vocational and specialist education in the EU than in the US. Ashton and Green (1996) have indeed documented such a difference.

Apart from risks and macro-economic complementarities, innate talents are also an important determinant of wages. We therefore also extend the two-period model to include talent heterogeneity, where talent is defined as
an efficiency unit of productivity for any specialisation. This generalisation is found not to alter the previous conclusions: we find that under most utility functions, those with highest total wages are then also the ones with least risks. Those with most talent in essence ‘buy’ higher security by taking low-risk jobs, whilst their total wages are still higher due to their greater talent. This extended model is therefore capable of reconciling the theoretical prediction that individuals would have to get higher wages when running higher risks and the empirical evidence that suggests that those with higher risks often get paid less (e.g. Hwang et al., 1992).

To study the dynamics of the model, we first extend the two-period model to an infinite period continuous-time OLG model where a higher degree of specialisation implies lower job-finding rates and higher job-destruction rates. For the most plausible parametric assumptions, the main conclusions from the two-period models are found to also apply to the continuous case. We then calibrate a dynamic 20-period OLG model by which we investigate the dynamics of the economy after unanticipated shocks. We find that, because educational choices are irreversible, changes in parameters tend to have delayed effects. In particular, the effects of reducing taxes and unemployment benefits on the unemployment rate takes far longer to work through the economy when there is a large long-run decrease in the number of specialists, simply because it takes several cohorts before the new equilibrium mix between generalists and specialists is achieved. This explanation of unemployment rates combines shocks with institutional factors, in line with the recommendations of Blanchard and Wolfers (2001).

General economic shocks turn out to have a longer persistence in an economy with high unemployment benefits compared to one without. The intuition behind this is that an economy with unemployment benefits has more specialists who have lower job-finding rates. Hence, following an economic shock it takes longer for these individuals to find a job again than it takes the generalists, of which there are more in the economy with low unemployment benefits. This might be another potential explanation for the sluggishness with which the EU unemployment rates fell following the oil price shocks of the early 1970’s (Blanchard and Wolfers, 1999). Sargent and Ljungqvist (1998) explained this same phenomenon by arguing that unemployment benefits increased the willingness of individuals to wait for better jobs. Sargent and Ljungqvist combined this with the assumption that individual loose skills in unemployment. In this case, unemployment benefits then aggravated the negative shocks in their model because the high un-
employment benefits indirectly and adversely changed the characteristics of individuals whilst they were already in the labour market. The key difference in our model is that there is no resort to a reduction in skills during unemployment, but rather an effect of unemployment benefits on the long-term composition of the workforce. Hence we stress that the US and EU labour market were already very different before the oil price shocks, even if at that moment unemployment benefits would have been equated. The evidence for the existence of loss-of-skills or stigma as a result of prolonged unemployment is still very limited (see e.g. Heckman and Borjas, 1980, Lynch 1989; Frijters, Van den Berg and Lindeboom 2001, Bonnal et al. 1997). In this paper we propose an explanation for persistent unemployment that does not require changes in the characteristics of individuals during unemployment.

Overall, we aim to contribute to the literature in a number of ways. Firstly, we explicitly allow for macro-economic production complementarities in a search model. Previous search models have typically taken the productivity of a specific match to be independent of the activities of individuals in different types of activities (e.g. Burdett and Mortensen 1998; Moens 1997; Acemoglu and Shimer 1998; Marimon and Zilibotti 1999; Pissarides 1990; Fredriksson and Holmlund 2001; Petrongolo 2001). Secondly, we solve for entire wage distributions in the context of individual heterogeneity and competitive wage setting. This sets us apart from the extensions of the Burdett-Mortensen (1998) model that also solve for wage distributions and individual heterogeneity (e.g. Bontemps et al 2000), but assume that labour markets are segmented and that not all firms are at the production frontier. It also distinguishes us from Acemoglu and Shimer (1999) and Moens (1997), who also have competitive wage setting, but do not solve for distributions and have no heterogeneity in production talents. Finally, we believe that this paper original in solving a continuous time OLG model with risk-aversion, heterogeneity, arbitrary utility functions, job search and job destruction. The paper is set out as follows: In Secion 2 we describe our two-period general equilibrium OLG model, without and then with individual talent heterogeneity. We highlight the dynamics of the model in Section 3 by extending the model to the case of continuous time and by undertaking a number of calibration exercises. We also discuss some limitations of our model and possible future research directions. Section 4 concludes the paper.
2 A 2-period general equilibrium OLG model

Let there be a continuum of individuals each period of measure 1. A generation of ex ante identical individuals lives two periods, where one generation is in period 1 whilst the previous generation is in period 2. In the first period, individuals can choose a level of specialisation \( 1 \geq \theta \geq 0 \) as their type of skill in the second period. The cumulative density of individuals at time \( t \) choosing specialisation \( \theta \) is denoted by \( F_t(\theta) \), where \( \int_0^1 dF_t(\theta) = 1 \). Focusing only on steady states, we will for convenience drop the time subscripts.

In the second period, individuals search production facilities. The probability of finding and keeping a job is \( h(\theta) \) where \( h(0) = 1 \), \( h(1) > 0 \) and \( \frac{\partial h}{\partial \theta} < 0 \). More specialised jobs are by definition harder to find and to keep. A worker produces one unit of a specialisation-specific good which is an intermediary into a final good, where the production technology is CES. Total production is therefore:

\[
y = \left[ \int_0^1 (f(\theta)h(\theta))^\gamma d\theta \right]^{1/\gamma} \tag{1}
\]

where \( f(\theta)h(\theta) \) is the total amount of the intermediary good of type \( \theta \) that is produced, where \( f(\theta) \) is the p.d.f. of \( \theta \), and where \( 1 > \gamma > 0 \).

Individuals are assumed to be forward looking risk-averse rational utility maximizers. Expected utility is:

\[
E\{U(\theta)\} = h(\theta)u((1 - \tau)w(\theta)) + (1 - h(\theta))u(b) \tag{2}
\]

where \( \tau \) is the tax rate, and \( b \) is the level of unemployment benefits. A requirement for \( b \) to be feasible is that \( b = \frac{\gamma y}{\int_0^1 f(\theta)h(\theta)d\theta} \) in equilibrium. Wages are set competitively: \( w(\theta) = \frac{\partial y}{\partial f(\theta)h(\theta)} = (f(\theta)h(\theta))^{\gamma - 1}y^{1-\gamma} \) when \( f(\theta) \) exists and \( w(\theta) = 0 \) at positive mass points of \( F(\theta) \). Also, \( u(.) \) is convex, its derivative exists and is continuous, with \( u(\infty) = \infty \). Non-negative non-work incomes ensure that \( u(0) > -\infty \).\(^1\) We restrict initial attention to cases where \( b < w(\theta) \).

First, a standard argument holds that in equilibrium, \( f(\theta) \) has to be continuous if \( h(\theta) \) is continuous. The reason is that if \( f(\theta) \) is not continuous and, say, drops at some point \( x \), then wages will make a discontinuous jump

\(^1\)This provides a lower bound to the utility function. Without such a bound, we would run into the St. Petersburg paradox i.e. where individuals would not have complete preferences. For a discussion on this problem with Von Neumann - Morgenstern expected utility functions, which was first noted by Savage, see Aumann (1977).
at \( x \). There would then be a first order gain to be made for individuals just before \( x \) to change their choice of \( \theta \) to \( x \), with only a second-order loss of finding a job. It is also the case that \( f(\theta) > 0 \) for every \( \theta \) and every utility function because \( w(\theta) = \infty \) when \( f(\theta) = 0 \). Choosing \( \theta \) will hence be preferred over choices with wages less than infinite, which ensures that \( f(\theta) > 0 \) for any \( \theta \). These regularities in \( f(\theta) \) mean that \( w(\theta) \) is also continuous when \( h(\theta) \) is continuous.

