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A General Equilibrium Analysis


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A GENERAL EQUILIBRIUM ANALYSIS

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Abstract

That many industries exhibit highly concentrated market structures, even at the global level, calls for trade theoretic analyses which can accommodate this fact. We present a two-country, general equilibrium analysis in which high concentration levels can be sustained through the interaction between R&D and market structure, whilst emphasizing the effects of trade and industrial policy on wages and welfare. The world economy is characterized by asymmetric initial conditions and populations. If initial conditions are very different, free-trade reduces wages in a backward economy, relative to autarky. However, the advanced economy always achieves higher wages through trade. Welfare gains from trade arise when economies are either very similar or very different. In the intermediate case, when initial conditions are not too different, and the advanced economy’s population is not very large, the backward economy loses from trade, while the advanced economy gains. A compensation mechanism is feasible and would ensure that no nation loses from trade. The analysis provides formal criteria for the choice of trade partners and the formation of trade blocs. Moreover, industrial policy (an R&D subsidy) is shown to be neutral or ineffective, in the sense that it does not affect any real magnitudes.

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I. Introduction

There is a long standing debate on the role of trade and industrial policy for economic development and national welfare, especially when considering trade between backward and advanced nations. During the XVIII\textsuperscript{th} century, mainland Europe was faced with the problem of catching-up with England. Friedrich List (1841) put forward the infant industry argument, arguing for economies to remain closed during the catching-up phase, and to liberalize trade only after domestic industry could compete against foreign ones on a level ground. The debate was rekindled by dependency theorists in the second half of the XX\textsuperscript{th} century. On that occasion, the arguments centered on core-periphery issues, and the difficulties that backward nations face regarding entry into manufacturing industries. More recently, a third round of this debate has been ignited by disappointing results from structural reforms based on prescriptions of the ‘Washington consensus’, particularly in Latin America and Sub-Saharan Africa. This, together with studies detailing how East Asian economies diverged from such prescriptions and yet achieved outstanding performance, sparked a search in economics for a better understanding of how such heterodox policies could have worked, and whether they did, in fact, work.\textsuperscript{1} This is a complex problem with many facets.

On the one hand, general equilibrium analyses of trade assuming perfect or monopolistic competition reach the robust conclusion that trade benefits all participants (Dixit, 1984). Moreover, the greater the differences between economies, the larger the potential for gains from trade. On the other hand, there are two propositions that are often raised in policy discussions. The first claims that a backward economy stands to lose by trading with an advanced nation. The logic behind this argument is that the advanced nation’s industry will crush the backward economy’s, thereby justifying import substitution in the backward economy. The second makes the reverse argument: Trading with a backward nation will destroy the advanced nation’s industry since the backward nation features lower wages. This argument also provides justification for import substitution, this time in the advanced economy.\textsuperscript{2} Implicit in these views is some form of strategic interaction between domestic and foreign firms, together with an entry and exit process. In view of the forceful arguments made in traditional trade theory, it would be surprising if any of these propositions should be anything but false. Nonetheless, through the introduction of strategic interaction (oligopolistic firms) and endogenous market structure in


\textsuperscript{2}Other substantiations for import substitution include declining terms of trade, particularly since backward nations produce mostly income-inelastic primary goods, and find it hard to diversify production into income-elastic manufactured goods. Also, there is the problem of primary goods’ price instability, which induces volatility in the rest of the economy, curtailing investment and growth. Neither of these arguments will be analyzed here.
general equilibrium, we find that the first proposition can hold under certain conditions, while the second proposition does not hold.

Oligopoly theory has made considerable progress towards developing a theory of firm behaviour that offers a greater degree of realism (e.g., Sutton, 1998). The empirical success of this literature lies in its ability to explain the high levels of concentration observed across many industries, even at the global level. With the exception of Sutton (2007) and Yanes (2005), this empirical regularity has so far been overlooked by the trade literature. The opening of the economy is associated to an increase in market size. Traditional trade models feature exogenously given entry costs, and exhibit the property that growth in market size leads to further entry, so that the number of firms eventually becomes large, converging towards the perfectly competitive limit.3

By contrast, in the class of models that we use, entry costs (taking the form of R&D) are endogenous. The effect of R&D is twofold. First, it increases a firm’s product quality. Second, since R&D is a fixed and sunk cost, it raises entry costs.4 In this case increases in market size do not lead to further entry. Rather, growth in market size increases investment in R&D, thereby increasing entry costs. The resulting equilibrium outcome is that after R&D investment takes place, entry is not profitable, so the number of firms does not grow with market size. This is called the ‘non-convergence’ property (Shaked and Sutton, 1982, 1983). Hence, allowing for endogeneity of R&D provides us with a mechanism which can sustain concentration, even at the global level. The strategic interactions present in such a setting call for an oligopolistic (game theoretic) treatment of firms. Moreover, the exogenous entry costs framework can be shown to be a limiting (special) case of endogenous entry costs, in which investment in R&D becomes increasingly ineffective. In view of its greater realism and generality, we follow the endogenous R&D route to devise a general equilibrium model with oligopolistic interactions. This brings forth a formal analysis of trade regimes and industrial policy within a framework that moves closer to what is observed in many industries, and yet is sufficiently tractable to allow a parsimonious representation.

General equilibrium models with oligopolistic firms face significant problems.5 In particular, it is hard to justify firms acting as price takers in inputs markets whilst exerting market power in output markets (but see Ruffin, 2003). Neary (2003) proposes a way to circumvent this, and

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3Examples of this class of models include models of oligopoly (such as Cournot or Bertrand with product differentiation) with a fixed entry cost (e.g., Markusen and Venables, 1988), as well as monopolistic competition models (Krugman, 1979).

4Throughout the paper, R&D is assumed to be a fixed and sunk cost. For brevity we will refer to it as an entry cost or just ‘R&D’.

in doing so, address the major problems identified in the literature. The key lies in having many industries, each of which is small relative to the economy. It then becomes rational for firms in each industry to treat input prices as given, whilst exerting market power in their particular output market. For simplicity, we start with a single oligopolistic industry assuming that firms are price takers in the input market and exert market power in the output market. The model is then extended to the multiple industry case, and our results hold for this more general setting.

The world we consider features two kinds of asymmetry, and it is the interplay of these that generates some of our most important findings. First, we allow population size to differ between countries. Second, we introduce differences in ‘initial conditions’, which represent firms’ inherited product quality. An economy is considered ‘backward’ if its initial conditions are less favorable than those of the other (‘advanced’) economy.

The main innovation of this paper is the endogenous determination of the number of firms and product quality (R&D) in general equilibrium, in an asymmetric world economy. Section II develops a two-country model of the world, and explicit solutions are presented in section III. Consumers supply labour inelastically, own shares in firms, and consume two types of (tradeable) goods. The first type (‘X’) is produced by an oligopolistic industry. In this industry, single product firms produce horizontally and vertically differentiated goods under increasing returns to scale, using labour as the only input. Product quality is the vertical differentiation attribute. Increases in product quality are achieved by hiring workers who exhibit diminishing marginal productivity. Wages paid to these workers constitute R&D investment. Since R&D is a choice variable, the extent of scale economies is determined endogenously within the model. Free entry (zero profit) conditions then help determine the number of survivor firms. The second type of good (‘Y’) is a homogeneous good which acts as a residual, ensuring that the budget constraint holds. Consumers have an endowment of this good, and whether their consumption is above or below their endowment determines their net demand/supply.

There are two mechanisms which determine the demand for labour and thereby the wage rate. Suppose there is an increase in market size, as would occur when switching from autarky to free trade. This will increase quality at the firm level and, ceteris paribus, the demand for labour. However, increases in product quality are associated to rising entry costs, and hence the number of firms falls in each economy, thereby reducing labour demand. The first mechanism is the ‘quality effect’: changes in product quality affect labour demand at the firm level. The second is the ‘market structure effect’: changes in the number of firms will also affect labour demand, this time at the industry level. Which of the two effects is dominant determines whether wages rise or fall.

In section IV we show that, depending on relative initial conditions and population sizes, trade may lead to increases or falls in wages for each economy. If initial conditions are too
different, the backward economy will feature higher wages in autarky, relative to free trade, while the advanced economy exhibits higher wages under free trade. If the asymmetry is not too large, then both economies feature higher wages in a free trade regime. The rationale for this pattern is as follows. As a nation becomes more backward (initial conditions worsen), an increasing number of its firms exit when the economy is opened to trade. Nonetheless, in order to attain non-zero market share against foreign rivals, surviving firms must now achieve higher quality. When the asymmetry in initial conditions is sufficiently large, the loss of firms in the backward economy (market structure effect) dominates the quality effect, leading to a contraction in labour demand and wages (Yanes, 2005).

Section V deals with welfare. Welfare is ultimately determined by product quality, product variety and income. Product variety plays no role in welfare comparisons between autarky and free trade, since the number of products available to consumers (the number of single product firms) does not change with the opening of the economy, which in turn results from the ‘non-convergence’ property.

Welfare analysis shows that if economies are either relatively similar, or very different, both gain from trade. The case when economies are similar is straightforward. On the one hand, the increase in market size generates escalation in quality. This increases welfare through higher labour demand and wages, as well as consumption of higher quality goods. On the other hand, exit of firms reduces welfare through the contraction of labour demand (and wages). However the market structure effect is insufficient to counter the welfare gains from increasing quality, so that trade is welfare enhancing.

To see why very different economies gain from trade, consider the perspective of the backward economy. Under free trade, the backward economy exhibits lower wages than in autarky, which reduces welfare. However, given that initial conditions are substantially different, the gains from product quality achieved by consumers in the backward economy more than offset the fall in wages associated with trade. We label this an ‘impoverished consumerist economy’, to highlight the idea that consumers in such an economy are happy to gain access to high quality products, even though doing so decimates the local industry, reducing the demand for labour and wages. The advanced economy benefits since it gains access to the backward market, and faces little firm exit. Thus, its wages rise along with quality.

There is another possibility, in which the advanced economy gains from trade, while the backward economy loses from trade, labelled ‘unequal exchange’. In this case, the advanced economy’s population is not particularly large relative to the backward economy’s, and there is an intermediate level of asymmetry in initial conditions. Losses from trade arise from the interplay of the following forces. First, in such a setting quality gains for consumers in the backward economy are not substantial. Second, even though exit of backward firms is not
very pronounced, it is sufficient to lead to wages being smaller than in autarky. Finally, when the advanced economy joins the world market, it offers access to a not-particularly-large market relative to the backward economy’s, curtailing the benefits of market expansion for the backward economy. Meanwhile, the advanced nation reaps the benefits of trade through higher quality and wages without any of the disadvantages, since firm exit is insufficient to reduce its labour demand and hence wages. Notwithstanding this result, we show that even though the backward economy loses from trade, world welfare is increased. Hence, it is possible to implement a compensation scheme such that no nation loses by trading. The complex topic of how to design such a scheme is, however, left for future research.

In summary, there exists a range of admissible asymmetries within which free trade is welfare improving for both nations. Outside of this range, only the advanced nation reaps the benefits of trade. This provides an answer to the question of: Who should trade with whom? That is, which countries should be chosen as partners for the formation of trading blocs? A short answer is that trade is always beneficial, except for a mildly backward economy considering trade with a (relatively) not-very-large advanced economy. It must be emphasized that the logical conclusion to draw from this is not the closure of mildly backward, relatively large economies. On the contrary, by choosing appropriate partners, trade can be used to improve initial conditions such that the backward economy ceases to be so, and can then reap the gains from trade. In the absence of a compensation scheme, a non-staggered opening of a mildly backward, relatively large economy is likely to elicit a backlash against free trade, which can then lead to political instability and unsound economic policy.

The next question deals with industrial policy and catching-up. The first result relates to the ‘technology gap’, defined as the ratio of countries’ product qualities. If this gap is too wide, firms in the backward nation will face negative marginal benefit from investment in R&D. It is simply not optimal to try to catch-up with the advanced nation (Proposition 1). The second result concerns industrial policy, which takes the form of an *ad-valorem* subsidy to R&D, financed with a lump sum tax on consumers (section VI). Such a scheme leaves the number of firms, quality and welfare unchanged, while wages rise just enough to offset lump sum taxes. This finding means that industrial policy is essentially neutral, or ineffective, in both autarky and free trade. The result arises from the general equilibrium structure of the model. In general equilibrium, labour market clearing determines a level of quality, for a given the number of firms: For a fixed labour force and given number of firms, each firm achieves a quality level determined by how many workers are left for it to employ. Since the subsidy only changes the amount firms pay for product quality, but does not change the amount of labour required to achieve a certain quality level, equilibrium quality remains constrained by the labour market. Firms try to achieve higher quality, but this only translates into increased pressure in the labour
market, which is dissipated through the wage. The change in wages exactly offsets the subsidy, leaving net profits unaffected, so that the number of firms does not change. This finding calls for a reconsideration of how aid is administered. What is required then, is to change the efficiency with which firms transform labour (or more generally production factors) into product quality. The latter task requires ingenuity, rather than handouts. Indeed, foreign aid taking the form of monetary transfers will only serve to raise wages, leaving the recipient country’s technological structure unaltered. Once the aid is withdrawn, the country returns to its original income level. The logical conclusion from this is not to eliminate aid, but to re-design the form it takes. If it is to have long run effects, aid needs to look into technology transfer.

Another of our findings sheds light in a persistent puzzle in Taiwanese and South Korean development. In particular, in a comment on Rodrik (1995), Victor Norman notes that in these nations trade occurred prior to the investment boom. The received view proposes a reverse causality: Investment leads to expansion of output, which is then exported. How, then, do we explain that in these nations trade preceded investment? Our model clarifies this pattern. Consider an unexpected opening of the economy. Firms are then exposed to overseas competition and a larger market size. Firms then have to either increase R&D or exit, and this may lead to an investment boom (in our single factor setting, this corresponds to a rise in wages).

Section VII benchmarks the decentralized market equilibrium against the social optimum. Relative to the social optimum, the market equilibrium features lower quality and production, as well as more firms. We rank world welfare for different regimes. The regimes vary along the dimensions of free trade versus autarky, and social optimum versus decentralized market equilibrium. World welfare at the market equilibrium is not greater than at the social optimum (for both free trade and autarky). Interestingly, socially optimal wages are not necessarily higher than those pertaining to the market equilibrium. Relative to autarky, opening the economy induces no loss of (world) welfare, for both the decentralized market equilibrium and the social optimum. Section VIII extends the model to multiple industries, while section IX concludes.

II. Model

There are two countries, labelled 1 and 2. To avoid duplication of equations, it will be convenient to use subscripts $i, j = 1, 2$ with $i \neq j$ to label expressions which are identical for both countries\(^6\). Corresponding expressions for autarky are distinguished by subscript ‘$A’.$ In each economy there is a population of $L_i$ representative consumers ($i = 1, 2$), who supply labour to the oligopolistic industry. Since there is no international labour mobility, consumers earn the wage rate prevalent in their economy. There are two types of goods, $X$ and $Y$, both tradeable. Good $Y$ is a homogeneous product, a Hicksian composite commodity. This is chosen

\(^6\)For ease of reference, a table with symbol definitions is offered in Appendix 2.7.
as the numeraire, so its price is set to 1 in both economies. Type X goods are vertically and horizontally differentiated products made in the oligopolistic industry. Trade incurs zero transport costs.