Having checked that minimal regularity conditions apply, we can now characterise the equilibrium by noting that each choice of \( \theta \) must yield the same expected utility. Differentiating \( E[U(\theta)] \) with respect to \( \theta \) and setting to 0, gives the main solution equation of the model:

\[
-h'(\theta)(u((1 - \tau)w(\theta)) - u(b)) = (1 - \tau)w'(\theta)h(\theta)u'((1 - \tau)w(\theta)) \quad (3)
\]

for differentiable points of \( h(\theta) \). This equation clearly shows that wages are increasing in the risk \(( = 1 - h)\) and hence increasing in \( \theta \). In order to judge the efficiency of the outcome, consider what would be the output maximizing choice of \( f(\theta) \), denoted as \( f^e(\theta) \). Due to the properties of the CES-function, \( y \) is maximised when \( h(\theta)w(\theta) \) is constant. Hence \( w^e(\theta) \propto \frac{1}{h(\theta)} \) and \( \frac{dw^e}{dh} = \frac{w^e}{h} \).

This corresponds to \( f^e(\theta) \propto h(\theta)^{-\frac{\gamma}{\tau}} \). The equilibrium of the model is now characterised in Proposition 1.

\[
\text{Proposition 1. At } \tau = 0, \text{ the model has a unique equilibrium solution } f(\theta) \text{ which is inefficient. Any continuous distribution of observed wages } z(w) \text{ can be supported as long as } z(x) = 0 \text{ for all } 0 < x < w_{\min}.
\]

Proof: Existence and uniqueness is proven in the Appendix. Inefficiency can be seen by noting that we can differentiate \( E\{U(\theta)\} \) with respect to \( h \), obtaining \( \frac{dw}{d(1-h)} = \frac{u(w(\theta)) - u(0)}{h(\theta)u'(w(h))} > \frac{w}{h} \) because of the risk-aversion in \( u(.) \). This in turn implies inefficiency of the equilibrium. The shape of observed wages \( z(w) \) follows because we can write \( z(x) = \frac{1}{u(0)} f(\theta) \) where \( \theta = \arg \theta w(\theta) = x \). Since \( \frac{w(1)}{w(0)} \) is not bounded from above, any observed continuous density function that is bounded from below can then be supported by an appropriate choice of \( h(\theta) \) and \( u(.) \).
The intuition behind existence is that the main solution equation uniquely maps \( w(\theta) \) as a continuous function of \( w(0) \). Conversely, this leads to a unique \( f(\theta) \) and \( y \), both continuous in \( w(0) \). Because \( w(0) \) is itself uniquely determined by \( f(0) \) and \( y \), there is a closing equation for which a fixed point argument shows it has a solution for at least one \( w(0) \). Uniqueness then follows because it is not possible to change \( f(\theta) \) without increasing some wages and decreasing others (proven in Lemma 1 in the Appendix). Since equilibrium implies that individuals have equal utility, it cannot be the case that there is a second equilibrium in which there are some better off and others strictly worse off.

The question now is whether this outcome can be improved upon by introducing an unemployment benefit. Four general results can be obtained:

**Proposition 2.** i) At 0, an increase in \( \tau \) increases utility, unemployment and output. ii) There is a critical level \( \tau^* \) above which all individuals would prefer not to work where \( \tau^* \) solves: \( (1 - \tau)w(0) = b \) and where \( w(\theta) = w(0) \) and \( f(\theta) \propto h(\theta)^{-1} \). iii) There is no level of \( \tau \) that yields efficiency. iv) There exists at least one equilibrium \( b \) for any \( \tau < \tau^* \), and the equilibrium is unique in a region below \( \tau^* \).

Proof of iii): Suppose that there is a \( \tau \) that maximizes output, this would have to mean that \( \frac{w}{d(1-h)} = \frac{u(1-\tau)w(h)}{(1-\tau)hu'(1-\tau)w(h)} = \frac{w}{h} \) for any \( \theta \). In turn, this would imply that \( u(x) = u(b) = xu'(x) \) where \( x = (1 - \tau)w(\theta) \). This equation can then only hold for a continuum of \( x \) when \( u''(.) = 0 \) and \( \tau = 0 \), which implies that individuals would have to be risk-neutral which contradicts the assumptions of the model. The proofs of i), ii), and iv) are provided in the Appendix.

The intuition behind this result is that at low levels of tax, the introduction of a benefit induces individuals to take more risks, which increases output, unemployment and utility. Since the utility of each choice is the same, the utility increase following an increase in \( \tau \) is ex ante the same for each individual. Because of the irreversibility of specialisation, however, ex post some individuals will not want taxation. The individuals with \( \theta = 0 \) for instance, run no risks ex post and will therefore oppose any tax ex post, even though they have benefitted from it ex ante. The reason why there is no tax level that will yield the maximum output is that ex post individuals want different levels of insurance: given that individuals will choose different risks, the
optimal insurance should differ for different levels of risk. The ex post risk-pooling between individuals with different risks means that those who run little risks will be over insured and those who run high risks will be under insured. This points to a key difference between considering wage distributions instead of a single wage outcome, such as in Acemoglu and Shimer (1999), where there is a single tax that restores efficiency.

The key features of this static model are highlighted in Figures 1, 2 and 3. Figure 1 shows the relationship between $f(\theta)$ and $\tau$.

Figure 1: Relation between risks and the distribution of risk choices.

Parametric assumptions: $h(\theta) = 1 - 0.5 \times \theta$, $\gamma = 0.5$, and $u(y) = y^{0.1}$.

The straight lines correspond to model outcomes under various tax regimes, whereas the dotted line shows the efficient outcome. The distribution of risks when $\tau = 0$ is too skewed to the left in comparison to efficiency. As $\tau$ increases, the distribution becomes less tilted. At $\tau = 0.09$, the distribution is almost identical to the efficient distribution. When $\tau = \tau^* = 0.25$, we have the limit case, where all wages are equal and hence $f(\theta) \propto \frac{1}{h(\theta)}$. 
Figure 2: Relationship between wage profiles and taxation

Figure 2 shows the wage profile for the same tax regimes. At $\tau = 0$, wages rise very quickly with risks and hence with $\theta$. As $\tau$ increases, the wage profile becomes less skewed to the right. At $\tau = 0.09$, we obtain the almost efficient wage curve. In the very limit case of $\tau = 0.25$, wages are constant.

Figure 3: Effect of taxation on utility, production, and unemployment

Finally, Figure 3 highlights the relationship between $\tau$ and utility, pro-
duction and unemployment. Each rises fairly rapidly for very low levels of $\tau$. Production peaks quickly i.e. at $\tau = 0.09$, and then slowly levels off. Utility peaks much later at $\tau = 0.22$, and unemployment increases till the limit of $\tau = 0.25$.

In this homogeneous-worker model, the main mechanism is the trade-off between wages and risks. The higher the wages, the higher the risk of unemployment. This corresponds to the results of the classic hedonic wage literature that started with Rosen (1972), where individuals pay with lower wages for good job amenities, in this case low risks of unemployment. This trade-off is also present in Acemoglu and Shimer (1999). It is empirically very implausible, however, that the better paid run more chance of being unemployed, except perhaps for some very high paying jobs such as CEO's. Daniel and Sofer (1998) show that in the majority of recent empirical studies, it is found that the higher the wage, the better the amenities, which is counter to our simple model's prediction. Unemployment levels are, for instance, generally higher for individuals at lower earning potential levels.

Our basic model then appears to conflict with the available empirical evidence. A probable reason for this is that we have so far neglected individual heterogeneity i.e. individuals earning higher wages and enjoying lower amenities probably differ in more ways than merely their early life choices. If there is heterogeneous talent then it might well be the case that there is assortative matching taking place between talent and low-risk early life choices that impacts on the observed relationship between risks and wages. To investigate this possibility further, we therefore extend our basic model to include individual talent heterogeneity.