The $k^{th}$ firm in country $i$ produces a single product, labelled $x_{ki}$. To denote production of the $k^{th}$ firm from either country, subscript $i$ is dropped and we use the notation $x_k$ in this case. In each economy, there is a finite number of firms, denoted by $n_i$ ($i = 1, 2$). The worldwide number of firms is given by $N = n_i + n_j$. We introduce the following vector notation: $x_i = (x_{1i}, ..., x_{ni})$ and $x = (x_1, x_2)$. Product quality for the $k^{th}$ type X good is denoted by $u_{ki}$, with $u_k$ denoting product quality in either country. In vector notation, we have: $u_i = (u_{1i}, ..., u_{ni})$ and $u = (u_1, u_2)$. The consumer’s problem can be stated as follows:

\[
\begin{align*}
\max_{x, y} & \quad V_i = \sum_{k=1}^{N} \left( x_k - \frac{x_k^2}{u_k^2} \right) - 2\sigma \sum_{k=1}^{N} \frac{x_k}{u_k} \sum_{l \neq k}^{N-1} x_l + Y_i, \\
\text{subject to} & \quad \sum_{k=1}^{N} p_k x_k + Y_i \leq w_i + \bar{Y}_i + \sum_{k=1}^{n_i} s_{hki} \Pi_{ki},
\end{align*}
\]

where $\sigma \in (0, 1)$ measures substitutability between $X$ type goods, $p_k$ is the price of the $k^{th}$ X type good (in either country). For country $i = 1, 2$: $w_i$ is the wage rate, $Y_i$ is consumption of good $Y$, $\bar{Y}_i$ represents a per-capita endowment\footnote{The presence of an endowment of good Y serves a twofold objective. First, it ensures balanced trade flows, should trade in type X goods be unbalanced. Second, since consumption of good Y is always positive, the endowment guarantees that marginal utility of income is equal to unity, except in the borderline scenario in which one economy exports all of its endowment. Since our results are obtained without the need to consider this extreme case, we assume that the exporting country’s endowment is sufficiently large to ensure that it always exhibits non-zero consumption of good Y.} of good Y, $s_{hki}$ is the ownership share of consumer $h$ in firm $k$ (with $\sum_{h=1}^{n_i} s_{hki} = 1$), and $\Pi_{ki}$ denotes net profits of firm $k$.

The utility function in (1) is the underlying utility for the standard linear demand model, with some modifications. In particular, utility is an increasing and concave function of product quality. From the consumer’s problem we obtain inverse demand:

\[
p_{ki} = 1 - 2 \frac{x_{ki}}{u_{ki}^2} - 2\sigma \frac{\sum_{l \neq k}^{n_i} x_{li}}{u_{ki} + \sum_{l=1}^{n_i} x_{lj} u_{lj}}.
\]

It is clear that a firm’s demand expands with increases in its product quality. Demand for good $Y$ is obtained as a residual from the budget constraint. Consumers in economy $i$ will be importers of $Y$ when their demand is above their endowment. If their demand is below their endowment, they export $Y$.

Firms hire workers from the local workforce, and play a three stage game. We seek a
Subgame Perfect Nash Equilibrium. In keeping with backward induction, we proceed to the
description of the final stage in the firms’ decision problem.

In the third stage, firms choose quantities à la Cournot. For simplicity, variable costs are
set to zero, although the model can be easily extended to include these. Whence, gross profits
are given by \( \pi_{ki} = p_{ki}x_{ki} \). The first order condition for the \( k^{th} \) firm in country \( i \) is:

\[
p_{ki} + \frac{\partial p_{ki}}{\partial x_{ki}}x_{ki} = 0.
\]

This is a system of \( N \) linear equations in \( x_1 \) and \( x_2 \). In Appendix 2.1, we find the unique Nash
Equilibrium. Substituting this into gross profits, we obtain a solved-out payoff which is used to
solve the second stage of the game. The solved-out payoff is:

\[
\pi_{ki}(u) = \frac{u_{ki}^2}{2(2-\sigma)^2} \left[ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j - 1)} \left( \sum_{l=1}^{n_i} \frac{u_{li}}{u_{ki}} + \sum_{l=1}^{n_j} \frac{u_{lj}}{u_{ki}} \right) \right]^2.
\]

Total gross profit of the \( k^{th} \) firm in country \( i \) is given by \( (L_i + L_j)\pi_{ki}(u) \). Autarky solved-out
payoffs are obtained by setting \( n_j \) and \( L_j \) equal to zero.

In the second stage, firms compete in product quality, achieved through R&D expenditure. The firm’s net profit is given by

\[
\Pi_{ki} = (L_i + L_j)\pi_{ki}(u) - F(u_{ki}, w_i).
\]

\( F(u_{ki}, w_i) \) is R&D expenditure: \( F(u_{ki}, w_i) = w_i f(u_{ki}) \), where \( w_i \) is the wage rate prevailing
in country \( i \), and \( f(u_{ki}) = (u_{ki}/u_{oi})^{\beta_i} \) is the amount of labour required to achieve \( u_{ki} \). \( u_{oi} \)
represents an initial (inherited) product quality, which is an exogenous parameter. In section
IV, \( u_{oi} \) is used as a means of modelling differences in initial conditions across countries. \( \beta_i > 2 \)
is required for second order conditions to hold (see Appendix 2.2), so \( f(u_{ki}) \) is a convex function
of \( u_{ki} \). We also assume \( u_{ki} > u_{oi} \geq 1 \). The first order conditions to maximize (6) with respect
to \( u_{ki} \) are:

\[
(L_i + L_j) \frac{\partial \pi_{ki}}{\partial u_{ki}} = \frac{w_i \beta_i}{u_{ki}} \left( \frac{u_{ki}}{u_{oi}} \right)^{\beta_i},
\]

which constitute a system of \( N \) non-linear equations in \( u_1 \) and \( u_2 \).

In the first stage firms enter so long as net profits are positive. Assume there is a sufficiently
large pool of potential entrants. This leads to the following non-negative-profit condition

\[
(L_i + L_j)\pi_{ki} \geq w_i \left( \frac{u_{ki}}{u_{oi}} \right)^{\beta_i}.
\]
Ignoring integer effects, entry occurs until (8) holds with equality. This completes the description of the industry. The following lemma provides a useful result.\textsuperscript{8}

**Lemma 1.** In each economy, the Subgame Perfect quality level is unique and identical for all firms: \( u_{ki} = u_i \), for \( k = 1, \ldots, n_i \) and \( i = 1, 2 \).

Turning to the labour market, each consumer supplies a fixed amount of labour which is set to unity. Hence, total labour supply is fixed at each country’s population \( (L_i) \). Labour demand is given by \( \sum_{k=1}^{n_i} f(u_{ki}) \). The wage rate adjusts to ensure labour market clearing:

\[
L_i = \sum_{k=1}^{n_i} f(u_{ki}).
\]

As noted in the introduction, labour demand and wages are determined by two effects. The ‘quality effect’ notes that changes in product quality affect \( f(u) \) and thus labour demand at the firm level. The ‘market structure effect’ notes that changes in the number of firms will also affect labour demand, this time at the industry level. *Ceteris paribus*, an increase in product quality implies higher entry costs, which reduces the number of firms. Conversely, a fall in product quality reduces entry costs, leading to a rise in the number of firms. Which of these two effects is dominant determines whether wages rise or fall.

We now discuss the trade balance. The trade balance for good \( Y \) is labelled \( TB_{yi} \), and is given by the difference between the aggregate endowment and aggregate demand. Exports of type \( X \) goods in country \( i \) are given by \( L_i n_i p_i x_i \), while imports are \( L_i n_j p_j x_j \). It is clear that there will always be intra-industry trade, unless any of \( n_i, L_i, p_i \) or \( x_i \) \((i = 1, 2)\) are equal to zero. The trade balance for type \( X \) goods is labelled \( TB_{xi} \). The trade balance for each economy is given by \( TB_i = TB_{xi} + TB_{yi} \). In this model trade is always balanced, hence the trade balance for type \( X \) goods must be equal to the negative of the trade balance for good \( Y \) \((TB_{xi} = -TB_{yi})\). Moreover, because there are only two countries in the model, one economy’s trade balance for type \( X \) goods must equal the negative of the other economy’s trade balance for type \( X \) goods \((TB_{xi} = -TB_{xj})\), which in turn is the negative of the foreign trade balance for good \( Y \) \((TB_{xj} = -TB_{yj})\), hence \( TB_{yi} = -TB_{yj} \). To recap: \( TB_{yi} = -TB_{yj} = TB_{xj} = -TB_{xi} \). This completes the description of the model.\textsuperscript{9}

\textsuperscript{8}Longer proofs (not in the text) are provided in Appendix 1.

\textsuperscript{9}As an extension, Appendix 2.6 discusses value added and the trade balance in the context of this model.
III. General Equilibrium

General equilibrium exhibits labour market clearing for each economy, world markets clearing for type $X$ goods as well as good $Y$, and a Subgame Perfect Nash Equilibrium (SPNE) in the oligopolistic industry. The SPNE features Nash quantities, Nash qualities, and free entry. General equilibrium is characterized by three conditions for each country. From Lemma 1 we know that, within each economy, firms set identical qualities. Thus, the first equilibrium condition is a mapping from $u_j$ to $u_i$, such that along this mapping no individual firm wishes to deviate from its strategy. The mapping is defined by $\partial \Pi_{ki}/\partial u_{ki} = 0$ (equation 7):

\[
(OR_i) \quad \frac{(L_i + L_j) [2 + \sigma (n_i + n_j - 2)]}{(2 - \sigma)^2 [2 + \sigma (n_i + n_j - 1)]} \left\{ 1 - \frac{\sigma (n_i + n_j u_j/u_i)}{2 + \sigma (n_i + n_j - 1)} \right\} = w_i \beta_i u_i^{\beta_i - 2} u_{oi}^{\beta_i - 2}.
\]

Second, we have the free entry condition, as stated in (8). After substituting gross profits from (5), this simplifies to:

\[
(ZP_i) \quad \frac{(L_i + L_j)}{2(2 - \sigma)^2} \left\{ 1 - \frac{\sigma [n_i + n_j u_j/u_i]}{2 + \sigma (n_i + n_j - 1)} \right\}^2 = w_i u_i^{\beta_i - 2} u_{oi}^{\beta_i - 2}.
\]

Third, we have labour market clearing, from (9):

\[
(LM_i) \quad L_i = n_i \left( \frac{u_i}{u_{oi}} \right)^{\beta_i}
\]

Market clearing for good $Y$ and type $X$ goods is ensured by construction. The six conditions contained in $(OR_i)$, $(ZP_i)$ and $(LM_i)$ determine the general equilibrium values of quality ($\hat{u}_i$, $\hat{u}_j$), the number of firms ($\hat{n}_i$, $\hat{n}_j$) and the wage rate ($\hat{w}_i$, $\hat{w}_j$). Autarky counterparts for country $i$ are obtained by setting $n_j$ and $L_j$ equal to zero. Equilibrium condition $(OR_i)$ embeds the following result:

**Proposition 1: Technology Gap.**

The net marginal benefit of investment in R&D is decreasing in the technology gap ($u_i/u_j$). Whence, there exists a threshold level of $u_i/u_j$ above which investment in R&D is not optimal for the backward economy.

**Proof.** The left hand side of $(OR_i)$ is the marginal benefit of investment in $u_i$. Since this is decreasing in $u_j$, there is a threshold, given by $u_j^* = [2 + \sigma (n_j - 1)] u_i/\sigma n_j$, above which the marginal benefit becomes negative, and investment in R&D cannot be optimal $\Box$.

A fundamental observation on the threshold $u_j^*$ is that, being a property of marginal revenue, it is independent of $w_i$ or any other marginal cost consideration. Consequently, regardless of
how low wages in country \( i \) are, if \( j \)'s product quality is above \( u_j^* \), firms in country \( i \) cannot compete in the world economy. Accordingly, if country \( i \) opens to trade, its industry, together with demand for labour, would collapse. In this scenario the only source of sustenance for consumers in \( i \) would be to either export or consume their endowment of good \( Y \) (Yanes, 2005). A similar result can be found in Sutton (2007). The key driving force behind this is that if a firm offers a price-quality ratio which is lower than its rivals’, it will achieve greater market share.

Given the non-linearity of the system in \((OR_i), (ZP_i)\) and \((LM_i)\), a natural question to pose pertains to the possibility of multiple equilibria. The following proposition clarifies this.

**Proposition 2. Uniqueness.**

The general equilibrium in the open economy and in autarky is unique.

A unique general equilibrium requires a unique equilibrium in each of the three markets under consideration: type \( X \) goods, good \( Y \) and the labour market. In the oligopolistic industry, uniqueness of the Nash Equilibrium in each of the three stages implies uniqueness of the Subgame Perfect Nash Equilibrium. Uniqueness of the Nash Equilibrium for stage 3 (Cournot competition) follows directly from the linearity of the first order conditions for this stage (see Appendix 2.1). For uniqueness in stage 2 (competition in product quality), optimal reply functions must intersect exactly once. Figure I shows the open economy optimal reply schedules, \( u_i^{OR}(u_j) \) and \( u_j^{OR}(u_j) \), which are obtained from condition \((OR_i)\). Their unique (admissible) intersection occurs at \((\bar{u}_i, \bar{u}_j)^{10}\).

As shown in Appendix 1, uniqueness of the Nash Equilibrium in stage 1 (entry) follows by noting that the equations which determine \((\bar{n}_i, \bar{n}_j)\) are either linear or quadratic. In the linear case, uniqueness is straightforward. In the quadratic case, the negative root of these equations violates second order conditions, implying uniqueness of \((\bar{n}_i, \bar{n}_j)\). Uniqueness in the labour market can be seen by noting that labour supply is constant, while labour demand is decreasing in \( w_i \), leading to a unique equilibrium wage rate.

Next we solve for the general equilibrium. In general, explicit solutions to \((OR_i), (ZP_i)\) and \((LM_i)\) cannot be found, and the system must be solved numerically. Nonetheless, the following lemma provides the basis for an explicit solution.

---

10There is another intersection at \((0, 0)\), but at this point any firm has an incentive to deviate to a higher quality level. Moreover, \((0, 0)\) is not contained in the admissible space of \((u_i, u_j)\), which requires \( u_i > 1 \).

As will be shown in Lemma 2, \((\bar{u}_i, \bar{u}_j)\) lies on the 45° line whenever \( \beta_i = \beta \). Optimal reply schedules exhibit the same properties in autarky. However, for a closed economy the equilibrium always lies on the 45° line, since all firms share the same value of \( \beta \), and hence the schedules are symmetric. Note that the optimal reply schedules depicted in Figure I imply that depending on whether their intersection occurs in the upward or downward sloping sections, product qualities will be strategic complements or substitutes, respectively. The equilibrium may correspond to either case, depending on parameter values.
Lemma 2. (i) $\hat{u}_i > \hat{u}_j \Leftrightarrow \beta_i < \beta_j$.
(ii) $\hat{u}_i < \hat{u}_j \Leftrightarrow \beta_i > \beta_j$.
(iii) $\hat{u}_i = \hat{u}_j = \hat{u} \Leftrightarrow \beta_i = \beta_j = \beta$.

For tractability, we assume $\beta_i = \beta$ henceforth. In this case, explicit solutions can be found by algebraic manipulation of conditions $(OR_i)$, $(ZP_i)$ and $(LM_i)$, leading to the general equilibrium solutions presented in Table I.\footnote{Detailed derivations are offered in Appendix 2.3. Although explicit solutions cannot be found when $\beta_i \neq \beta_j$, comparative statics analysis can be readily implemented (available upon request). Results are the same as those presented below.}

<table>
<thead>
<tr>
<th>Table I. General Equilibrium Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Free Trade</strong></td>
</tr>
<tr>
<td>$\hat{n}<em>i = \frac{(\beta - 2)(2 - \sigma) + 1}{2\sigma(\frac{u</em>{oi}}{n_{oi}})^{\frac{1}{\beta}} + 1}$</td>
</tr>
<tr>
<td>$\hat{u} = u_{oi} \left( \frac{L_i}{n_i} \right)^{1/\beta}$</td>
</tr>
<tr>
<td>$\hat{\omega}<em>i = \frac{L_i + L_j}{L_i} \frac{u</em>{oi}^2 L_i^{2/\beta}}{2L_i} \frac{\hat{n}_i^{-2/\beta}}{\left[ \beta \left(1 - \frac{\sigma}{2} \right)^{2/\beta} \right]^{2}}$</td>
</tr>
</tbody>
</table>

The solution in Table I exhibits some interesting features, to which we now turn. First we discuss what happens with the number of firms and product quality. Free trade increases market size, which induces greater investment in R&D. This increases R&D, generating exit at the national level. The exit process will be more severe for a relatively small and backward country. The world number of firms under free trade ($N$) is equal to $n_i + n_j$. Using $\hat{n}_i$ from Table I, $\hat{N}$ simplifies to:

$$\hat{N} = \frac{(\beta - 2)(2 - \sigma)}{2\sigma} + 1.$$  \hspace{1cm} (10)

This is the same as $\hat{n}_A$: for two countries, the world number of firms in autarky is twice that under free trade. If $L_j/L_i (u_{oj}/u_{oi})^\beta > 1$, economy $i$ will contain less than half of the world’s firms. Conversely for $L_j/L_i (u_{oj}/u_{oi})^\beta < 1$. Furthermore, population size ($L_i$) and initial conditions ($u_{oi}$) only affect the distribution of firms between the economies, but not the worldwide number of firms, which depends only on $\beta$ and $\sigma$. The independence of world concentration from market size is closely related to an important and well documented finding in the vertical product differentiation literature. In this literature, the standard result that
market size increases, concentration falls, converging towards a perfectly competitive market structure as the market grows indefinitely is shown not to hold. This is the ‘non-convergence’ property, introduced by Gabszewicz et al. (1981) and Shaked and Sutton (1982, 1983), albeit in a partial equilibrium setting. Non-convergence also arises in the present general equilibrium framework.

We now turn to wages. The net effect of free trade on labour demand depends on the trade-off between rising quality and a falling number of firms. We can investigate this issue further by comparing the free trade and autarky wage rates presented in Table I. It is easy to see that in a symmetric world (with $u_{oj}/u_{oi} = L_j/L_i = 1$), wages are strictly higher under free trade. In this case the quality effect dominates the market structure effect. With asymmetric economies, $\hat{w}_i = \hat{w}_{iA}$ occurs along the following locus:

$$
(11) \quad \left( \frac{u_{oj}}{u_{oi}} \right)^{\beta} = \frac{L_i}{L_j} \left[ \left( 1 + \frac{L_j}{L_i} \right)^{\beta} - 1 \right].
$$

Reversing indices $i$ and $j$ gives the corresponding locus for $\hat{w}_j = \hat{w}_{jA}$. To gain some intuition on the mechanisms operating behind (11), suppose that the left hand side is greater than the right hand side. Then the free trade wage is lower than the autarky wage for country $i$ ($\hat{w}_i < \hat{w}_{iA}$), while the opposite holds for country $j$ ($\hat{w}_j > \hat{w}_{jA}$). Moreover, if the left hand side is greater than one, firms in country $j$ feature better initial conditions relative to those of economy $i$. This leads to fewer firms and a smaller labour demand in country $i$. However, notice that the right hand side of (11) is increasing in $L_j/L_i$. Accordingly, in order to restore $\hat{w}_i = \hat{w}_{iA}$, $L_j/L_i$ needs to rise. There are two ways to achieve this, either through an increase of $L_j$ or a reduction of $L_i$. Suppose $L_j$ rises. In this case, country $j$ not only joins the world economy with an advantage in initial conditions, it also offers access to a large market. If $L_j$ is sufficiently large, it may be sufficient to support a quality level that offsets the loss of labour demand in country $i$ from having fewer firms. Country $j$’s better initial conditions and larger population reduce $\hat{n}_i$ and increase $\hat{n}_j$, but also raise quality for both countries. If the advanced economy is sufficiently large, the resulting increase in quality can raise country $i$’s wage to its autarky level.\(^\text{12}\) Now suppose that the required rise in $L_j/L_i$ is achieved through lower $L_i$. In this case economy $i$’s autarky wage is small to begin with, and free trade implies a relatively large expansion of market size. In order to restore $\hat{w}_i = \hat{w}_{iA}$, the increase in market size must be sufficient to achieve a level of quality that more than offsets the fall in $\hat{n}_i$. A low wage in autarky is, of course, conducive to this outcome.

\(^\text{12}\)This is, of course, subject to the caveat that country $i$ exhibits more than zero firms in equilibrium. Otherwise labor demand in country $i$ collapses, $\hat{w}_i$ becomes zero, and consumers’ sole source of sustenance is their endowment of good $Y$.
IV. ASYMMETRIC INITIAL CONDITIONS

We describe how equilibrium outcomes change with initial conditions \((u_{oi})\). The following is in essence a global comparative statics analysis of the solutions presented in table I. The results hold for all of the admissible parameter space \((\beta > 2, \sigma \in (0, 1), u_{oi} \geq 1\) and \(L_i > 0\).\(^{13}\) Figure II depicts the effect of changes in country \(i\)'s initial conditions, \textit{ceteris paribus}.

In the top left graph, we can see that increases in \(u_{oi}\) will increase \(\hat{u}_{i,A}\) linearly, will not affect \(\hat{u}_{j,A}\), and will increase \(\hat{u}\) convexly. The effects in autarky are clear from Table I. To see why \(\hat{u}\) increases convexly, note that there are two effects at work. First, there is a multiplicative effect of \(u_{oi}\). This is the same effect that operates in autarky. The second effect operates through the number of firms (bottom left graph). As \(u_{oi}\) increases, \(\hat{n}_i\) also rises, dampening the increase in \(\hat{u}\). Meanwhile, \(\hat{n}_j\) falls. To see this, note that a rise in \(u_{oi}\) reduces the marginal cost of R&D in economy \(i\). This leads to a rise in country \(i\)'s quality. From Lemma 2 (part iii), firms in country \(j\) must match country \(i\)'s quality if they are to maintain a positive market share. Rising R&D in economy \(j\) induces exit in that country. In order to exhaust net profits, entry is required in country \(i\), such that the world number of firms remains constant. The net effect of the changes in quality and market structure on the demand for labour can be observed in the top right graph. \(\hat{w}_i\) is rising convexly, since not only is the demand for labour increasing from the rise in \(\hat{u}\), it is also increasing through the rise in \(\hat{n}_i\). Meanwhile, \(\hat{w}_j\) falls since the rise in \(\hat{u}\) is more than offset by fall in \(\hat{n}_j\). Note that the fall in \(\hat{w}_j\) is slower than the rise in \(\hat{w}_i\). This asymmetric behaviour around \(u_{oi} = u_{oj}\) can be ascertained from Table I: As \(u_{oi}\) rises, \(\hat{w}_i\) rises at a quadratic rate through \(u_{oi}^2\) as well as through the increase in \(\hat{n}_i\). Instead, \(\hat{w}_j\) falls only through the contraction in \(\hat{n}_j\), since \(u_{oj}\) is constant. In autarky, \(\hat{w}_{j,A}\) is unaffected by \(u_{oi}\), and \(\hat{w}_{i,A}\) rises at a quadratic rate. The different slopes of \(\hat{w}_{i,A}, \hat{w}_i, \hat{w}_{j,A}\), and \(\hat{w}_j\) give rise to crossings between these schedules. Two crossings are of particular interest. First, the crossing between \(\hat{w}_i\) and \(\hat{w}_{i,A}\), which occurs at \(u_{oi}^*\). Second, the crossing between \(\hat{w}_j\) and \(\hat{w}_{j,A}\), occurring at \(u_{oi}^{**}\). These give rise to the following result:

\textbf{Proposition 3. Initial Conditions, Trade and Wages.}

(i) For \(u_{oi} < u_{oi}^*: \hat{w}_i < \hat{w}_{i,A}, \hat{w}_j > \hat{w}_{j,A}\).

(ii) For \(u_{oi} = u_{oi}^*: \hat{w}_i = \hat{w}_{i,A}, \hat{w}_j > \hat{w}_{j,A}\).

(iii) For \(u_{oi}^* < u_{oi} < u_{oi}^{**}: \hat{w}_i > \hat{w}_{i,A}, \hat{w}_j > \hat{w}_{j,A}\).

(iv) For \(u_{oi} = u_{oi}^{**}: \hat{w}_i > \hat{w}_{i,A}, \hat{w}_j = \hat{w}_{j,A}\).

(v) For \(u_{oi}^{**} < u_{oi}: \hat{w}_i > \hat{w}_{i,A}, \hat{w}_j < \hat{w}_{j,A}\).

\(^{13}\text{Remaining comparative statics (for } L_i, \sigma \text{ and } \beta \text{ are presented in Appendix 2.4.}\)
Proof.\textsuperscript{14} In a symmetric world economy, with $u_{oi} = u_{oj}$, $L_i = L_j$, it is straightforward to show that $\hat{w}_i = \hat{w}_j > \hat{w}_{iA} = \hat{w}_{jA}$. For economy $i$, we have that $\hat{w}_i$ and $\hat{w}_{iA}$ are both convex and increasing functions of $u_{oi}$. Since $\hat{w}_i$ is increasing at a higher rate than $\hat{w}_{iA}$, these schedules cross. The crossing occurs at $u^*_{oi}$ (shown in Figure II). This, together with the fact that $\hat{w}_i > \hat{w}_{iA}$, implies that $u^*_{oi}$ lies to the left of $u_{oi} = u_{oj}$. Setting $\hat{w}_i = \hat{w}_{iA}$ and simplifying yields (11), from which we solve for

\begin{equation}
\begin{aligned}
    u^*_{oi} &= u_{oj} \left\{ \frac{L_i}{L_j} \left[ \left( 1 + \frac{L_j}{L_i} \right)^{\frac{\beta_i}{\beta}} - 1 \right] \right\}^{-1/\beta} .
\end{aligned}
\end{equation}

For economy $j$, we have that $\hat{w}_j$ is a convex and decreasing function of $u_{oi}$, while $\hat{w}_{jA}$ is independent of $u_{oi}$. Hence, there exists a crossing of $\hat{w}_j$ and $\hat{w}_{jA}$. Since $\hat{w}_j > \hat{w}_{jA}$ at $u_{oi} = u_{oj}$, this crossing lies to the right of $u_{oi} = u_{oj}$, at $u^{**}_{oi}$ (also shown in Figure II). Setting $\hat{w}_j = \hat{w}_{jA}$ and simplifying we obtain the counterpart of (11) for economy $j$, from which we obtain

\begin{equation}
\begin{aligned}
    u^{**}_{oi} &= u_{oj} \left\{ \frac{L_j}{L_i} \left[ \left( 1 + \frac{L_i}{L_j} \right)^{\frac{\beta_j}{\beta}} - 1 \right] \right\}^{1/\beta} .
\end{aligned}
\end{equation}

Having found $u^*_{oi}$ and $u^{**}_{oi}$, regions (i)-(v) in the proposition follow immediately $\square$.

Proposition 3 states that if two economies are not too different in their initial conditions, free trade generates higher wage rates for both. However, if initial conditions are very dissimilar, the backward economy achieves a higher wage rate in autarky. The advanced economy achieves a higher wage rate under free trade. If the advanced economy wishes to maximize national income, it has incentives to negotiate free trade agreements which could depress wages in the backward economy.

It is worth noting in Figure II that as initial conditions become very dissimilar, with $u_{oi}$ becoming either very small or very large (relative to $u_{oj}$), the number of firms in the advanced economy converges towards $\tilde{N}$, while that of the backward economy converges to zero. Accordingly, product quality converges to its autarky counterpart (either $\hat{u}_{iA}$ if $u_{oi}$ is very large, or $\hat{u}_{jA}$ if $u_{oi}$ is very small). Meanwhile, wages in the backward economy converge to zero.

It remains to analyse the trade balance. From the bottom right graph of Figure II, it is clear that the backward economy is a net importer of type $X$ goods and a net exporter of good $Y$. The advanced economy exhibits the reverse pattern. This follows from the relationship between trade balances obtained in section II: $TB_{yi} = -TB_{yj} = TB_{xj} = -TB_{xj}$. Moreover, the larger the gap in initial conditions, the more pronounced sectorial trade imbalances will be.