2.1 Including individual talent heterogeneity

We assume that individuals have an innate talent $1 > q > 0$, that is drawn from a differentiable population distribution $G(q)$. We interpret this talent as an efficiency unit. This implies that the wage for an individual who works with talent $q$ and choice of specialisation $\theta$, is $qw(\theta)$. We now conjecture that it will be the case that all individuals with a certain talent will choose a particular $\theta$ and that this implicit function $q(\theta)$ is either increasing or decreasing on its domain (a no-crossing conditions). The validity of this conjecture will be confirmed ex post. The efficiency units of labour supplied for speciality $\theta$
is then \( g(q(\theta))\frac{dq(\theta)}{d\theta}q(\theta)h(\theta) \).\(^2\) Total output can therefore be written as:

\[
y = \left[ \int_0^1 \left( \frac{dq(\theta)}{d\theta} g(q(\theta))q(\theta)h(\theta) \right)^{\gamma} d\theta \right]^{1/\gamma}
\]

with the wage per efficiency unit \( w(\theta) = \frac{\partial y}{\partial g(q(\theta))\frac{dq(\theta)}{d\theta}q(\theta)h(\theta)} = \left( \frac{dq(\theta)}{dq} \right)_g g(q(\theta))q(\theta)h(\theta) \) \( y^{1-\gamma} \).

Individual utility maximization will now mean that at \( \theta \), an individual with quality \( q(\theta) \), is indifferent in his or her choice:

\[
\frac{\partial \{ h(\theta)u((1 - \tau)qw(\theta)) + (1 - h(\theta))u(b) \}}{\partial \theta} \bigg|_{q(\theta)} = 0
\]

For each quality level \( q \), this requirement leads to an indifference curve on the \( \{\theta, w\} \) space. Applying the argument in the seminal paper by Rosen (1972), this means that in equilibrium the wage curve \( w(\theta) \) will be the envelope of these indifference curves, which in turn determines \( q(\theta) \). In order to calculate the equilibrium in any practical instance, we note that we can simply trace the condition above for any \( w(0) \) to arrive at a \( w(\theta) \). This determines every other outcome, and we then choose the \( w(0) \) that is itself implied by the outcomes. Proposition 3 provides conditions for uniqueness and of the main relationship of interest.

Proposition 3. i) The model with individual heterogeneity has a unique \( w(\theta) \), and hence a unique level of \( E\{U(q)\} \). ii) The equilibrium is inefficient for any \( \tau \), through \( \frac{dw}{d\tau} \bigg|_{\tau=0} > 0 \) and \( \frac{dU(q)}{d\tau} \bigg|_{\tau=0} > 0 \). iii) Any continuous distribution of observed wages is supported. iv) Individuals with \( q < \frac{b}{w(1)} \) are voluntarily unemployed. v) Total wages always increase with talent \( \frac{dwq}{dq} > 0 \), but will only decrease with risk \( \frac{dwq}{d(1-h)} < 0 \) when \( \left\{ u(x) - u(b) \right\} \left( -\frac{w'}{w(x)} x - 1 \right) + xu'(x) > 0 \), which is certain to hold when the degree of relative risk aversion does not cross 1.

Proof: The proofs can be found in the Appendix.

We here note that \( \frac{dwq}{d(1-h)} < 0 \) holds for most popular utility functions, such as \( u(w) = \ln(a+w) \) with \( a>0 \), \( u(w) = w^\alpha \) with \( 0 < \alpha < 1 \), and \( u(w) = 1 - e^{-\alpha w} \).
The intuition is that individuals with high innate talents can ‘buy’ security by choosing less risky occupations that pay less per efficiency unit. Observed total wages \( (qw(q)) \) are then increasing in talent but decreasing in risks. In such cases, the observed relationship between risks and wages is spurious and due to innate talent heterogeneity. This is slightly different to the explanation given by Hwang et al. (1998) and Daniel and Sofer (1998). In their models, the same empirical prediction arises because workers with high bargaining power extract surplus from efficient firms by having both higher wages and better amenities. In Hwang et al. (1998) this bargaining power derives from the possibility of on-the-job-search in the presence of heterogeneity in the productivity of firms, whereas in Daniel and Sofer (1998) the bargaining power derives from the presence of a trade union. In our model, however, individual heterogeneity can cause a similar relationship but we do not rely on the presence of a market-distortion such as some firms not being on the production frontier or the presence of trade unions. Our explanation rather stresses unobserved differences in individual characteristics which increase potential wages and thereby the marginal utility for amenities. Due to the spurious relationship between wages and risks on the individual level, we expect different relationships between wages and risks on the aggregate than on the individual level. Supporting evidence for this can be found in Van der Berg and Van Vuuren (2001), who note that risks and wages are positively related at the sector level in the Netherlands, whereas risks and wages are negatively related at the individual level (e.g. Frijters et al., 2001).

The main testable prediction from these static models is that the proportion of individuals in less specialised occupations will be higher in economies with lower levels of welfare. In this respect, it is widely recognised that the welfare system is considered more generous and elaborate in the European and richer Asian countries than in the US. Ashton and Greene (1996) provide some evidence in support of our predictions through a detailed and extensive review of the education and training systems in the OECD countries as well as in a number of the faster growing Asian countries. They note that the education system in the US is indeed quite generalist, with both secondary and tertiary education being based on a wide variety of subjects, and very little specialisation taking place in the formal education system. In contrast, most of the OECD countries have education systems that are far more specialised in nature, with tertiary education in particular being much more focussed on a small subset of subjects. Even much of secondary education
is occupation-specific, with the most extreme case being the vocational system in Germany where large proportions of the population learn only very specific skills.

3 Dynamics

We would now like to explore these issues in a more dynamic framework. To do this we begin by extending the two-stage model above into an infinite horizon, continuous time OLG model, where we focus exclusively on steady states. Following this, in order to investigate what happens after unanticipated shocks to an economy, we then set up an empirical version of the model which we calibrate.

3.1 An analytical dynamic model

Consider an infinite-period continuous overlapping generations model with a total measure of individuals equal to 1. Ex-ante homogenous individuals can be unemployed, employed or can die. At a mortality rate \( m \) individuals die, who are immediately replaced by a new individual. This new individual chooses his or her specialisation and then starts out being unemployed. There is a common discount rate \( \rho \). The distribution of specialisation chosen by the individuals who enter into the economy at time \( s \) is denoted as \( f_s(\theta) \). Calendar time is denoted by \( t \). The job-arrival rate for the unemployed with specialisation \( \theta \) is denoted as \( \lambda(\theta) > 0 \), and the job-destruction rate by \( \delta(\theta) > 0 \). We define specialisation as \( \delta(\theta) < 0 \) and \( \delta(\theta) > 0 \).

The total measure of workers and unemployed with specialisation \( \theta \) at time 0 will equal \( \int_{-\infty}^{0} me^{ms} f_s(\theta) ds \), where \( me^{ms} \) is the density of the individuals alive at 0 who were born at time \( s < 0 \). The density of individuals with specialisation \( \theta \), who entered at time \( s \), employed at time \( t > 0 \), is then \( p(\theta, t - s) f_s(\theta) me^{m(s-t)} \), where the probability \( p(\theta, t - s) \) is defined by the differential equation:

\[
\frac{\partial p(\theta, t - s)}{\partial t} = \lambda(\theta)\{1 - p(\theta, e, t - s)\} - \delta(\theta) p(\theta, e, t - s)
\]

and the initial condition \( p(\theta, 0) = 0 \). Solving this equation leads to:

\[
p(\theta, t - s) = \frac{\lambda(\theta)}{\lambda(\theta) + \delta(\theta)} (1 - e^{-(t-s)(\lambda(\theta)+\delta(\theta))})
\]
This probability has standard properties: \( \frac{\partial p(\theta,t-s)}{\partial t} > 0, \frac{\partial^2 p(\theta,t-s)}{\partial t^2} < 0 \), and 
\[
\lim_{t \to \infty} p(\theta, t - s) = \frac{\lambda(\theta)}{\lambda(\theta) + \delta(\theta)}.
\]
The total population measure of individuals with specialisation \( \theta \) employed at time \( t \) is denoted as \( G_t(\theta) \) and equal to 
\[
\int_{-\infty}^{t} p(\theta, t - s) f_s(\theta) me^{m(s-t)} ds.
\]
Total production in each period is then:
\[
y_t = \left( \int \{G_t(\theta)\}^\gamma d\theta \right)^{\frac{1}{\gamma}}
\]
and wages solve \( w_t(\theta) = \frac{\partial y_t}{\partial G_t(\theta)} \).

We can now find the Euler equations for the value of unemployment and employment that characterise maximising behaviour. Denoting the expected utility value of unemployment as \( V^{UN} \) and the utility value of employment as \( V^{EM} \), there holds:
\[
(\rho + m)V^{UN} = u(b) + \lambda(\theta) \{ V^{EM} - V^{UN} \}
\]
\[
(\rho + m)V^{EM} = u((1 - \tau)w(\theta)) + \delta(\theta) \{ V^{UN} - V^{EM} \}
\]
Solving these equations for \( V^{UN} \) yields
\[
(\rho + m)V^{UN} = \frac{\rho + m + \delta(\theta)}{\lambda(\theta) + \delta(\theta) + \rho + m} u(b) + \frac{\lambda(\theta)}{\lambda(\theta) + \delta(\theta) + \rho + m} w((1 - \tau)w(\theta))
\]
or
\[
(\rho + m)V^{UN} = (1 - a(\theta))u(b) + a(\theta)u((1 - \tau)w(\theta))
\]
where \( a(\theta) = \frac{\lambda(\theta)}{\lambda(\theta) + \delta(\theta) + \rho + m} \). Looking only at RE stationary steady states, \( V^{UN} \) has to be the same for every \( \theta \). This means the solution has to satisfy:
\[
\frac{da(\theta)}{d\theta} [u((1 - \tau)w(\theta)) - u(b)] = -w'(\theta)(1 - \tau)a(\theta)u'((1 - \tau)w(\theta))
\]
where \( \frac{da(\theta)}{d\theta} < 0 \). Writing the continuous time model in this way has consequently provided a solution equation that is virtually the same as in the case of the static model, and for which the arguments in Proposition 1 apply. The solution equation again maps out a unique \( w(\theta) \) from any starting point \( w(0) \). From \( w(\theta) \), we can then map out a unique \( \frac{G(\theta)}{G(0)} \), which in turn uniquely determines a static \( f(\theta) \) and \( y \). Since these in turn lead to a unique \( w(0) \),
there is a closing equation that is continuous and which has to have at least one fixed point. Existence is thereby assured. The arguments on uniqueness are the same because it is again not possible to change \( G(\theta) \) (via \( f(\theta) \)) without making some individuals strictly better and some others strictly worse off, which is not possible. Obviously, \( w(\theta) \) increases in \( \delta(\theta) \) and decreases in \( \lambda(\theta) \).

When it concerns efficiency and the role of \( \tau \), this dynamic model is more problematic because there is no single risk parameter but rather now two, i.e. \( \lambda(\theta) \) and \( \delta(\theta) \). For efficiency to hold, it has to be the case that \( \frac{\partial y}{\partial f(\theta)} = f(0) \gamma_{-1} y_{1-\gamma} = w(0) \). Denoting the efficient solution as \( f^e(\theta) \) and \( w^e(\theta) \), there then holds \( \frac{f^e(\theta)}{f^e(0)} = \bar{p}(\theta)^{\gamma_{-1}} \) where \( \bar{p}(\theta) = \int_0^\infty p(\theta,e,t-s)me^{ms}ds \). Also, \( \frac{d w^e(\theta)}{d(1-\bar{p}(\theta))} = \frac{w^e(\theta)}{\bar{p}(\theta)} \). We can hence examine the efficiency of the equilibrium by checking whether \( w(\theta) = \frac{w(0)}{\bar{p}(\theta)} \) can optimize \( V^{UN} \). First we calculate:

\[
\bar{p}(\theta) = \int_{-\infty}^0 \frac{\lambda(\theta)}{\lambda(\theta) + \delta(\theta)} (1 - e^{s(\lambda(\theta) + \delta(\theta)))}me^{ms}ds
\]

\[
= \frac{\lambda(\theta)}{\lambda(\theta) + \delta(\theta) + m}
\]

and

\[
\frac{d \bar{p}(\theta)}{d \theta} = \frac{\lambda'(\theta) - \lambda(\theta) \frac{\lambda'(\theta) + \delta'(\theta)}{\lambda(\theta) + \delta(\theta) + m}}{\lambda(\theta) + \delta(\theta) + m} < 0
\]

For efficiency to be possible, there would have to hold that:

\[
\frac{dw(\theta)}{d(1-\bar{p}(\theta))} = -\frac{\partial y}{\partial \theta} \left[ u((1-\tau)w(\theta)) - u(b) \right] \frac{d \bar{p}(\theta)}{d \theta} = \frac{\lambda(\theta) + \delta(\theta) + m}{} w(\theta) \frac{\lambda(\theta)}{\lambda(\theta)}
\]

which after some manipulations can be written as the condition that:

\[
xu'(x) = A(\theta) \{ u(x) - u(b) \}
\]

where:

\[
x = (1-\tau)w(\theta)
\]

\[
A = \frac{-\lambda'(\theta) + \lambda(\theta) \frac{\lambda'(\theta) + \delta'(\theta)}{\lambda(\theta) + \delta(\theta) + m}}{-\lambda'(\theta) + \lambda(\theta) \frac{\lambda'(\theta) + \delta'(\theta)}{\lambda(\theta) + \delta(\theta) + m}} > 0
\]
from which it is clear that, under any \( \tau \), this condition can only be satisfied by a linear utility function at particular parameter values. Therefore, in general, the equilibrium is once again never efficient under any tax system.

In order to see whether we can follow the same arguments in the dynamic case as in the two-period case about the effect of increasing \( \tau \) at \( \tau = 0 \), we can note that:

\[
\frac{d w(\theta)}{d(1 - \bar{p}(\theta))}|_{\tau = 0} > A(\theta) \frac{w(\theta)}{\bar{p}(\theta)}
\]

Now, in the case that \( \rho \lambda'(\theta) \leq -\rho \delta'(\theta) \), then \( A(\theta) = 1 \) and hence \( \frac{d w(\theta)}{d(1 - \bar{p}(\theta))}|_{\tau = 0} > \frac{w(\theta)}{\bar{p}(\theta)} \), which corresponds to inefficiently low risk-taking at \( \tau = 0 \), and whose characteristics would correspond to the two-period case. Looking at this condition, this will hold whenever the negative change in job-finding rates is lower than the positive change in job-destruction rates. In these circumstances, the introduction of an unemployment benefit would again improve efficiency. To gauge the likelihood of this condition being true, we have to consider the evidence on \( \lambda'(\theta) \) and \( \delta'(\theta) \). Layard, Nickell and Jackman (1991) report in their survey of duration of work that there is little heterogeneity in job-destruction rates for different individuals whilst there is a great deal of heterogeneity in job-finding rates. A lack of heterogeneity means that the difference in rates between individuals with different characteristics is low. Hence, we take the survey evidence presented in Layard et al. (1991) to mean that \( \lambda'(\theta) < -\delta'(\theta) \approx 0 \).

In the (less likely) case that \( \rho \lambda'(\theta) > -\rho \delta'(\theta) \), there holds that \( A(\theta) < 1 \). For low degrees of risk-aversion, it then becomes possible that \( \frac{d w(\theta)}{d(1 - \bar{p}(\theta))}|_{\tau = 0} < \frac{w(\theta)}{\bar{p}(\theta)} \), in which case individuals take too much risk, even at \( \tau = 0 \). Therefore, we cannot a priori say for sure whether taxes will be output increasing or not. The reason behind this unexpected result is due to discounting: individuals attach more value to what happens in the near future than output maximisation dictates. Then, simply because job-destruction can only occur at a later date than getting a job, individuals attach a disproportionate importance to \( \lambda'(\theta) \) and not enough to \( \delta'(\theta) \). It is then possible that to higher a proportion of individuals choose specialisations with a high job-destruction rate from an output maximising point of view. Therefore, perhaps surprisingly, whether aggregate choices are too risky or not depends on the precise shape of \( \lambda(\theta) \) and \( \delta(\theta) \), although in all the parametric examples looked at in this paper, \( A(\theta) > 1 \).
3.2 A calibrated dynamic model

In order to explore what happens when unanticipated shocks occur to the underlying parameters, we set up a calibrated version of the model to highlight the main dynamics at work.