\textsuperscript{14}An alternative proof for the case of $\beta_i \neq \beta_j$, based on comparative statics in the neighborhood of the equilibrium, is available upon request.
V. Welfare Analysis

Positive analysis (in particular, Proposition 3) reveals that an economy may suffer lower wages as a result of free trade. This raises the question of whether such a fall in income could lead to welfare losses. In this section we turn our attention to the welfare consequences of opening the economy, in the presence of asymmetries in initial conditions and population size. The first step is to construct a welfare indicator, to then compare welfare in autarky versus free trade. The welfare indicator is obtained by substituting into the utility function (equation 1) the demand for good $Y$ (solved as a residual from the consumer’s budget constraint, equation 2), quantity and price of type $X$ goods (equations (A2.1.4) and (A2.1.6) in Appendix 2.1). After simplifying, this yields:

$$\hat{V}_i = \frac{\tilde{w}_i^2 \tilde{N}}{4[2 + \sigma (\tilde{N} - 1)]^2} + \hat{w}_i + \hat{Y}_i, \quad \hat{V}_{iA} = \frac{\tilde{w}_i^2 \tilde{N}_A}{4[2 + \sigma (\tilde{N}_A - 1)]^2} + \hat{w}_{iA} + \hat{Y}_i,$$

where a circumflex ($\hat{}$) over a variable denotes a general equilibrium outcome (corresponding to Table I). From these expressions it is clear that welfare is determined by product variety and quality, wages and the endowment of good $Y$. With symmetric economies, welfare indicators simplify to:

$$\hat{V}_i = \frac{3}{2} \hat{w}_i + \hat{Y}_i, \quad \hat{V}_{iA} = \frac{3}{2} \hat{w}_{iA} + \hat{Y}_i$$

In this case, given $\hat{Y}_i$, welfare is determined solely by wages. Comparing $\hat{V}_i$ and $\hat{V}_{iA}$ from (14) yields the following result:

**Proposition 4A. Gains from Trade: Single Economy.**

$$\hat{V}_i \geq \hat{V}_{iA} \iff \left(1 + \frac{L_j}{L_i} \frac{u_{oj}}{u_{oi}}\right)^{2/\beta} \left[1 + 2 \frac{1 + \frac{L_j}{L_i} u_{oj}^{\beta}}{1 + \frac{L_j}{L_i} u_{oj}^{\beta}}\right] \geq 3. $$

**Proof.** Substitute $\hat{V}_i$ and $\hat{V}_{iA}$ from (14) into the left hand side of (15). $\hat{w}_i$ and $\hat{w}_{iA}$ (Table I) can be expressed as follows:

$$\hat{w}_i = \frac{\tilde{w}_i^2 \tilde{N}_i}{4[2 + \sigma (\tilde{N}_i - 1)]^2} \frac{L_i + L_j}{L_i}, \quad \hat{w}_{iA} = \frac{\tilde{w}_{iA}^2 \tilde{N}_A}{4[2 + \sigma (\tilde{N}_A - 1)]^2}.$$

Noting that $\tilde{N} = \tilde{N}_A$, and simplifying the left hand side of (15) yields the inequality on the right hand side of (15). □

The corresponding expression for economy $j$ is obtained by reversing indices $i$ and $j$. If (15) holds with equality, we obtain an indifference surface in $(L_j/L_i, u_{oj}/u_{oi}, \beta)$-space. Points
lying above this surface imply $\hat{V}_i > \hat{V}_{iA}$, while points below the surface yield $\hat{V}_i < \hat{V}_{iA}$. Holding $\beta$ constant, we can project the indifference surfaces for each economy onto indifference loci in $(L_j/L_i, u_{oj}/u_{oi})$-space, showing the set of points for which $\hat{V}_i = \hat{V}_{iA}$ and $\hat{V}_j = \hat{V}_{jA}$. This is presented in Figure III.\footnote{The inequality in (15) depends also on $\beta$, and the effect of changing $\beta$ is discussed in Appendix 2.5.}

The shaded areas in Figure III show regions for which autarky yields higher welfare than free trade (country $i$ is shown on the left side of the diagram, country $j$ on the right side). The boundaries of these areas represent the indifference loci. The remaining (unshaded) area features higher welfare under free trade for both economies. This includes the case of symmetric economies, with $L_j/L_i = u_{oj}/u_{oi} = 1$, shown at the center of the diagram.\footnote{With symmetric economies (15) simplifies to $2^{2/\beta} > 1$, which holds with strict inequality (recall $\beta > 2$).}

Figure III also plots the schedules $u_{oi}^*/u_{oj}$ and $u_{oi}^{**}/u_{oj}$, which were obtained in (12) and (13), respectively. $u_{oi}^*/u_{oj}$ corresponds to combinations of $L_i/L_j$ and $u_{oi}/u_{oj}$ such that $\hat{w}_i = \hat{w}_{iA}$. To the left of $u_{oi}^*/u_{oj}$ we have $\hat{w}_i < \hat{w}_{iA}$, whereas $\hat{w}_i > \hat{w}_{iA}$ holds to the right of $u_{oi}^*/u_{oj}$. Likewise, $u_{oi}^{**}/u_{oj}$ shows pairs $(L_i/L_j, u_{oi}/u_{oj})$ for which $\hat{w}_j = \hat{w}_{jA}$. To the left of $u_{oi}^{**}/u_{oj}$ we have $\hat{w}_j > \hat{w}_{jA}$, and to the right we have $\hat{w}_j < \hat{w}_{jA}$. Thus, income reductions brought about by free trade do not necessarily coincide with welfare losses. The reasons for this will become clear below.

An important feature of Figure III is that the shaded regions never overlap, that is, it is never the case that both economies are better off in autarky. This is a consequence of Proposition 4A, and its implication is captured in the following corollary.

**COROLLARY 1. UNEQUAL EXCHANGE.**
For any point $(u_{oi}/u_{oj}, L_i/L_j)$, if $\hat{V}_i < \hat{V}_{iA}$, then $\hat{V}_j > \hat{V}_{jA}$ ($i, j = 1, 2$, $i \neq j$).

The advanced economy, which is better off under free trade, has incentives to seek a free trade agreement, even though this may not be in the interest of the backward economy. This finding clarifies the nature of free trade negotiations, and provides clear criteria for bilateral free trade agreements. Free trade agreements will be welfare improving to both economies so long as neither economy is located in the shaded regions in Figure III, that is, if initial conditions are either quite similar or very different, or the population of the advanced economy is sufficiently large, relative to that of the backward economy. It is in the shaded regions, where initial conditions are neither very similar nor very different, and population of the advanced economy is not sufficiently large relative to that of the backward economy, that trade will reduce welfare in the backward economy.

The intuition behind the shaded regions is as follows. Consider the shaded region for which $\hat{V}_i < \hat{V}_{iA}$ (left side of the diagram). This represents an area in which economy $j$ is advanced relative to $i$, but not exceedingly so ($u_{oi}/u_{oj} < 1$, but not too low). It also corresponds to
economy $j$ having a population that is not very large relative to economy $i$’s ($L_i/L_j$ is not too low).

On the one hand, economy $j$’s advantage in initial conditions tends to lower wages (and thus welfare) in economy $i$, and raise wages (and welfare) in economy $j$. This can be corroborated by noting that the shaded region with $\hat{V}_i < \hat{V}_{iA}$ lies left of the $u^*_{oi}/u_{oj}$ and $u^{**}_{oi}/u_{oj}$ schedules, implying $\hat{w}_i < \hat{w}_{iA}$ and $\hat{w}_j > \hat{w}_{jA}$, respectively. Free trade also increases product quality, which tends to increase welfare in both economies. However, the rise in product quality is insufficient to offset the welfare reduction due to lower wages in economy $i$.

On the other hand, $L_j$ not being overly large relative to $L_i$ means that when economy $j$ opens to trade, it does not offer access to a particularly large market, which does little for economy $i$’s welfare gains. In summary, the loses from trade in economy $i$ arise through economy $j$’s combination of relatively advanced initial conditions and a not-particularly-large market size. The analysis for the other shaded region, with $\hat{V}_j < \hat{V}_{jA}$, is identical, with the roles of $i$ and $j$ reversed.

It remains to analyse the non-shaded region, in which free trade improves welfare for both economies. For the central section of this region it is easy to see that wages tend to rise in both economies, or should free trade reduce wages in any economy, the contraction will be of a relatively small magnitude. This, together with higher product quality, leads to welfare improvements for both economies. What about the non-shaded regions on the right and left extremes of the diagram? In these regions, it is easy to see that the advanced economy benefits from trade through rising wages and higher product quality. Why the backward economy also benefits from trade is less clear. Essentially, the rise in product quality is sufficient to offset the fall in the wage rate. The gains from trade in the backward economy arise from a consumption effect: Even though wages fall, consumers in the backward economy can now access products with a level of quality that is much higher than what would have been available in autarky. We label this an ‘impoverished consumerist economy’. As examples of this, we can think of poor nations where consumers are happy to purchase high quality imports, even though this implies a contraction of local industry and labour demand.

It is perhaps surprising that in comparing welfare under free trade and autarky, no reference has been made to changes in product variety (the number of firms). The derivations leading to (15), as well as this expression itself, make clear that any such changes are of no concern to welfare comparisons between autarky and free trade, since any terms relating to the number of firms contained in the welfare indicators under free trade and autarky cancel out. This is a consequence of the fact that the number of product varieties available to consumers is identical in autarky and free trade ($\hat{n}_{iA} = \hat{N}$), which follows from the non-convergence property.

The idea that free trade might, under the circumstances specified above, benefit one economy
at the expense of the other raises the question of whether the world as a whole will gain from trade. A response to this requires the construction of indicators for world welfare. Under the representative consumer assumption, total world welfare is given by \( \sum_{i=1}^{2} L_i \tilde{V}_i \) under free trade, and by \( \sum_{i=1}^{2} L_i \tilde{V}_{iA} \) in autarky. Substituting general equilibrium outcomes (Table I) into (14), world welfare under free trade can be simplified to

\[
\sum_{i=1}^{2} L_i \tilde{V}_i = \frac{3}{2} \left( L_1 + L_2 \right) \tilde{N}^{1-2/\beta} \left( L_1 u_{\alpha \alpha}^\beta + L_2 u_{\alpha \beta}^\beta \right)^{2/\beta} + \sum_{i=1}^{2} L_i \tilde{Y}_i.
\]

Similar calculations yield the corresponding expression for autarky. Prior to comparing world welfare under free trade and autarky, it is convenient to introduce the following lemma.

**Lemma 3.** Let \( \Lambda = \left( \frac{1}{1+L_j/L_i} + \frac{L_j^{2/\beta} u_{\alpha j}}{L_j^{2/\beta} u_{\alpha i}} \right) / \left( \frac{1}{1+L_j/L_i} \right) \left( 1 + \frac{L_j u_{\alpha j}}{L_i u_{\alpha i}} \right)^{2/\beta} \). Then \( \Lambda \leq 1 \) holds for all admissible parameter values.

**Proof.** By inspection of \( \Lambda \). □

**Proposition 4b. Gains from Trade: World Economy.**

\[
\sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \tilde{V}_{iA}.
\]

**Proof.** Substitute (16) and its autarky counterpart into \( \sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \tilde{V}_{iA} \). Simplification yields \( \Lambda \leq 1 \), which follows from Lemma 3. □

Thus, even though a country might lose from trade, the gains from trade accruing to the world as a whole are sufficient to offset losses to an individual nation, provided a suitable compensation mechanism is implemented.

**VI. Industrial Policy**

Is it possible to subsidize investment in R&D in order to help an economy achieve higher welfare? Consider an *ad-valorem* subsidy to R&D, \( \delta_i \), financed with a lump sum tax on consumers. R&D expenditure per firm becomes \( \delta_i w_i (u_i/u_{\alpha i})^\beta \), while consumers pay a per capita lump sum tax of \( T_i = n_i (1 - \delta_i) w_i (u_i/u_{\alpha i})^\beta / L_i \). The only change in the consumer’s problem is that income is reduced by \( T_i \). Accordingly, demand for type \( X \) goods is unchanged, while demand for good \( Y \) is reduced by \( T_i \). In the firm’s problem, choice of \( x_{ki} \) and the resulting solved out payoff (equation 5) in stage 3 are unaffected. First order conditions in stage 2 become:

\[
(L_i + L_j) \frac{\partial \pi_{ki}}{\partial u_{ki}} = \delta_i w_i (u_{ki} / u_{\alpha i})^\beta,
\]
and the stage 1 free entry condition is now given by

\[(L_i + L_j) \pi_{ki} \geq \delta_i w_i \left( \frac{u_{ki}}{u_{oi}} \right)^\beta.\]

Since the subsidy does not affect the labour requirement, given by \( f(u_{ki}) = (u_{ki}/u_{oi})^\beta \), the labour market clearing condition (equation 9) is unchanged. Equilibrium outcomes under industrial policy are differentiated from outcomes without intervention by subscript ‘\( t \)’. Solving for equilibrium in a similar manner to section III, we obtain the following results.

**Proposition 5. Neutrality of Industrial Policy.**

In free trade: \( \hat{n}_{it} = \hat{n}_i, \hat{\mu}_t = \hat{\mu}, \hat{\mu}_{it} = \hat{\mu}/\delta_i, \hat{V}_{it} = \hat{V}_i. \)

In autarky: \( \hat{n}_{At} = \hat{n}_A, \hat{\mu}_{At} = \hat{\mu}_A, \hat{\mu}_{At}/\delta_i, \hat{V}_{At} = \hat{V}_A. \)

**Proof.** Using (18) and (19) instead of (7) and (8), equilibrium conditions \((OR_t)\) and \((ZP_t)\) are replaced by:

\[
(OR_{it}) \quad \frac{(L_i + L_j) [2 + \sigma (n_i + n_j - 2)]}{(2 - \sigma)^2 [2 + \sigma (n_i + n_j - 1)]} \left\{ 1 - \frac{\sigma [n_i + n_j u_j/u_i]}{2 + \sigma (n_i + n_j - 1)} \right\} = \delta_i w_i \beta \frac{u_i^{\beta-2}}{u_{oi}}.
\]

\[
(ZP_{it}) \quad \frac{(L_i + L_j)}{2(2 - \sigma)^2} \left\{ 1 - \frac{\sigma [n_i + n_j u_j/u_i]}{2 + \sigma (n_i + n_j - 1)} \right\}^2 = \delta_i w_i \beta \frac{u_i^{\beta-2}}{u_{oi}}.
\]

while \((LM_t)\) remains as before. Explicit solutions for \( \hat{n}_{it}, \hat{\mu}_t \) and \( \hat{\mu}_{it} \) can be found by following the procedure set out in Appendix 2.3. Comparing these outcomes with their counterparts in Table I, it follows that the number of firms and quality are unchanged by industrial policy, while wages rise by a factor of \( (1/\delta_i). \) Making appropriate substitutions into (14), it follows that welfare is not affected by the subsidy. \( \Box \)

That quality is not affected by the subsidy follows from the fact that the labour requirement, \( f(u_{ki}), \) is unchanged. Even though firms do try to increase their quality in response to a lower marginal cost, in general equilibrium, the quality level achievable with economy’s labour force is the same as without the subsidy, and it is the wage which rises in response to increased pressure in the labour market. The rise in wages fully offsets savings in R&D, and, in general equilibrium, the free entry condition is unchanged. Hence the number of firms is also not affected. With product quality and the number of firms unchanged, the only remaining effect on welfare must then operate through income. However, after tax income is also unaffected since the rise in wages is offset by the lump sum tax.\(^{17}\)

\(^{17}\)The neutrality of industrial policy hinges on labor market clearing in the context of a single industry. A
VII. Social Planner

A social planner maximizes aggregate world welfare, \(\sum_{i=1}^{2} L_i V_i\), where \(V_i\) is utility in country \(i\) (equation 1). First best (social optimum) solutions are denoted by a tilde (\(\tilde{\cdot}\)). Substituting \(\Pi_{ki} = (L_i + L_j) p_{ki} x_{ki} - w_i (u_{ki}/u_{oi})^{\beta_i}\), the world resource constraint simplifies to:

\[
2 \sum_{i=1}^{2} L_i Y_i \leq 2 \sum_{i=1}^{2} \left\{ L_i \tilde{Y}_i + w_i \left[ L_i - \sum_{k=1}^{n_i} \left( \frac{u_{ki}}{u_{oi}} \right)^{\beta_i} \right] \right\}
\]