Let’s assume that there are only two occupations: one with a high unemployment risk and one with a low risk. Newborn, ex ante identical individuals, choose their future occupation at \( t = 0 \), attend education for two-periods, and then enter the labour market as unemployment at \( t = 2 \). After 20 periods they then die. The population grows at a rate \( n \), implying that the cohort that dies is replaced by a new cohort that is a fraction \((1 + n)^{20}\) bigger.\(^3\) A fraction \( \beta_s \) of the cohort born at time \( s \) will choose low-risk future occupations, and a fraction \((1 - \beta_s)\) will choose high-risk future occupations. Those with low-risk (generalist) future occupations have job-finding rates equal to \( \lambda^L \) and job-destruction rates equal to \( \delta^L \). Those with high-risk (specialised) future occupations have job-finding rates equal to \( \lambda^H = \lambda^L \) and job-destruction rates equal to \( \delta^H > \delta^L \). The timing is such that at the beginning of the period those individuals working the previous period can become unemployed, after which they can search for a new job. The unemployed search jobs during the period, whereby production takes place at the end of each period. Individuals are assumed to have rational expectations and have polynomial per-period utility functions: \( u(x) = \ln(x + A) \). Given that we calibrate working lives, each period corresponds to about 2.5 years. Taking standard estimates for the discount rate from empirical studies (e.g. Frijters and Van der Klaauw, 2001), we take \( \rho \) to be equal to 10\% a year. There is no mortality before period 20.

A rational expectation individual \( i \) born at time \( s \) hence maximizes:

\[
E\{U_t\} = \sum_{t=s}^{s+T} P_t(working|\theta_i, s) \ln[A+(1-\tau_t)w_t(\theta_i)] + (1-P_t(working, \theta_i))u(A+b_t)
\]

where \( P_t(working|\theta_i, s) \) denotes the probability that someone with choice \( \theta_i \in \{H, L\} \) who is born at time \( s \) works at time \( t \). This probability follows the Markov-rule:

\(^3\)Due to the constant returns to scale production function, population growth will not affect much in the steady state. Population growth will however serve to ‘dampen out’ fluctuations.
\[ P_t(\text{working}|\theta_i, s) = (1 - \delta_{i,t})P_t(\text{working}|\theta_i, s) + \lambda_{i,t} \times (1 - P_t(\text{working}|\theta_i, s)) \]

\[ P_{s+2}(\text{working}|\theta_i, s) = 0 \]

By definition, for \( t < s + 2 \).

Total production equals:

\[ Y_t = \{(X_t^H)^\gamma + (X_t^L)^\gamma\}^{1/\gamma} \]

where \( X_t^H = \sum_{s=t-T}^{s=t} P_t(\text{working}|H, s) \) denotes the total measure of individuals working in high-risk occupations at time \( t \) and \( X_t^L \) denotes the total measure of individuals working at low-risk occupations at time \( t \). Wages are competitive: \( w_t(H) = \frac{\partial Y_t}{\partial X_t^H} \) and \( w_t(L) = \frac{\partial Y_t}{\partial X_t^L} \). Benefits solve the budget constraint of the government: benefits are equal to total amount of taxes collected divided by the number of unemployed. Throughout, we assume that individuals have rational expectations and hence correctly predict future wages, benefits, and working probabilities.

We first show some baseline calculations of the steady state, where we construct two different baseline economies. The choice of the key variables \( \{\gamma, T, n, \lambda^H, \lambda^H, \delta^L, \delta^L\} \) is calibrated on statistics from the US and the EU. In line with Sargent and Ljungqvist (1998), we do not allow job-destruction rates or basic production processes to differ. We have hence set \( \{\gamma, \lambda^L, \lambda^H, \delta^L, \delta^H\} \) equal for both economies, in order to allow for proper comparisons of the dynamics in the shock experiments. In addition, we set the parameters at the level of specialists to be much higher in one economy, in order to be assured that our risk-arguments have some relevance for the two economies. The statistics we took into account are average job-finding rates, recent unemployment rates, population growth rates in the last 30 years, job-destruction rates, and level of unemployment benefit. The underlying data is from the OECD.

Table 1 shows the baseline functions.
Table 1: baseline calibrations for a high tax and a low tax economy.

<table>
<thead>
<tr>
<th>Specific inputs</th>
<th>‘EU’</th>
<th>‘US’</th>
</tr>
</thead>
<tbody>
<tr>
<td>n per year</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.08</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>‘EU’</th>
<th>‘US’</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.59</td>
<td>0.83</td>
</tr>
<tr>
<td>( w^L \times (1 - \tau) )</td>
<td>1.04</td>
<td>1.03</td>
</tr>
<tr>
<td>( w^H \times (1 - \tau) )</td>
<td>1.17</td>
<td>1.49</td>
</tr>
<tr>
<td>benefits</td>
<td>0.74</td>
<td>0.32</td>
</tr>
<tr>
<td>Average production</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>unemployment</td>
<td>0.11</td>
<td>0.07</td>
</tr>
<tr>
<td>utility</td>
<td>0.05</td>
<td>-1.31</td>
</tr>
</tbody>
</table>

Common inputs: \( \Lambda = 0.1 \), \( \rho \) per period = 0.1, \( \gamma = 0.8 \), \( \lambda^L = 0.9 \), \( \lambda^H = 0.5 \), \( \delta = 0.25 \)

In the ‘EU’ baseline model, we see that taxes and unemployment levels are higher than in the ‘US’ baseline model. Welfare is slightly higher in the ‘EU’, whilst wage-differentials are higher in the ’US’. Output per unit input is virtually the same for the ‘EU’ and ‘US’. The sensitivity to taxes is fairly large: even though basic production possibilities are set equal for both economies, the differences in the taxes translates to large differences in unemployment and skill distributions, although the differences in benefits are not that large.

We now turn to the effect of unanticipated shocks. Starting from the steady state, we perturb the economy at \( t=0 \). We then assume that expectations instantaneously realign themselves i.e. after the shock all individuals know what will happen next. We focus on three different types of shocks:

1. A skill-biased recession. At \( t = 0 \), \( \delta \) is equal to 0.5, and \( \lambda^L \) and \( \lambda^H \) are halved. At \( t = 1 \), \( \delta \) and \( \lambda^H \) return to baseline.

2. A permanent biased search shock. At \( t = 0 \), \( (1 - \lambda^H) \) is halved.

3. A welfare system change. At \( t = 0 \), \( \tau \) is halved.

We show the effects of each of these shocks in Figures 4, 5 and 6, respectively.
Figure 4: Response to a skill-biased unemployment shock

The effect of a skill-biased unemployment shock is highlighted in Figure 4 separately for the ‘EU’ and ‘US’. Interestingly, the effect of a recession corresponds somewhat to the historical data on unemployment dynamics after the oil shocks of the 1970’s (see Blanchard and Wolfers, 1999), where there is a similar initial drop in employment and output, but in the ‘EU’ the recovery in production and output takes about twice as long because the number of specialists is far higher. The different experiences of the two economies are especially pronounced when we look at the development of $\beta$: the recession temporarily increases the value of being a generalist. In the ‘US’, where most were generalists to begin with, this changed little, but in the ‘EU’, this led
to big increases in $\beta$, which themselves created a ‘ripple-effect’ in subsequent periods. At the aggregate level, this means that the levels returning to work are smaller in the ‘EU’. This is another potential explanation for the sluggishness with which the EU unemployment rates came down after the oil price shocks (Blanchard and Wolfers, 1999). Sargent and Ljungqvist (1998) explained this same phenomenon by arguing that unemployment benefits increased the willingness of individuals to wait for better jobs. They combined this argument with the assumption that individuals loose skills in unemployment. Therefore, unemployment benefits aggravated the negative shocks in their model because the high unemployment benefits adversely changed the characteristics of individuals whilst they were already on the labour market. As previously noted, the main difference with this paper is that we do not require a reduction in skills during unemployment, but rather an effect of unemployment benefits on the long term composition of the workforce. The figures also clearly show an increasing wage divergence after the shock, which is especially strong in the US.

In a sense, this calibrated model is set up such that the recovery is relatively quick, because new cohorts can immediately enter the labour force. If we were to allow for a waiting period between the choice of education and entering the labour market, the recovery of geographical areas with many specialists would take much longer.
Figure 5 documents the response of the economies to a decreased risk of unemployment for specialists. Clearly, a permanent biased search shock has stronger effects on the ‘EU’ than on the ‘US’, this is simply because the proportion of specialists at the time of the shock is much higher in the ‘EU’. The risks associated with being a specialist go down, which encourages more specialists and reduces wage gaps. As an immediate effect, entire cohorts become specialists in the ‘US’ and the ‘EU’. This in turn causes a ripple effect for the subsequent periods that slowly dies down. In the longer term, the
The proportion of specialists rises in both economies. Also, production increases. Unemployment actually increases in the ‘US’, despite the objective decrease in the riskiness of specialised jobs. In the ‘EU’, there is an initial sharp drop in unemployment as more unemployed specialists find a job. In the longer term, however, the increased proportion of specialists, who still run higher risks than the generalists, means that the initial employment gains are reduced.

Figure 6a: Response of the ‘US’ to a halving of tax rates

Figure 6b: Response of the ‘EU’ to a halving of tax rates

Figure 6: Response to a halving of tax rates

Finally, the response to a halving of the tax rate is show in Figure 6. Here, the welfare system change clearly increases the incentives to become generalist. Since in the ‘US’ the proportion of generalists was already very
high, the effect of lowering $\tau$ on unemployment and $\beta$ is small: production and unemployment decrease very slightly. All that really changes is a large increase in the wages of specialists and hence a greater wage inequality. In contrast, there are relatively big effects in the ‘EU’, where the changes in $\beta$ are quite large, leading to long-lasting ripples. The effect on production and output is very delayed however, simply because it takes time for new cohorts of generalists to come through the education system and substantially change the composition of the labour force. In the figure it takes about 8 periods ($\approx$20 years) before production and unemployment have reached their new steady state. The long-term effects are that production increases and unemployment decreases because of the reduced numbers of specialists in the economy. These predictions seem to mirror the sluggishness with which unemployment levels have been found to react to changes in the level of unemployment benefits, and indeed to other welfare changes (see Blanchard and Wolfers 1999, 2001; Dolado et al., 1996; and Gruber and Wise, 1997).

3.3 Limitations and future directions

Before we conclude the paper we would like to note a number of limitations to our analysis, which in turn point to future research directions.

Firstly, the model is based on the existence of a link between different types of jobs and different probabilities of finding and maintaining them. If this is not the case, where nothing about the probability of finding and holding a job is job-specific, then our model falls apart. The empirical evidence to support such a link is mainly to be found in the job search literature, where job-finding and job-severing probabilities have been found to be strongly related to job characteristics. These include industry sector characteristics (for which van den Berg and Van Vuuren, 2003, find large differences in search frictions), characteristics of the profession and the job (various chapters in Ashenfelter and Card, 1999, review hundreds of empirical studies documenting the importance of these for individual transitions) and product market characteristics (Amable and Gatti, 2001, review evidence of the importance of market characteristics for transition rates). A consistent finding in this literature has been that there is an enormous amount of heterogeneity in job-finding and job-severing probabilities. Within this heterogeneity, we can simply define specialised jobs as those associated with low job finding probabilities and high job severing probabilities. In this paper, we have made
no attempt at making these probabilities endogenous, so at present the only ‘micro-reason’ for \( h(\theta) \) that is encapsulated within our model is heterogeneity in exogenous technological shocks.

Secondly, our model assumes that each individual spends an equal amount of time in education and can costlessly choose a specialisation. However, introducing a cost to education would not make any material difference to the model, since it would simply mean adding a known cost-function to equation (2), which in turn would mean that wages would have to compensate for these costs. Whilst clouding the argumentation, this would have no impact on the basic risk-compensation story we develop.

Thirdly, anything observed that has a one-to-one correspondence with \( \theta \) would serve as a perfect indicator to base a first-best insurance system on. Within this model, one could therefore base \( \tau \) and \( b \) on wages or unemployment risks to obtain better outcomes. This, for instance, would rationalise greater unemployment benefits for higher wages in combination with higher taxation on higher wages. However, such a system in practice would face many information problems, if only because some potential workers have never find a job to begin with.

4 Conclusion

In this paper we have contributed to the recent literature that has focussed on the potential economic advantages of unemployment benefits (for example, Gruber, 1997; Acemoglu and Shimer, 1999, 2000; Mortensen and Pissarides, 1999). Our contribution comes from the development of a model where unemployment benefits increase the incentives of individuals to make riskier educational investments, which at low levels of benefit increases expected welfare, total output and unemployment. Our model predicts that a geographical area or country with high unemployment benefits will also have a higher number of specialists who have lower job-finding rates. This prediction fits well with the detailed evidence presented by Ashton and Greene (1996), that the education system in the US, which has relatively low unemployment benefits, produces a less specialised work force than the more specialised education systems in the EU. In our model, areas with higher benefits are also more vulnerable to general shocks, because unemployment levels return less quickly to their natural rate. Moreover, increasing unemployment benefits increases unemployment, but with a large delay since it
takes time before the more specialist newer cohort, who are more frequently unemployed, appear in the labour market. This is one potential explanation for the lack of responsiveness in unemployment rates that is frequently found for changes in unemployment benefit levels (see, for example, Dolado et al., 1996; Blanchard and Wolfers, 2001). It also provides a possible explanation for why the US unemployment rates returned faster to a lower level after the oil price shocks in the 1970’s, than it did for a number of European countries.

In addition, the theoretical framework we have developed is also conducive to understanding the mechanics and effects of the wider system of social security and welfare benefits. For example, other risk-pooling schemes such as disability benefits and early retirement benefits, whose take-up is also to some extent affected by the early life choices of participants, are good applications. In particular, risk-pooling in these outcomes may lead to exactly the same dynamics as for unemployment benefits, where there is a delay between policy changes and changes in average outcomes, which may affect both the distribution and efficiency of actual outcomes.

More generally, when there are several risk-pooling systems for the same risks, the introduction or expansion of one system may well affect investments into other systems. One speculative possibility is that maintaining close family and community ties has as a likely benefit that one can count on support in the event of unemployment or other financial setbacks. In this sense, the maintenance and development of certain forms of social ties is a form of risk sharing. Welfare benefits then change the incentives for investments in these social ties and hence change the ‘social fabric’ of an economy.

Finally, the importance of long-term composition effects and general equilibrium effects found in this paper casts some doubt upon the usefulness of looking at the partial effects of changes in policy on individuals with given early life choices.

**Literature**


3. Amable, B., and D. Gatti (2001), ‘The impact of product market competition on employment and wages’, *Social Science Research Centre*


Appendix

First, we highlight a useful result:

**Lemma 1.** Considering two distributions $F_1(\theta)$ and $F_2(\theta)$ with a continuous CES production function, wages will be higher for at least one $\theta$ and lower for at least one $\theta$.