The autarky counterpart for country \(i\) is obtained by setting \(L_j = n_j = 0\). The last term in (20), namely, \(L_i - \sum_{k=1}^{n_i} (u_{ki}/u_{oi})^{\beta_i}\), is labour supply minus labour demand in country \(i\). If the planner makes full use of resources, this will be zero at a social optimum. Likewise, full resource utilization ensures that (20) holds with equality. Substituting (20) with equality into \(\sum_{i=1}^{2} L_i V_i\) and simplifying yields:

\[
2 \sum_{i=1}^{2} L_i \left\{ \sum_{k=1}^{N} \left( x_{ki} - \frac{x_{ki}^2}{u_{ki}} \right) - 2\sigma \sum_{k=1}^{N} u_{ki} \sum_{l \neq k}^{N-1} \frac{x_{li}}{u_{li}} + \frac{w_i}{L_i} \left[ L_i - \sum_{k=1}^{n_i} \left( \frac{u_{ki}}{u_{oi}} \right)^{\beta_i} \right] \right\}.
\]

The social planner maximizes (21) by choosing \(x_{ki}\), \(u_{ki}\), and \(n_i\) \((i = 1, 2)\). When choosing each of these variables the planner takes other variables as given. Hence, the solution method follows the three stage procedure used before. However, the planner realizes that \(w_i\) could, in principle, depend on \(x_{ki}\), \(u_{ki}\), and \(n_i\), and takes this into consideration when seeking the social optimum. Nonetheless, because \(w_i\) only appears in (21) as a factor multiplying the term \(\left[ L_i - \sum_{k=1}^{n_i} (u_{ki}/u_{oi})^{\beta_i} \right]\), labour market clearing will ensure that such feedbacks do not affect the social optimum. It follows that all terms involving differentiation of \(w_i\) cancel out. To reduce cluttering, these are not shown. First order conditions for \(x_{ki}\) are then given by:

\[
1 - 2 \frac{x_{kh}}{u_{kh}^2} - 4\sigma \sum_{l \neq k}^{N-1} \frac{u_l}{u_{ki}} = 0
\]

The natural question to pose is: To what extent would allowing for other sources of employment affect this result? The simplest setting for this is one in which we add a constant returns to scale industry producing good \(Y\), resulting in a two sector economy, as in Yanes (2005). With labor as the sole input, constant returns to scale imply a linear (Ricardian) technology. Assume this technology is freely available, so that it becomes workers’ outside option. The marginal product of labor in industry \(Y\) then constitutes a minimum wage. So long as employment in industry \(Y\) is zero, wages will be strictly higher than the minimum wage. With positive employment in industry \(Y\) (and hence minimum wages), it can be shown that country \(i\)’s welfare-maximizing subsidy is given by \(\delta^* = \beta / 3\) in autarky, and by \(\delta^* = \beta(L_i + L_j)/(3L_i + 2L_j)\) under free trade. Such a subsidy shifts workers from industry \(Y\) to the oligopolistic industry. However, if the subsidy is sufficient to drive employment in industry \(Y\) to zero (such that wages rise above the minimum), the neutrality result kicks in immediately. Detailed derivations are available upon request.

\(^{18}\) Full expressions are available upon request. The solution to the social planner’s problem is not affected by this exclusion.
This is a system of \( N \) linear equations in \( x_1 \) and \( x_2 \). Following similar steps to those outlined in Appendix 2.1 yields a unique socially optimal output vector \( \tilde{x} \), with typical element:

\[
(22) \quad \tilde{x}_{ki} = \frac{u_{ki}^2}{2(1 - 2\sigma)} \left[ 1 - \frac{2\sigma}{1 + 2\sigma(N - 1)} \sum_{l=1}^{N} \frac{u_l}{u_{ki}} \right].
\]

Substituting \( \tilde{x}_{ki} \) into the planner’s objective and simplifying, we obtain aggregate world welfare in terms of product quality and the number of firms:

\[
(23) \quad \sum_{i=1}^{2} L_i \left\{ \frac{\sum_{k=1}^{N} u_k^2 - \frac{2\sigma}{1 + 2\sigma(N - 1)} \left( \sum_{k=1}^{N} u_k \right)^2}{4(1 - 2\sigma)} + \frac{1}{L_i} \left( \frac{u_{ki}^2}{u_{oi}^2} \right) \frac{\beta_i}{\beta_i} \right\}.
\]

This replaces the solved-out payoff in equation (5). The planner now chooses \( u_{ki} \) in order to maximize (23). The associated first order conditions are:

\[
(24) \quad \frac{(L_i + L_j) u_{ki}}{2(1 - 2\sigma)} \left[ 1 - \frac{2\sigma}{1 + 2\sigma(N - 1)} \sum_{l=1}^{N} \frac{u_l}{u_{ki}} \right] - \frac{w_i \beta_i}{u_{ki}} \left( \frac{u_{ki}}{u_{oi}} \right)^{\beta_i} = 0
\]

Expression (24) contains a system of \( N \) non-linear equations in \( u_1 \) and \( u_2 \). Before solving this system, assume that a unique solution vector \( \tilde{u} = (\tilde{u}_1, \tilde{u}_2) \) exists and its elements share the following property: \( \tilde{u}_{ki} = \tilde{u}_i \). This (hypothetical) solution is then substituted into (23) to obtain aggregate world welfare in terms of the number of firms, which simplifies to:

\[
(25) \quad \sum_{i=1}^{2} L_i \left\{ K + \frac{1}{L_i} \left[ L_i - n_i \left( \frac{u_{ki}}{u_{oi}} \right)^{\beta_i} \right] \right\}.
\]

where \( K = \frac{n_1 u_1^2 + n_2 u_2^2 - \frac{2\sigma}{1 + 2\sigma(N - 1)} (n_1 u_1 + n_2 u_2)^2}{4(1 - 2\sigma)} \).

This objective is then maximized by choosing \( n_i \) (\( i = 1, 2 \)). The hypothetical solution, \( \tilde{u} \), is a function of \( n_i \), and this needs to be kept in mind when obtaining first order conditions. However, as follows from the envelope theorem, all terms involving differentiation of \( \tilde{u} \) with respect to \( n_i \) cancel out since they are multiplied by the first order conditions in (24), which are equal to zero at \( \tilde{u} \). Optimality conditions for \( (\tilde{n}_1, \tilde{n}_2) \) can be written as:

\[
(26) \quad 1 + \frac{4\sigma \left( n_i + n_j \frac{u_j}{u_i} \right)}{1 + 2\sigma(N - 1)} \left[ \frac{\sigma \left( n_i + n_j \frac{u_j}{u_i} \right)}{1 + 2\sigma(N - 1)} - 1 \right] = \frac{4(1 - 2\sigma) w_i \beta_i \left( \frac{u_{ki}}{u_{oi}} \right)^{\beta_i - 2}}{L_i + L_j}.
\]

A solution to this system is a pair \( (\tilde{n}_1, \tilde{n}_2) \) such that (26) holds for \( i, j = 1, 2, i \neq j \).
There are three conditions for a social optimum, which are similar to the equilibrium conditions set out in section III (namely, \( OR_i, ZP_i \) and \( LM_i \)). The first condition relates to the socially optimal quality level. This corresponds to (24). Evaluating this at \( \bar{u} = (\bar{u}_1, \bar{u}_2) \), (24) reduces to a pair of equations in \( \bar{w}_1 \) and \( \bar{w}_2 \), and replaces equilibrium condition \( OR_i \). The second condition relates to the socially optimal number of firms. This corresponds to (26), and replaces \( ZP_i \). The third condition is labour market clearing, given, as before, by \( LM_i \).

Let us now return to the (hypothetical) solution of (24). Using the three conditions, and following similar arguments to those used in the proofs of Lemmas 1 and 2 and Proposition 2, it is straightforward to show that: (1) the social optimum is unique, (2) \( \bar{u} = (\bar{u}_1, \bar{u}_2) \) has typical element \( \bar{u}_{ki} = \bar{u}_i \), and (3) when \( \beta_i = \beta \), we have \( \bar{u}_i = \bar{u} \). As before, we assume \( \beta_i = \beta \) in what follows. This brings forth an explicit solution for the social optimum. Following a similar procedure to that shown in Appendix 2.3, solutions are obtained by manipulation of the optimal plan conditions, namely, (24) evaluated at \( \bar{u} = (\bar{u}_1, \bar{u}_2) \), (26), and \( LM_i \). The results are presented in Table II.

<table>
<thead>
<tr>
<th>Table II. Social Optimum (First Best)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FREE TRADE</strong></td>
</tr>
<tr>
<td>( \bar{n}<em>i = \frac{(\beta-2)(1-2\sigma)}{4\sigma \frac{L_i}{L_i} \frac{\frac{u</em>{oi}}{n_i}}{\frac{\sigma}{L_i}}} + 1 )</td>
</tr>
<tr>
<td>( \bar{u} = u_{oi} \left( \frac{L_i}{\bar{n}_i} \right)^{1/\beta} )</td>
</tr>
<tr>
<td>( \bar{w}<em>i = \frac{L_i + L</em>{ij}}{L_i} u_{oi}^2 L_{ij}^{2/\beta} \frac{\bar{n}_i^{1-2/\beta}}{\bar{n}_i^{2/(1-2\sigma)}} )</td>
</tr>
</tbody>
</table>

As before, we have \( \bar{N} = \bar{n}_A = \bar{n}_1 + \bar{n}_2 \). The next step is to benchmark the social optimum with the market (decentralized) equilibrium, in autarky as well as free-trade. The following lemma will be useful in establishing our results.

**Lemma 4.** Let \( \Phi = \left\{ \frac{(\beta-2)(1-2\sigma)}{2(\beta(2-\sigma)-4(1-\sigma))} \right\}^{1-2/\beta} \) and \( \Upsilon = \frac{6\beta(1-2\sigma)}{\beta(2-\sigma)+2\sigma} \). Then \( \sigma > 0 \Rightarrow \Phi / \Upsilon > 1 \).

**Proof.** Consider \( \Phi / \Upsilon \) as a function of \( \beta \) and \( \sigma \). The surface given by \( \Phi / \Upsilon \) lies below 1 only for \( \sigma < 0 \), otherwise it lies strictly above 1. \( \square \)

**Proposition 6a. Social Optimum versus Market Equilibrium**

(i) \( \bar{w}_i \geq \bar{w}_i \) and \( \bar{w}_{iA} \geq \bar{w}_{iA} \Leftrightarrow \Phi / \Upsilon \geq \frac{\beta}{\beta} \).

(ii) \( \bar{N} < \bar{N} \) and \( \bar{n}_A < \bar{n}_A \).
(iii) \( \tilde{u} > \hat{u} \) and \( \tilde{u}_{iA} > \hat{u}_{iA} \).
(iv) \( \tilde{x} > \hat{x} \) and \( \tilde{x}_{iA} > \hat{x}_{iA} \).

\[ \Phi / \Upsilon \geq \frac{\beta}{\tilde{N}}. \]

Proof. (i): Substitute \( \tilde{w}_i \) (from Table II) and \( \hat{w}_i \) (from Table I) into \( \tilde{w}_i \geq \hat{w}_i \). Simplifying one obtains \( \Phi / \Upsilon \geq \frac{\beta}{\tilde{N}} \). Simplification of \( \tilde{w}_{iA} \geq \hat{w}_{iA} \) yields the same result.

(ii) and (iii): The inequalities given by \( \tilde{N} < \tilde{N}, \tilde{n}_A < \tilde{n}_A, \tilde{u} > \hat{u} \) and \( \tilde{u}_{iA} > \hat{u}_{iA} \) (as defined in Tables I and II) all simplify to \( \sigma > -3 \beta - 2 / 4 \). This holds since \( \sigma \in (0, 1) \) and \( \beta > 2 \).

(iv): Inspection of (22) and (A2.1.4) reveals that production is increasing in product quality and decreasing in the number of firms. \( \tilde{x} > \hat{x} \) and \( \tilde{x}_{iA} > \hat{x}_{iA} \) then follow from (ii) and (iii). \( \square \)

Thus, the social optimum features higher quality and fewer firms, relative to the free market equilibrium. As a consequence, there is higher production per firm. Discussion of wages will be offered after Proposition 6B (on welfare).

Let us turn our attention to world welfare. In the market equilibrium this is given by (16). At the social optimum, world welfare is obtained by evaluating (25) at the outcomes set out in Table II. Simplifying, this leads to

\[ \sum_{i=1}^{2} L_i \tilde{V}_i = \frac{(L_1 + L_2) \tilde{N}^{1-2/\beta}}{2 \beta (1 - 2 \sigma)} \left( L_1 u_{\alpha_1}^\beta + L_2 u_{\alpha_2}^\beta \right)^{2/\beta} + \sum_{i=1}^{2} L_i \tilde{Y}_i. \]

Similar substitutions yield the corresponding expression for autarky. Comparison of world welfare under different regimes gives rise to a ranking of regimes in terms of welfare. This is summarized in the following proposition.

**Proposition 6B. Welfare Ranking**

(i) \( \sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \hat{V}_i \geq \sum_{i=1}^{2} L_i \hat{V}_{iA} \).

(ii) \( \sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \hat{V}_{iA} \geq \sum_{i=1}^{2} L_i \hat{V}_{iA} \).

(iii) \( \sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \hat{V}_{iA} \Leftrightarrow \Phi \Lambda / \Upsilon \geq 1. \)

Proof. (i): Substituting (27) and (16) into \( \sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \hat{V}_i \) and simplifying yields \( \Phi / \Upsilon \geq 1 \) (implied by Lemma 4). \( \sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \hat{V}_{iA} \) is Proposition 4B. This simplifies to \( \Lambda \leq 1 \) (which follows by Lemma 3).

(ii): Substitute (27) and its autarky counterpart into \( \sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \hat{V}_i \). Simplification yields \( \Lambda \leq 1 \) (Lemma 3). \( \sum_{i=1}^{2} L_i \hat{V}_{iA} \geq \sum_{i=1}^{2} L_i \hat{V}_{iA} \) simplifies to \( \Phi / \Upsilon \geq 1 \) (Lemma 4).

(iii): Substitute (16) and the autarky counterpart to (27) into \( \sum_{i=1}^{2} L_i \tilde{V}_i \geq \sum_{i=1}^{2} L_i \hat{V}_i \). Simplification yields \( \Phi \Lambda / \Upsilon \geq 1. \) \( \square \)

Proposition 6B offers a ranking of world welfare along two dimensions: Free trade versus autarky and social optimum versus decentralized market equilibrium. The first inequality in part (i) states that, under free trade, the social optimum achieves no less welfare than the market.
The second inequality affirms that, relative to autarky, the market achieves no less welfare under free trade (Proposition 4B). Part (ii) states that, at the social optimum, welfare is no less under free trade, relative to autarky, which in turn is no less than the autarky market outcome. To highlight the intuition behind part (iii), let us first analyse the inequalities \( \Phi / \Upsilon \geq 1 \) and \( \Lambda \leq 1 \). We can interpret \( \Phi / \Upsilon \) as the wedge between welfare at the social optimum and welfare at the market equilibrium. Similarly, \( \Lambda \) is the wedge between welfare at the social optimum versus the decentralized market equilibrium. The finding in (iii), then, is that welfare at the decentralized market equilibrium under free trade is no less than welfare under a social planner in autarky, if and only if \( \Phi \Lambda / \Upsilon \geq 1 \). It follows that \( \Phi \Lambda / \Upsilon \geq 1 \) will be the case when the wedge between welfare at the social optimum and welfare at the market equilibrium \( (\Phi / \Upsilon) \) is sufficiently large, or the wedge between welfare at the social optimum versus the decentralized market equilibrium is sufficiently small \((\Lambda \approx 1)\).

We now discuss wages. An important consequence of Proposition 6a (part i) is the following:

**Corollary 2. Wage Separation Locus.**

\[
\tilde{w}_i = \bar{w}_i \text{ and } \tilde{w}_{iA} = \bar{w}_{iA} \iff \Phi / \Upsilon = \frac{\beta}{3}.
\]

This corollary implies that the social optimum does not always feature higher wages than the market equilibrium. Figure IV plots the separating locus along which \( \tilde{w}_i = \bar{w}_i \) and \( \tilde{w}_{iA} = \bar{w}_{iA} \) in \((\beta, \sigma)\)-space.

Points above the \( \Phi / \Upsilon = \beta / 3 \) locus imply \( \tilde{w}_i > \bar{w}_i \) and \( \tilde{w}_{iA} > \bar{w}_{iA} \), while points lying below the locus are characterized by \( \tilde{w}_i < \bar{w}_i \) and \( \tilde{w}_{iA} < \bar{w}_{iA} \). On the one hand, if \( \sigma \) is sufficiently high and \( \beta \) is sufficiently low, the social optimum features higher wages than the market equilibrium. In this case, high \( \sigma \) means that the welfare gains from product variety are somewhat diminished, since the term \(-2\sigma \sum_{k=1}^{N} x_{ik} / u_{ik} \sum_{j \neq k}^{N-1} x_{ij} / u_{ij}\) in (21) carries a heavier weight. A low \( \beta \) implies that raising quality is relatively cheap. The planner internalizes both considerations, and this results in fewer firms each with higher quality, such that socially optimal wages are higher relative to the market equilibrium.

On the other hand, a sufficiently low \( \sigma \) and a sufficiently high \( \beta \) imply that the social optimum features lower wages than the market equilibrium. This highlights the situation when a country may attain higher welfare in spite of exhibiting lower income. In this scenario, the planner recognizes that increasing quality is quite costly. Moreover, low \( \sigma \) enhances welfare gains from product variety. Both effects mean that the social optimum features reduced quality and greater number of firms,\(^{19}\) which result in low wages relative to the market equilibrium.

\(^{19}\)Noting, of course, that parts (iii) and (iv) of Proposition 6a continue to hold: \( \bar{N} < \tilde{N}, \bar{n}_A < \tilde{n}_A, \) and \( \bar{u} > \tilde{u}, \bar{u}_{iA} > \tilde{u}_{iA} \).
VIII. Multiple Industries

We have considered a single oligopolistic industry. This raises the following question: Is it justifiable to have firms acting as price takers in input markets, whilst exerting market power in the product market? Does this call for an oligopsonistic treatment of firms? This is an old problem that has plagued models of oligopoly in general equilibrium. Neary (2003) has proposed a solution which involves consideration of many oligopolistic industries, each characterized by few firms. Since each industry is small relative to the economy, firms are justified in acting as price takers in the input market, whilst exhibiting market power within their industry. Under some simplifying assumptions, the results presented above can be extended to the multiple industry case. The assumptions we have in mind are of two types.

Firstly, on the demand side assume that products in each of these industries are non-substitutable with products in other industries. Meanwhile, the product of any firm in a given industry continues to be substitutable with products of other firms in the same industry, to a degree determined by \( \sigma \). This can be illustrated by letting consumers solve a two-step resource allocation problem. In the first step, consumers solve the following problem:

\[
\max_{V_i} \min (V_{1i}, V_{2i}, \ldots, V_{Ri}),
\]

\[
\text{subject to } \sum_{r=1}^{R} \sum_{k=1}^{N_r} p_{kr} x_{kr} + Y_i \leq w_i + \bar{V}_i + \sum_{r=1}^{R} \sum_{k=1}^{n_{ri}} s_{hkr} \Pi_{kri},
\]

where subscript \( r \) identifies the industry \( (r = 1, \ldots, R) \) and other subscripts are as before. \( V_{ri} \) is a sub-utility function identical to that in (1), and denotes country \( i \) consumers’ utility from consumption of industry \( r \) goods and good \( Y \). \( V_i \) is a vector with typical element \( V_{ri} \). In this simple setting, the optimal choice features \( V_{1i} = V_{2i} = \ldots = V_{Ri} \). In step 2 consumers maximize per-industry utility, as set out in (1), now indexed by \( r \), and subject to (29). This leads to the same inverse demand functions as in (3), now also indexed by \( r \).

Secondly, on the supply side assume that there are no supply-side linkages between industries (no intermediate goods or economies of scope). Assume further that all industries exhibit the same initial conditions within each economy: \( u_{ori} = u_{oil} \), for \( i = 1, 2 \) and \( r = 1, \ldots, R \). In this case the equilibrium within each industry is the same as before, such that all industries exhibit identical production, quality and number of firms. Each additional industry replicates the others, ensuring consistency between steps 1 and 2 of the consumers’ problem. This arises because \( V_{ri} \) is a strictly concave function of \( x_{kri} \). Optimality in step 1 requires identical utility (and hence consumption of type \( X \) goods) across all industries. Optimality in step 2 and industry equilibrium ensures that this is indeed the case.

The absence of demand and supply linkages between industries implies that these are only
linked through the labour market. Labour market clearing then becomes:

\[ L_i = \sum_{r=1}^{R} \sum_{k=1}^{n_{ri}} \left( \frac{u_{kri}}{u_{ori}} \right)^{\beta}. \]

Following similar steps as before, we can solve for the wage rate:

\[ \bar{w}_i = \frac{L_i + L_j u_{oi}^2 L_i^{2/\beta}}{2} \left( \frac{R\bar{n}_i}{\beta} \right)^{1-2/\beta} \left[ \beta \left( 1 - \frac{\sigma}{2} \right) + \sigma \right]^2, \]

which is essentially the same as in Table I, scaled up by a constant \((R^{1-2/\beta})\). It is straightforward to see that the analysis leading to Propositions 1-6 (and their corollaries) continues to hold in this simplified case where industries are replicas of each other.

This, of course, raises questions about what happens when industries are not simple replicas. For example, industries might exhibit differences in initial conditions, both within and across economies. Substitutability (\(\sigma\)) and the elasticity of R&D with respect to quality (\(\beta\)) could also be allowed to differ between industries and countries, setting \(\sigma_{ri}\) and \(\beta_{ri}\) instead of the constants \(\sigma\) and \(\beta\). Furthermore, industrial targeting, through differential subsidies could also be introduced (setting \(\delta_{ri}\) instead of \(\delta_{i}\)). These are not trivial questions, and the answers will have to wait for future research (but see Dixit and Grossman’s (1986) prescient analysis).

**IX. Concluding Remarks**

This paper presented a two-country general equilibrium model, with oligopolistic interactions and endogenous product quality, market structure, wages, and terms of trade. We found a unique general equilibrium, featuring subgame perfection at the industry level. When both countries produce oligopolistic goods, there is intra-industry trade. Inter-industry trade arises when economies are not symmetric. Both types of trade may occur simultaneously. Relative to autarky, free trade results in higher product quality, as well as a more concentrated market structure for each country. Upon facing a larger market, firms in both nations raise their product quality, leading to increases in R&D and national concentration. The opening of the economy to foreign competition forces firms to raise product quality or exit. This mechanism sheds light on why the opening of the economy preceded the investment boom in South Korea and Taiwan (see the comment by Victor Norman in Rodrik, 1995).

With few exceptions, symmetric analysis is of little relevance to actual economies. For this reason, asymmetric initial conditions and population sizes were introduced. The idea that a backward economy may lose by trading with an advanced economy is shown to be valid only under certain circumstances. Gains from trade arise if initial conditions are either very different
or quite similar. On the other hand, if the gap in initial conditions takes intermediate values and the advanced economy’s population is not (relatively) very large, free trade generates losses for the backward economy. Meanwhile, the advanced economy reaps welfare gains.

The notion that an advanced economy may lose from trade because of the backward economy’s low input prices is shown to be false. It is never the case that an advanced economy loses from trade. Furthermore, it is also shown (Proposition 3) that trade generates higher wages for the advanced economy, relative to autarky. The backward economy may also achieve higher wages through trade, provided the asymmetry in initial conditions is not too pronounced.

The possibility of incompatible national interests gives rise to questions of unfair trade negotiations, in which an advanced country may pressure a backward nation into a trade agreement where the latter will face welfare losses. This result is summarized in Proposition 4A and corollary 1, which provide a formal basis for the choice of trade partners, and for the formation of trade blocs. Nonetheless, world welfare always increases with free trade (Proposition 4B), so in principle, it should be possible to devise a compensation scheme such that no nation loses from trade. This raises questions about the design of an optimal compensation scheme, and about the political process underlying such policies.\(^{20}\)

With asymmetries in initial conditions, the question arises of whether catching-up is feasible and welfare enhancing. In Proposition 1, the marginal benefit of investment in R&D was found to be decreasing in rivals’ product quality. This implies the existence of a threshold for the ratio of product qualities (the technology gap), above which the marginal benefit of investment in R&D becomes negative. Hence, catching up in quality will be feasible only if the technological gap is not too wide. Moreover, the threshold is independent of marginal cost considerations, and in particular wages. Consequently, regardless of how low wages are in the backward country, if the technology gap is too wide, backward firms will not survive in the open economy.

We further analysed this issue by introducing industrial policy in the form of an *ad-valorem* subsidy to R&D, funded with a lump sum tax on consumers. This policy was shown to be neutral or ineffective, in the sense that product quality, the number of firms, after tax income and welfare are not affected by it (Proposition 5). The key lesson is that it is not sufficient to offer R&D subsidies. This will redistribute such resources towards production factors, only to be taken away through taxes.

The social optimum features higher product quality and fewer firms than the market equilibrium, in both autarky and free trade (Proposition 6A). Wages need not necessarily be higher at the social optimum (Corollary 2), although world welfare certainly is (Proposition 6B).

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\(^{20}\)These issues lie outside the scope of this study, and are left for future research. Furusawa and Konishi (2005) discuss trade agreements with transfers. Yi (1996), Das and Ghosh (2006) and Aghion, Anrâs and Helpman (2007) present theories about the formation of trade blocs. Detailed treatment of political aspects can be found in Grossman and Helpman (1994, 1995).
The model was extended to multiple oligopolistic industries, which were replicas of each other, linked solely through the labour market. Allowing exclusively for labour market linkages rules out demand-side linkages (goods are not substitutable across industries), as well as supply-side linkages (no intermediate products or economies of scope). Under these simplifying assumptions our results readily extend to the multiple industry case.

Appendix 1. Longer Proofs.

Proof of Lemma 1.
Considering each economy in autarky, the first order conditions for product quality choice, given by (7), simplify to:

\[
(A1.1) \quad L_i \frac{2 + \sigma (n_i - 2)}{(2 - \sigma)^2 [2 + \sigma (n_i - 1)]} \left[ 1 - \frac{\sigma}{2 + \sigma (n_i - 1)} \sum_{l=1}^{n_i} \frac{u_{li}}{u_{ki}} \right] = w_i \beta_i \frac{u_{ki}^{\beta_i-2}}{u_{oi}^{\beta_i}}.
\]

The free entry condition becomes:

\[
(A1.2) \quad \frac{L_i}{2(2 - \sigma)^2} \left\{ 1 - \frac{\sigma}{2 + \sigma (n_i - 1)} \sum_{l=1}^{n_i} \frac{u_{li}}{u_{ki}} \right\}^2 = w_i \frac{u_{ki}^{\beta_i-2}}{u_{oi}^{\beta_i}}.
\]

Squaring (A1.1), dividing this by (A1.2) and rearranging, yields:

\[
(A1.3) \quad \frac{2L_i}{\beta_i^2 (2 - \sigma)^2 [2 + \sigma (n_i - 1)]^2} = w_i \frac{u_{ki}^{\beta_i-2}}{u_{oi}^{\beta_i}}.
\]

The right hand sides of (A1.2) and (A1.3) are identical. Equating the left hand sides and simplifying, we obtain:

\[
(A1.4) \quad \frac{2 + \sigma (n_i - 1)}{\sigma} \left[ 1 - \frac{2 + \sigma (n_i - 2)}{3 \beta_i^2 \sigma (n_i - 1)} \right] = \sum_{l=1}^{n_i} \frac{u_{li}}{u_{ki}}.
\]

The left hand side of (A1.4) is a constant for any given firm. Thus we have that

\[
(A1.5) \quad \sum_{l=1}^{n_i} \frac{u_{li}}{u_{ji}} = \ldots = \sum_{l=1}^{n_i} \frac{u_{li}}{u_{ki}}.
\]

Dividing (A1.5) by \(\sum_{l=1}^{n_i} u_{li}\) yields \(u_{ji} = u_{ki} = u_i\), as required \(\Box\).

Proof of Proposition 2.
For a unique general equilibrium, equilibrium in each of the three markets under consideration (type X goods, good Y and the labour market) must be unique. This entails a unique
Subgame Perfect Nash Equilibrium in the oligopolistic industry, a unique price and quantity for good $Y$, and a unique wage rate and labour allocation. The proof proceeds by showing how each of these attain. We analyse the open economy case (the proof for autarky follows identical steps).

First, we show uniqueness of the Subgame Perfect Nash Equilibrium in the oligopolistic industry. This follows if the Nash Equilibrium in each stage is unique. Uniqueness in stage 3 (Cournot competition), follows immediately from the linearity of the first order conditions for this stage. This can be observed in (A2.1.1) in Appendix 2.1, which is a linear, non-homogeneous, system of $N$ equations in $N$ variables ($x_{ki}$). Uniqueness follows from the property that a linear system of equations can only have zero, one or infinitely many solutions. Infinitely many solutions can only arise if the coefficient matrix associated with system (A2.1.1) does not have full rank (the system has at least one degree of freedom). Since this is not the case, the system (A2.1.1) has either one or zero solutions. The explicit solution found in (A2.1.4) is therefore, unique.