Proof: consider the level $\theta^* = \arg \max \{f_2(\theta)\}$. If $F_2 \neq F_1$, then it has to hold that $\frac{f_2(\theta^*)}{f_1(\theta^*)} > 1$. There holds:

$$\frac{w_2(\theta^*)}{w_1(\theta^*)} = \frac{(h(\theta^*)f_2(\theta^*))^{\gamma-1} \left[\int (h(\theta)f_2(\theta))^{\gamma}d\theta\right]^\frac{1}{\gamma-1}}{(h(\theta^*)f_1(\theta^*))^{\gamma-1} \left[\int (h(\theta)f_1(\theta))^{\gamma}d\theta\right]^\frac{1}{\gamma-1}}$$

$$= \left[\frac{f_2(\theta^*)}{f_1(\theta^*)}\right]^{\gamma-1} \left[\int (h(\theta)f_2(\theta))^{\gamma}d\theta\right]^\frac{1}{\gamma-1} \left[\int (h(\theta)f_1(\theta))^{\gamma}d\theta\right]^\frac{1}{\gamma-1}$$

Now,

$$\int (h(\theta)f_1(\theta))^{\gamma}d\theta = \left(\frac{f_2(\theta^*)}{f_1(\theta^*)}\right)^{-\gamma} \star \int (h(\theta)f_2(\theta))^{\gamma}d\theta$$

$$\geq \left[\frac{f_2(\theta^*)}{f_1(\theta^*)}\right]^{-\gamma} \int (h(\theta)f_2(\theta))^{\gamma}d\theta$$
Hence:
\[
\frac{w_2(\theta^*)}{w_1(\theta^*)} < \left[ \frac{f_2(\theta^*)}{f_1(\theta^*)} \right]^{\gamma - 1} \left[ \frac{f_2(\theta^*)}{f_1(\theta^*)} \right]^{\frac{1}{\gamma} - 1} < 1
\]

Which implies that \( w_2(\theta^*) < w_1(\theta^*) \). Using the same argument for \( \theta^* = \text{arg min}\{\frac{f_2(\theta)}{f_1(\theta)}\} \), we can see that there is also a \( \theta^* \) for which \( w_2(\theta^*) > w_1(\theta^*) \). When \( f(\theta) \) is continuous for both \( F_1(\theta) \) and \( F_2(\theta) \), the strict inequality implies that any change in \( f(\theta) \) means that a whole range of wages must decrease and another range of wages must increase.

Proof of proposition 1.

Uniqueness of \( w(\theta) \). First, we note that the differential equation:
\[
-h'(\theta)(u((1 - \tau)w(\theta)) - u(0)) = (1 - \tau)w'(\theta)h(\theta)u'((1 - \tau)w(\theta))
\]
defining the equilibrium \( w(\theta) \) is well-behaved in the sense that \( w(\theta) \) is uniquely determined by a \( w(0) \). Also, \( \frac{\partial w(\theta)}{\partial w(0)} \) is continuous and bigger than 0. In turn, the equation \( \frac{w(\theta)}{w(0)} = \frac{(h(\theta)f(\theta))^{\gamma - 1}}{(h(0)f(0))^{\gamma - 1}} \) means that \( \frac{w(\theta)}{w(0)} \) uniquely determines \( \frac{f(\theta)}{f(0)} \) and \( f(0) \) is then solved by \( \int f d\theta = 1 \). The function \( f(\theta, w(\theta)) \) is also continuous in \( w(\theta) \). By implication of \( w(\theta) \) being continuous in \( w(0) \), the implicit function \( f(\theta, w(0)) \) must therefore also be continuous in \( w(0) \).

Finally, equilibrium requires that the level \( w(0) \) also solves:
\[
(h(0)f(0, w(0)))^{\gamma - 1} \left[ \int (h(\theta)f(\theta, w(0)))^\gamma d\theta \right]^{\frac{1}{\gamma} - 1} = w(0)
\]

For \( w(0) \downarrow 0 \), we first can note that at \( \lim_{w(0) \downarrow 0} \) the defining condition
\[
-h'(\theta)(u(w(\theta)) - u(0)) = (1 - \tau)w'(\theta)h(\theta)u'(w(\theta))
\]
reduces to \( \frac{w'(\theta)}{w(\theta)} = -\frac{h'(\theta)}{h(\theta)} \).
Translating this into \( f(\theta) \), implies that the left-hand side of the above expression will converge to some positive number. For \( w(0) \uparrow \infty \), we can note that in the limit, \( u'(w(\theta)) \) becomes constant, which in turn again pins down the the left-hand side of the expression above to a finite number. Due of the continuity of \( f(\theta, w(0)) \), the fixed-point theorem then applies and there must be at least some level \( w(0) \) for which the condition is satisfied.

Considering uniqueness, suppose there are two wage function \( w_1 \) and \( w_2 \) that are an equilibrium. Without loss of generality, take \( w_2(0) > w_1(0) \).
Then, it has to be the case for $w_2$ that all will prefer $\theta = 0$ above any other level $\theta$ unless $w_2(\theta) > w_1(\theta)$ for any $\theta > 0$. If this does not hold, then there is a positive mass of quality that will choose 0 and there will be no mass choosing a quality slightly above 0. This would mean $w_2(0) = 0$, which cannot be the case. Therefore, there can only be a second equilibrium if $w_2(\theta) > w_1(\theta)$ for all $\theta > 0$. As we know from Lemma 1, this is an impossibility. If there is an equilibrium, it has to be unique.

In this proof it is not necessary to assume that $w_0(\theta)$ or $f(\theta)$ is continuous. This means we can include cases where $h(\theta)$ is not-continuous. In such cases, we can apply the same proof, but simply note that $w_2(\theta)$ is then defined by the more general requirement that $E\{U(\theta)\} - E\{U(0)\} = 0$. In the text all the formulas are given for the continuous case for ease of exposition.

Proof of proposition 2.

Our strategy for proving i) is to first prove that when taxation increases from 0 to some arbitrarily small $\varepsilon$, that there will be a $\theta^* > 0$ for which $w(\theta)$ remains constant. We will then show that for all $\theta < \theta^*$, $w(\theta)$ has increased. For all other $\theta$, $w(\theta)$ has decreased. Since $w'(\theta)$ will remain greater than $\frac{w}{h}$, we can write the changes in $w(\theta)$ as a succession of production-increasing changes, which establishes production improvements. For $\theta = 0$, we then show that his utility has increased, which implies it has to have increased for all choices.

Looking at $\frac{dw}{d(1-h)}$ it holds:

$$
\frac{d}{d\tau} \left[ \frac{dw}{d(1-h)} \right] = \frac{1}{(1-\tau)hw((1-\tau)w(h))} \left\{ \left[ -w(h)u'((1-\tau)w(h)) - \frac{db}{d\tau} u'(b) \right] + \left\{ u((1-\tau)w(h)) - u(b) \right\} \right\} * \frac{1}{1-\tau} + \frac{w(h)u''((1-\tau)w(h))}{u'((1-\tau)w(h))}
$$

Using that $\left. \frac{db}{d\tau} \right|_{\tau=0} = \int (1-h)f\theta d\theta > \frac{h(0)w(0)}{1-h(1)} > w(0)$ and that $u((1-\tau)w(\theta)) - u(b) < ((1-\tau)w(\theta) - b)u'(b)$, we therefore know that $\frac{d}{d\tau} \left[ \frac{dw}{d(1-h)} \right] < 0$ at $\theta = 0$ for $\tau$ small, say $\tau = \varepsilon$. This in turn establishes that the whole wage function changes when $\tau$ changes. We denote the changed wage function as $w(\theta, \varepsilon)$. Using the same argument as in Lemma 1, we then know that there has to be a whole range of $\theta$ for which $w(\theta)$ has increased. Resulting from
unemployment has decreased, taking more risks must have become relatively
more attractive. This also means that \(\frac{\text{sign}\{w(\theta, \varepsilon) - w(\theta)\}}{\text{sign}\{\theta^* - \theta\}}\) because the importance of the risk of
unemployment has decreased, taking more risks must have become relatively
more attractive. This also means that \(\text{sign}\{f(\theta, \varepsilon) - f(\theta)\}\) where \(\theta^{**} = \text{arg}\{f(\theta, \varepsilon) = f(\theta)\}\). For small \(\varepsilon\) we can write \(y(\tau = \varepsilon) - y(\tau = 0) = \int [f(\theta, \varepsilon) - f(\theta)]h(\theta)w(\theta)d\theta + \sigma(\varepsilon)\). Now, since \(\frac{dw}{d(1-h)} = \frac{\Delta w}{(1-h)h}u'(1-h)w(\theta)\),
is continuous in \(\tau\). \(\frac{dh(\theta)w}{d(1-h)} > \frac{dh(\theta)w(\theta, \varepsilon)}{d\theta} > 0\), implying that \(y(\tau = \varepsilon) - y(\tau = 0) > 1\).