We now focus on stage 2 (quality competition). To show uniqueness in this stage, we use Lemma 1, which implies that within each economy all firms set an identical quality level. It remains to show that the optimal reply schedules, implicitly defined by $OR_i$, intersect exactly once in the admissible region of $(u_i, u_j)$-space, that is, $u_i > 1$. $OR_i$ can be written as:

$$a_i - b_i \frac{u_j}{u_i} = c_i u_i^{\beta_i - 2},$$

(A1.6)

where $a_i$, $b_i$ and $c_i$ are constants in stage 2 of the game, defined as follows:

$$a_i = 2 + \sigma (n_j - 1)$$
$$b_i = \sigma n_j$$
$$c_i = \frac{(2 - \sigma)^2 [2 + \sigma (n_i + n_j - 1)] w_i \beta_i}{(L_i + L_j) [2 + \sigma (n_i + n_j - 2)] u_{oi} \beta_i}$$

Although (A1.6) cannot be solved explicitly for $u_i$, it can be solved for $u_j$. This yields:

$$u_j = \frac{1}{b_i} \left( a_i u_i - c_i u_i^{\beta_i - 1} \right).$$

(A1.7)

It is easy to see that the graph of $u_j$ has an inverted-U shape for $u_i \geq 0$ and $\beta_i > 2$. We can then invert the graph of $u_j$ to obtain the optimal reply schedules shown in Figure I, $u_i^{OR}(u_j)$ and $u_j^{OR}(u_i)$. In general, there will be two intersections of $u_i^{OR}(u_j)$ and $u_j^{OR}(u_i)$. The first occurs at $(\hat{u}_i, \hat{u}_j)$. The other intersection occurs at $(0, 0)$. However, the latter is not a Nash Equilibrium, since at $(0, 0)$ any firm has an incentive to deviate to a higher quality. Furthermore, $(0, 0)$ is
not in the admissible space, which requires \( u_i > 1 \). Setting \( u_i = 0 \) in (A1.7), it is clear that the inadmissible intersection always occurs at \((0, 0)\). Thus, \((\hat{u}_i, \hat{u}_j)\) is the unique Nash Equilibrium for stage 2.

We now show uniqueness of the Nash Equilibrium in stage 1 (entry). The solution to this stage is a pair \((\hat{n}_i, \hat{n}_j)\). If \( \beta_i = \beta_j \), this is solved from (A2.3.2) and (A2.3.3), two linear equations in \((n_i, n_j)\). By similar reasoning to that used for stage 3, the solution is unique. If \( \beta_i \neq \beta_j \), then the zero profit conditions \((ZP_i)\) are at most quadratic in \( n_i \). Accordingly, each zero profit condition will have at most two real roots. However, only the positive root satisfies the second order conditions, implying uniqueness of \((\hat{n}_i, \hat{n}_j)\).

As explained in section III, labour market clearing follows from \((LM_i)\). Uniqueness can be ascertained from the following observations:

i) \( OR_i \) implies that \( u_i \) is a hyperbolic function of \( w_i \), of order \(-1/(\beta_i - 2)\).

ii) \( LM_i \) implies that labour demand is a power function of \( u_i \), with power \( \beta_i \).

From (i) and (ii), it follows that labour demand is a hyperbolic function of \( w_i \), of order \(-\beta_i/(\beta_i - 2)\). Since labour supply is constant, it is clear that there is a unique wage rate, \( \hat{w}_i \), which clears the labour market in each economy. Since labour is perfectly immobile, there is a unique pair \((\hat{w}_i, \hat{w}_j)\) which clears both labour markets □.

**Proof of Lemma 2.**

The expression in \((OR_i)\) contains two equilibrium conditions, one for each economy. We refer to these as \((OR_i)\) and \((OR_j)\). Dividing \((OR_i)\) by \((OR_j)\) yields

\[
(A1.8) \quad \frac{2 + \sigma (n_j - 1) - \sigma n_j \frac{u_j}{n_j}}{2 + \sigma (n_i - 1) - \sigma n_i \frac{u_i}{n_i}} = \frac{w_i \beta_i \beta_j^{\beta_i - 2} u_j^{\beta_j - 2} u_{ij}}{w_j \beta_j \beta_i^{\beta_j - 2} u_j^{\beta_i - 2} u_{ij}}.
\]

Dividing the zero profit conditions in \((ZP_i)\), we obtain

\[
(A1.9) \quad \left[\frac{2 + \sigma (n_j - 1) - \sigma n_j \frac{u_j}{n_j}}{2 + \sigma (n_i - 1) - \sigma n_i \frac{u_i}{n_i}}\right] = \frac{w_i \beta_i \beta_j^{\beta_i - 2} u_j^{\beta_j - 2} u_{ij}}{w_j \beta_j \beta_i^{\beta_j - 2} u_j^{\beta_i - 2} u_{ij}}.
\]

Squaring (A1.8), and dividing the result by (A1.9), yields

\[
(A1.10) \quad 1 = \frac{w_i}{w_j} \left(\frac{\beta_i}{\beta_j}\right)^2 \frac{\beta_j^{\beta_i - 2}}{\beta_i^{\beta_j - 2} u_{ij}^{\beta_i - 2}}.
\]

Substituting (A1.10) into either (A1.8) or (A1.9), we obtain the following expression

\[
(A1.11) \quad 2 + \sigma (n_j - 1) - \sigma n_j \frac{u_j}{n_j} = \frac{\beta_j}{\beta_i} \left[2 + \sigma (n_i - 1) - \sigma n_i \frac{u_i}{n_i}\right].
\]
Adding and subtracting $\sigma$ on the left hand side, and $\sigma \beta_j / \beta_i$ on the right hand side, this expression can be rewritten as follows

\[(A1.12) \quad (2 - \sigma) + \sigma n_j \left(1 - \frac{u_j}{u_i}\right) = \frac{\beta_j}{\beta_i} \left[ (2 - \sigma) + \sigma n_i \left(1 - \frac{u_i}{u_j}\right) \right].\]

Rearranging the above expression, we obtain

\[(A1.13) \quad (2 - \sigma) \left(1 - \frac{\beta_j}{\beta_i}\right) = \sigma (u_j - u_i) \left(\frac{n_j}{u_i} + \frac{\beta_j}{\beta_i} \frac{n_i}{u_j}\right).\]

From (A1.13), it is clear that if $\beta_j > \beta_i$, the left hand side is negative. The right hand side will be negative if and only if $u_j < u_i$. Likewise, if $\beta_j < \beta_i$, the left hand side is positive, and the right hand side will be positive if and only if $u_j > u_i$. If $\beta_i = \beta_j = \beta$, then the left hand side is zero. The right hand side will also be zero if and only if $u_j = u_i = u \square$.

**APPENDIX 2. FURTHER DERIVATIONS AND EXTENSIONS.**

2.1. Solution of the Final Stage Subgame

In this appendix we solve the final stage subgame (Cournot competition), in order to obtain a solved-out payoff. Recall the first order conditions for this stage of the game (equation 4):

\[p_{ki} + \frac{\partial p_{ki}}{\partial x_{ki}} x_{ki} = 0; \quad \text{for } i = 1, 2 \text{ and } k = 1, \ldots, n_i.\]

Substitute inverse demand (equation 3) and the associated derivatives ($\partial p_{ki}/\partial x_{ki}$ for $i = d, f$) into the first order conditions. Adding and subtracting $2\sigma x_{ki}/u_{ki}^2$, we obtain

\[(A2.1.1) \quad 1 - 2(2 - \sigma) \frac{x_{ki}}{u_{ki}^2} - \frac{2\sigma}{u_{ki}} \left(\sum_{l=1}^{n_i} \frac{x_{li}}{u_i} + \sum_{l=1}^{n_j} \frac{x_{lj}}{u_j}\right) = 0.\]

Multiply (A2.1.1) through by $u_{ki}$ and reorganize the expression as follows

\[(A2.1.2) \quad \frac{x_{ki}}{u_{ki}} = \frac{u_{ki} - 2\sigma \left(\sum_{l=1}^{n_i} \frac{x_{li}}{u_i} + \sum_{l=1}^{n_j} \frac{x_{lj}}{u_j}\right)}{2(2 - \sigma)}.\]

Sum (A2.1.2) over $k$ and solve for $\sum_{l=1}^{n_i} x_{li}/u_i$. This yields

\[(A2.1.3) \quad \sum_{l=1}^{n_i} \frac{x_{li}}{u_i} = \frac{\sum_{l=1}^{n_i} u_{li} - 2\sigma n_i \sum_{l=1}^{n_j} \frac{x_{li}}{u_{lj}}}{2 [2 + \sigma (n_i - 1)]}.\]

The equations in (A2.1.3) constitute a pair of linear equations in $\sum_{l=1}^{n_i} x_{li}/u_i$ and $\sum_{l=1}^{n_j} x_{lj}/u_{lj}$. 

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Solving for these we obtain

\[(A2.1.3') \quad \sum_{l=1}^{n_i} \frac{x_{li}}{u_{li}} = \frac{2 + \sigma (n_j - 1)}{2 \cdot (2 - \sigma)} \frac{\sum_{l=1}^{n_i} u_{li} - \sigma n_i \sum_{l=1}^{n_j} u_{kj}}{\sum_{l=1}^{n_i} u_{li} + \sum_{l=1}^{n_j} u_{ij}}.\]

To obtain the solution for \(x_{ki}\), substitute the expressions in \((A2.1.3')\) back into \((A2.1.2)\). This yields the following solution for \(x_{ki}\)

\[(A2.1.4) \quad \hat{x}_{ki} = \frac{u_{ki}^2}{2(2 - \sigma)} \left[ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j - 1)} \left( \sum_{l=1}^{n_i} \frac{u_{li}}{u_{ki}} + \sum_{l=1}^{n_j} \frac{u_{ij}}{u_{ki}} \right) \right].\]

To solve for prices \((p_{ki})\), add and subtract \(2\sigma x_{ki}/u_{ki}^2\) to inverse demands \((3)\), to obtain

\[(A2.1.5) \quad p_{ki} = 1 - 2(1 - \sigma) \frac{x_{ki}}{u_{ki}^2} - \frac{2\sigma}{u_{ki}} \left( \sum_{l=1}^{n_i} \frac{x_{li}}{u_{li}} + \sum_{l=1}^{n_j} \frac{x_{lj}}{u_{lj}} \right) .\]

Next substitute \(\hat{x}_{ki}\) from equation \((A2.1.4)\) and the expressions in \((A2.1.3')\) into \((A2.1.5)\). This yields the solution for \(p_{ki}\):

\[(A2.1.6) \quad \hat{p}_{ki} = \frac{1}{2 - \sigma} \left[ 1 - \frac{\sigma}{2 + \sigma (n_i + n_j - 1)} \left( \sum_{l=1}^{n_i} \frac{u_{li}}{u_{ki}} + \sum_{l=1}^{n_j} \frac{u_{ij}}{u_{ki}} \right) \right].\]

The solved-out payoff is given by the product of \((A2.1.4)\) and \((A2.1.6)\). This yields equation \((5)\) in section II.

2.2. Second Order Conditions for the Second Stage Subgame

The second order conditions are obtained by differentiating the first order conditions (equation 7) with respect to \(u_{ki}\). This simplifies to:

\[(A2.2.1) \quad (L_i + L_j) \frac{1}{(2 - \sigma)^2} \left[ \frac{2 + \sigma (n_i + n_j - 2)}{2 + \sigma (n_i + n_j - 1)} \right]^2 \leq \frac{w_i \beta_i (\beta_i - 1) u_{ki}^{\beta_i - 2}}{u_{ii}^{\beta_i}}.\]

If we substitute out \(u_{ki}\), \(n_i\) and \(w_i\) with their equilibrium values, \((A2.2.1)\) provides a restriction on \((\sigma, \beta_i)\)-space. This restriction implies that \(\sigma\) cannot be too high and \(\beta_i\) cannot be too low (in any case we require \(\beta_i > 2\)). If any one of these parameters crosses its bound, the other will need to compensate by moving inward from its bound. This was taken into account throughout the paper, ensuring that \((A2.2.1)\) holds for all of the analysis.
2.3. Explicit Solutions for General Equilibrium

In this appendix we derive the results presented in Table I. Solving for \( u \) from \((OR_i)\) yields:

\[
(A2.3.1) \quad u = \left\{ \frac{u_{oi}^\beta (L_i + L_j) \cdot 2 + \sigma (n_i + n_j - 2)}{w_i \beta (2 - \sigma) \cdot [2 + \sigma (n_i + n_j - 1)]^2} \right\}^{\frac{1}{\pi - 2}}.
\]

The next step is to obtain the number of firms. Substituting \( u \) from (A2.3.1) into \((ZP_i)\), we obtain the following relationship:

\[
(A2.3.2) \quad n_i + n_j = \frac{(\beta - 2) (2 - \sigma)}{2\sigma} + 1.
\]

To obtain explicit solutions for \( n_i \) and \( n_j \), we use the labour market clearing conditions \((LM_i \) and \(LM_j\)). Division of \((LM_i)\) by \((LM_j)\) yields

\[
(A2.3.3) \quad \frac{L_i}{L_j} = \frac{n_i}{n_j} \left( \frac{u_{oj}}{u_{oi}} \right)^{\beta}.
\]

Expressions (A2.3.2) and (A2.3.3) constitute a system of linear equations in \( n_i \) and \( n_j \). Solving this system, we obtain its unique solutions:

\[
(A2.3.4) \quad \hat{n}_i = \frac{\frac{(\beta - 2) (2 - \sigma)}{2\sigma} + 1}{\frac{L_i}{L_j} \left( \frac{u_{oi}}{u_{oj}} \right)^{\beta} + 1}.
\]

Solutions for \( n_i \) and \( n_j \) are then substituted back into (A2.3.1). After simplifying, we obtain:

\[
(A2.3.5) \quad \hat{u} = \left\{ \frac{u_{oi}^\beta \cdot L_i + L_j}{2\hat{n}_i \left( \beta \left( 1 - \frac{\sigma}{2} \right) + \sigma \right)^2} \right\}^{\frac{1}{\pi - 2}}.
\]

We now determine the equilibrium wage rate. This is obtained by substituting (A2.3.5) into \((LM_i)\) and solving for \( w_i \):

\[
(A2.3.6) \quad \hat{w}_i = \frac{L_i + L_j \cdot u_{oi}^2 \cdot L_i^{\frac{2}{\hat{n}_i}} \cdot \hat{n}_i^{\frac{\beta - 2}{\sigma}}}{2 \hat{n}_i \left( \beta \left( 1 - \frac{\sigma}{2} \right) + \sigma \right)^2},
\]

where \( \hat{n}_i \) is given by (A2.3.4).

For the open economy, the general equilibrium solution for quality is obtained by substituting
(A2.3.6) into (A2.3.5):

\[(A2.3.7) \quad \hat{u} = u_{oi} \left( \frac{L_i}{\hat{n}_i} \right)^{\frac{1}{\beta}}.\]

This completes the general equilibrium solution for the open economy. Equilibrium outcomes for all remaining variables are obtained by substituting the solutions obtained above. Autarky solutions are obtained by following the same procedure, with \(L_j\) and \(n_j\) set to zero.