Since \(\frac{dw}{d(1-h)}|_{h=1, \tau=0} / \frac{d\tau}{d\theta} \mid_{\tau=0, h=1} < -1\) and due to average wages having
increased, we also know that \(h(\theta^*) < (1 - \varepsilon)\) and therefore that \(w(0, \varepsilon) > (1 + \varepsilon)w(0)\). This in turn means that the wage increase at \(\theta = 0\) more than
offsets the tax increase. This establishes a utility increase for all \(\theta\).

ii) Presuming existence and uniqueness, we simply define \(\tau^z\) as the \(\tau\) that
solves \((1 - \tau)w(0, \tau) = b(\tau)\). For this level of \(\tau\), \(\frac{dw}{d(1-h)}\) has to equal 0 since
otherwise someone could increase their expected utility by changing \(\theta\). Hence
\(w(\theta) = w(0)\) and \(f(\theta) \propto h(\theta)^{-1}\). Also, \(\frac{d\tau}{d\theta} \mid_{\tau=\tau^z} > 0\). This in turn implies
if \(\tau > \tau^z\), that \(\frac{dw}{d(1-h)} < 0\) and that individuals would want to take as much
risk of unemployment as possible, which in turn means they would not want
to work at all.

iv) Uniqueness and existence of equilibrium when \(\tau > 0\). For any given
\(\tau\) and \(b\), Proposition 1 implies that we know that there is a unique \(w(\theta)\)
that solves the model. There is a closing equation \(b = \frac{\tau_y}{f(1-h)f_{\theta}}\) with \(y\) and
\(f\) endogenous functions of \(b\). Looking only at \(\tau < \tau^z\), what is sufficient
for existence is that: 1) \(y(b = 0) > 0\) and \(\int (1 - h)f(0)b(\theta)d\theta > 0\), 2) both
\(y(b)\) and \(\int (1 - h)f(b)d\theta\) are continuous functions of \(b\), and 3) at \(b = -\infty, \frac{\tau_y}{f(1-h)} < \infty\). Continuity follows because \(w'(\theta) = \frac{-h'(\theta)u'(w(\theta)) - u'b(\theta)}{(1-h)}w(\theta)\) defines
\(w(\theta)\) as a continuous function of \(w(0)\), \(\tau\) and \(b\). Combined these imply at
least one value of \(b\) solves \(b = \frac{\tau_y}{f(1-h)f_{\theta}}\) for any \(\tau\). Uniqueness follows when
any level of \(\tau\) leads to only one level of \(b\). Suppose this does not hold, and
that there are at least 2 levels of \(b\) that solve \(b = \frac{\tau_y}{f(1-h)f_{\theta}}\). Denoting the
distributions of \(\theta\) that leads to these levels as \(f_1(.)\) and \(f_2(.)\) where \(b_1 < b_2\),
there would then hold that \(f_1(.) \neq f_2(.)\) for at least a range of \(\theta\). Using
Lemma 1, continuity means that there is a \(\theta^*\) for which \(w_1 = w_2\). Due to
\(b_1 < b_2\), \(U(b_2) > U(b_1)\) for any \(\theta\). Now, the condition that \(U(b_1)\) is the

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same for all \( \theta \) can then only hold if those with \( w_1 > w_2 \) run higher risks. By induction, \( \text{sign}\{w_1(\theta) - w_2(\theta)\} = \text{sign}\{\theta^* - \theta\} \). In turn, this lower risk taking means that \( \int (1-h)f(b_1)d\theta < \int (1-h)f(b_2)d\theta \). Then, it would have to hold that \( y(b_1) < y(b_2) \). When \( b = w(1) \), which occurs by definition at \( \tau = \tau^* \), it holds that \( \frac{dy}{db} < 0 \) and hence \( \frac{\partial y}{\partial b} < 0 \). Therefore, at high \( \tau \) we have uniqueness.

For other levels of \( \tau \) existence cannot be proven. The reason for this is that the model has an externality: someone becoming unemployed has an effect on the unemployment benefits of someone else which he or she does not take into account. This translates to an implicit \( y(\tau, b) \) that is always decreasing in \( \tau \), but is only increasing in \( b \) at low levels of \( b \), which leads to an indeterminancy of uniqueness.

Proof of proposition 3.

i) Existence. We can follow the proof of the homogenous case by noting that the main solution equation uniquely tracks out \( w(\theta) \) as a function of \( w(0) \). What is now uniquely defined by \( \frac{w(\theta)}{w(0)} \) is \( \frac{dg(\theta)q(q(\theta))h(\theta)}{dg(0)q(q(0))h(0)} \) and it is now the function \( |dq(\theta)| \) that is uniquely, continuously and implicitly defined by \( w(0) \) and by the requirement that \( \{q(\theta)\} = \{0 \leq x \leq 1\} \cup \{qw(\theta) \geq b\} \). Voluntary unemployment arises for those \( q \) where \( qw(1) < b \).

Uniqueness. Suppose the equilibrium is not unique. Without loss of generality, take \( w_2(0) > w_1(0) \). Then, it has to be the case that all with quality \( q \) in a small region near \( q(0) \) will prefer \( \theta = 0 \) above any other level \( \theta \) unless \( w_2(\theta) > w_1(\theta) \) for \( \theta > 0 \) also. If this does not hold, there is a positive mass of quality that will choose 0 and there will be no mass choosing a quality slightly above 0. This would then invalidate the initial assumption. By forward induction, there can hence only be a second equilibrium if \( w_2(\theta) > w_1(\theta) \) for all \( \theta > 0 \). As we know from Lemma 1, this is an impossibility. If there is an equilibrium, it therefore has to be unique.

iii) and iv) The arguments on taxation and the observed wage distribution trivially carry over from the homogeneous case.

v) First we will prove that it can never be the case that \( \frac{d\omega q}{dq} < 0 \). For this we note that \( E\{U(q_1)\} > E\{U(q_2)\} \) when \( q_1 > q_2 \). It thus has to be the case that \( h(q_1)\{u((1-\tau)q_1w(q_1)) - u(b)\} + u(b) > h(q_2)\{u((1-\tau)q_2w(q_2)) - u(b)\} + \phi(q_2) \int (1-h)f(b_1)d\theta < \int (1-h)f(b_2)d\theta \).
Now, because of utility maximisation, we also know that \( h(q_1)\{u((1 - \tau)q_2w(q_1)) - u(b)\} + u(b) \leq h(q_2)\{u((1 - \tau)q_2w(q_2)) - u(b)\} + u(b) \). Subtracting the second inequality from the first, we get \( h(q_1)\{u((1 - \tau)q_1w(q_1)) - u((1 - \tau)q_2w(q_2))\} > 0 \). In turn, this means that \( q_1w(q_1) > q_2w(q_2) \). Hence we indeed know that \( \frac{dwq}{dq} > 0 \), i.e. total wages will be higher for individuals with higher talents. We then know that \( \frac{d^{2}w_{q}}{d(1-h)} < 0 \) iff \( \frac{d^{2}h}{dwdq} > 0 \) because then individuals with higher talent have a greater preference for less risk than those with less talent. There now holds that

\[
\frac{d^{2}h}{dwdq} = u'(x)\frac{\{u(x) - u(b)\}( - \frac{u''(x)}{u'(x)}x - 1) + xu'(x)}{\{u(x) - u(b)\}^{2}}
\]

with \( x = (1 - \tau)w(h) \). Noting that \( -\frac{u''(x)}{u'(x)}x \) is the degree of relative risk aversion (=\( \sigma \)), it immediately holds that \( \frac{d^{2}h}{dwdq} > 0 \) if \( \sigma \geq 1 \). Also, because \( \{u(x) - u(b)\} \leq xu'(x)(1 - x \min b < a < x \{\frac{u''(a)}{u'(a)}\}) \), the condition will also hold when \( -\frac{u''(x)x}{u'(x)} \) is constant or always less than 1.