2.4. Additional Comparative Statics

Population Size \((L_i)\)

To analyse the effect of changes in population size it is convenient to begin from an asymmetric world economy\(^{21}\), with \(u_{oi} \neq u_{oj}\) and \(L_i = L_j\). Figure V illustrates the case of \(u_{oi} < u_{oj}\). This figure is similar to Figure II, with the horizontal axes measuring \(L_i\).

The top left graph in Figure V shows that as \(L_i\) rises, quality increases at a decreasing rate, in both the open economy and in the autarkic economy \(i\). That \(\hat{n}_{iA} < \hat{n}_{jA}\) at \(L_i = L_j\) follows immediately from \(u_{oi} < u_{oj}\). In Table I we can see that \(\hat{n}_{jA}\) does not depend on \(L_i\). The bottom left graph shows that \(\hat{n}_i\) is an increasing and concave function of \(L_i\), while \(\hat{n}_j\) is decreasing and convex. That \(\hat{n}_i < \hat{n}_j\) at \(L_i = L_j\) again follows from \(u_{oi} < u_{oj}\). To see the mechanisms behind the patterns in product quality and the number of firms, note that a larger population involves two aspects. Firstly, it represents an increase in the available labour force. This allows a greater number of firms to settle in economy \(i\), since the labour market is, \textit{ceteris paribus}, less tight. To see this, divide \((LM_i)\) by \((LM_j)\). Noting that \(\hat{u}_i = \hat{u}_j\) in equilibrium, this yields \(L_i/L_j = \hat{n}_i/\hat{n}_j (u_{oj}/u_{oi})^{\beta}\). It follows that a rise in \(L_i\) must be accompanied by a rise in \(\hat{n}_i\) or a fall in \(\hat{n}_j\). Since \(\hat{n}_i + \hat{n}_j = \hat{N}\), equilibrium is achieved through a rise in \(\hat{n}_i\) and a fall in \(\hat{n}_j\). The second aspect of a rise in \(L_i\) entails a larger market size for type \(X\) products, which generates a rise in quality, such that \(\hat{u}_i = \hat{u}_j\). This reinforces the contraction of \(\hat{n}_j\), since firms in economy \(j\) demand more labour in order to match \(\hat{n}_i\), but their labour force has not expanded. A similar effect operates in economy \(i\), but economy \(i\) features a rise in the supply of labour. Accordingly, economy \(i\) can accommodate more firms, relative to economy \(j\), and \(\hat{n}_i\) rises.

The net effect on wages of the changes in quality and market structure can be observed in the top right graph of Figure V. \(\hat{w}_j\) is an increasing and concave function of \(L_i\). Thus, the increase in quality more than offsets the fall in the number of firms. Likewise, \(\hat{w}_i\) is also increasing and concave in \(L_i\). That \(\hat{w}_i\) is rising is hardly surprising, since both product quality and the number of firms in economy \(i\) are rising, with both effects increasing labour demand. What is not obvious is why \(\hat{w}_i\) rises at a decreasing rate (as opposed to rising at an \textit{increasing} rate).

\(^{21}\)A detailed explanation of why this is the case is offered below. For the moment, simply note that whether \(L_i = L_j\) is not important, the key asymmetry arises from \(u_{oi} \neq u_{oj}\).
This follows from the observation that labour supply is rising in parallel with the increase in market size, curtailing the rise in \( \hat{w}_i \).

The top right graph, showing wages, exhibits some features worthy of note. First, in accordance with the analysis in section IV, we have \( \hat{w}_i < \hat{w}_j \), since \( u_{oi} < u_{oj} \). Second, there is a crossing between \( \hat{w}_{iA} \) and \( \hat{w}_i \). This point is shown at \( \hat{w}_{iA} = \hat{w}_i \) in the diagram, and lies to the right of \( L_i = L_j \). However, this need not always be the case. To see whether the crossing will occur to the right of, to the left of, or at \( L_i = L_j \), evaluate \( u_{oi}' = u_{oj} \left\{ \frac{L_i}{L_j} \left( 1 + \frac{L_j}{L_i} \right)^{\frac{\beta}{(\beta-2)} - 1} \right\}^{-1/\beta} \) (as defined in the proof of Proposition 3) at \( L_i = L_j \), and denote this value by \( u_{oi}' \). If \( u_{oi} > u_{oi}' \), the crossing will occur to the right of \( L_i = L_j \), if \( u_{oi} < u_{oi}' \), it will occur to the left of \( L_i = L_j \), and if \( u_{oi} = u_{oi}' \), it will occur at \( L_i = L_j \).

The trade balance is shown in the bottom right graph. As \( L_i \) rises, economy \( i \) incurs a rising trade deficit in type \( X \) goods, with the corresponding trade surplus in good \( Y \). The reverse pattern holds for economy \( j \), such that the overall trade balance in each economy is zero. Thus, the advanced economy \( (j) \) is a net exporter of type \( X \) goods, while the backward economy \( (i) \) is a net exporter of good \( Y \). This pattern is accentuated as \( L_i \) rises: A bigger population in economy \( i \) demands an increasing quantity of type \( X \) goods, and even though the number of firms in economy \( j \) is contracting, net exports of type \( X \) goods from economy \( j \) rise (net imports of type \( X \) goods from economy \( i \) rise). Accordingly, economy \( i \)'s net exports of good \( Y \) (economy \( j \)'s net imports of good \( Y \)) rise in order to cover its deficit in type \( X \) goods.

To close the discussion, let us return to the assumption made at the beginning of this section, namely, that the analysis starts from an asymmetric world economy, with \( u_{oi} \neq u_{oj} \). Substituting \( \hat{n}_i \) into \( \hat{w}_i \) (Table I), we can express \( \hat{w}_i \) as follows:

\[
\hat{w}_i = \frac{L_i + L_j}{L_i + L_j (u_{oj}/u_{oi})^\beta} \frac{u_{oi}^2}{2} \left[ \hat{n}_i^{1-2/\beta} \left( \beta \left( 1 - \frac{\sigma}{2} \right) + \sigma \right)^{2/\beta} \right].
\]

If \( u_{oi} = u_{oj} \), then \( \hat{w}_i \) is a function of \( L_i + L_j \) and \( \hat{w}_i = \hat{w}_j \) for any value of \( L_i \) or \( L_j \). Rather than center the analysis around this knife-edge scenario, it was deemed more illuminating to show the more general case of \( u_{oi} \neq u_{oj} \). We have illustrated the case of \( u_{oi} < u_{oj} \). The analysis of the opposite case, where \( u_{oi} > u_{oj} \), follows by reversing indices \( i \) and \( j \), with a caveat relating to the behaviour of \( \hat{w}_i \) and \( \hat{w}_j \) (after reversing \( i \) and \( j \), all other variables in Figure V exhibit an identical pattern to that shown). The caveat is the following. From (A2.4.1), it is clear that \( \hat{w}_i \) is increasing in \( L_i \) through the effect on the numerator, and decreasing through the effect in the denominator. Accordingly, there exists a threshold \( L_i^* \) at which \( \partial \hat{w}_i / \partial L_i = 0 \). This threshold can be solved to yield: \( L_i^* = L_j/2 \left\{ \beta \left[ 1 - (u_{oj}/u_{oi})^\beta \right] - 2 \right\} \). Calculations for \( \hat{w}_j \) yield identical results: \( \partial \hat{w}_j / \partial L_i \rvert_{L_i = L_i^*} = 0 \). If \( u_{oi} < u_{oj} \), \( L_i^* \) is always negative, so \( \partial \hat{w}_i / \partial L_i > 0 \) for \( L_i > 0 \), as
shown in Figure V. If \( u_{oi} > u_{oj} \), \( L_i^* \) will be positive. In this case, \( \partial \hat{w}_i / \partial L_i < 0 \) for \( L_i < L_i^* \), and \( \partial \hat{w}_i / \partial L_i > 0 \) for \( L_i > L_i^* \). Likewise for \( \hat{w}_j \). Thus, \( \hat{w}_i \) and \( \hat{w}_j \) are at first decreasing in \( L_i \), reach a minimum at \( L_i^* \), to then become increasing in \( L_i \). In conclusion, the only change introduced in the analysis by considering \( u_{oi} > u_{oj} \) is that \( \hat{w}_i \) and \( \hat{w}_j \) become \( \cup \)-shaped.

**Substitutability (\( \sigma \))**

From Table I, we can readily ascertain that the number of firms is a decreasing and convex function of \( \sigma \), while product quality is an increasing and concave function of \( \sigma \). These opposing patterns result in \( \cup \)-shaped wages. In both autarky and the open economy, the minimum of the wage schedule is achieved at \( \sigma^* = \left( 4 - 8\beta + 3\beta^2 - \sqrt{16 - 64\beta + 24\beta^2 + \beta^4} \right) / 2\beta (\beta - 4) \). For \( \sigma < \sigma^* \), the falling number of firms dominates the rise in quality, leading to falling wages, while the reverse holds for \( \sigma > \sigma^* \). These results are independent of whether we consider the case of a symmetric or asymmetric world economy. Introducing asymmetry in the world economy would only result in shifts of schedules in accordance with the effects identified in the previous section of this appendix and section IV in the main text, but no change in behaviour with respect to \( \sigma \).

**Elasticity of R&D with Respect to Product Quality (\( \beta \))**

Drawing again on the results in Table I, it is easy to see that quality and wages are decreasing and convex functions of \( \beta \). The number of firms in autarky, as well as the world number of firms, increases linearly with \( \beta \). \( \hat{n}_i \), however, can be non-monotonic with respect to \( \beta \). To see this, note that \( \beta \) appears in both the numerator and denominator of \( \hat{n}_i \). The numerator is always increasing in \( \beta \). The denominator, however, can be decreasing, increasing or constant with respect to \( \beta \). When \( u_{oi} = u_{oj} \), the effect in the denominator vanishes, and we have that \( \hat{n}_i \) increases linearly with \( \beta \) (as in autarky). If \( u_{oi} > u_{oj} \), then the denominator is decreasing in \( \beta \), and this makes \( \hat{n}_i \) an increasing and convex function of \( \beta \). If \( u_{oi} < u_{oj} \), then the denominator is increasing in \( \beta \). In this case, we can find a threshold for \( \beta \), denoted by \( \beta^* \), which will make \( \partial \hat{n}_i / \partial \beta = 0 \). For \( \beta < \beta^* \), we have \( \partial \hat{n}_i / \partial \beta > 0 \), and the opposite holds for \( \beta > \beta^* \). This implies that \( \hat{n}_i \) is \( \cap \)-shaped.

2.5. **Effect of \( \beta \) on the Gains-from-Trade Area**

The inequality in (15) depends also on \( \beta \). We now analyse how changes in \( \beta \) affect the shaded regions in Figure III. This is shown in Figure VI.

As \( \beta \) rises, the shaded regions expand and contract along certain sections of their boundaries. Consider the \( \hat{V}_i < \hat{V}_{iA} \) region (the analysis for the \( \hat{V}_j < \hat{V}_{jA} \) region is identical). An increase in \( \beta \) generates an expansion of this region along all of the boundary except in the section located close to the top left part of the diagram. This shifts inwards, leading to a reduction in the \( \hat{V}_i < \hat{V}_{iA} \) region, as can be seen by the successive crossings of the indifference loci. Nonetheless,
this contraction is insufficient to offset the overall expansion of the shaded area. Thus, rises in \( \beta \) tend to expand the losses-from-trade region in \((L_i/L_j, \frac{u_{oi}}{u_{oj}})\)-space. This result emerges through the following channels. First, a rise in \( \beta \) magnifies the effect of asymmetries in initial conditions. It can also be shown that as \( \beta \) rises, the \( u_{oi}^*/u_{oj} \) and \( u_{oi}^{**}/u_{oj} \) schedules shift closer to the vertical line at \( u_{oi}/u_{oj} = 1 \) (to reduce cluttering in Figure VI, this has not been shown). This means that as \( \beta \) rises, a smaller asymmetry in initial conditions is sufficient to generate a wage reduction from free trade relative to autarky, since backward economy firms incur higher R&D expenditure in order to match advanced economy firms’ quality, magnifying the market structure effect. Second, a higher \( \beta \) is associated with lower quality level (in both autarky and free trade), and this dampens the increase in quality associated to the larger market size under free trade, curtailing this source of gains from trade. The two effects lead to a contraction of the gains-from-trade region.

2.6. Value Added and the Trade Balance

Consider value added on the production, income and expenditure sides. Since there are no intermediate goods, value added in the oligopolistic industry measured on the production side is equal to world-wide revenue from type \( X \) goods, which is given by \((L_i + L_j) n_ip_i x_i \). Dividing by \( L_i \) yields per-capita value added (as measured by production): \( VAP_{xi} = VA_{xi} = (L_i + L_j) n_ip_i x_i/L_i \). As measured by income, value added in the oligopolistic industry is given by workers’ wages \((VAI_{xi} = w_i L_{xi})\). Moreover, employment in the oligopolistic industry \((L_{xi})\) is given by \( n_i f(u_i) \), where \( f(u_i) = (u_i/u_{oi})^\beta \). The free entry condition can be written as \((L_i + L_j) \pi_i = w_i f(u_i)\), so \( f(u_i) = (L_i + L_j) \pi_i/w_i\). Gross profits are \( \pi_i = p_i x_i \). Substituting this into \( VAI_{xi} \), yields the same result as before: \( VAI_{xi} = VA_{xi} = (L_i + L_j) n_i p_i x_i /L_i \). As measured by expenditure, value added in the oligopolistic industry is made up by consumption expenditure and net exports of good \( X \) (that is, the trade balance in type \( X \) goods).

Consumption of type \( X \) goods is \( L_i [n_i p_i x_i + n_j p_j x_j] \) in country \( i \). Exports of type \( X \) goods are equal to \( L_j n_i p_i x_i \), while imports are \( L_i n_j p_j x_j \). The trade balance for the oligopolistic industry is then given by: \( TB_{xi} = L_j n_i p_i x_i - L_i n_j p_j x_j \). Adding consumption expenditure and the trade balance, dividing by population and simplifying, we obtain the same result previously found in per-capita terms: \( VAEx_{xi} = VA_{xi} = (L_i + L_j) n_i p_i x_i /L_i \). Value added for good \( Y \) is equal to aggregate supply of good \( Y \), since the price of good \( Y \) has been set to 1: \( VA_{yi} = L_i \bar{Y}_i \). Measured by expenditure, value added in good \( Y \) is constituted by consumption and the trade balance for good \( Y \). Consumption expenditure is obtained as a residual from the consumer’s budget constraint: \( L_i Y_i = L_i [w_i + \bar{Y}_i - n_i p_i x_i - n_j p_j x_j] \). The trade balance for good \( Y \) is given by the difference between aggregate supply and aggregate demand of good \( Y \): \( TB_{yi} = L_i \bar{Y}_i - L_i [w_i + \bar{Y}_i - n_i p_i x_i - n_j p_j x_j] \). Adding \( L_i Y_i \) and \( TB_{yi} \) we obtain: \( VA_{yi} = L_i \bar{Y}_i \). Total (per-capita) value added is the sum of value added for both goods: \( VA_i = VA_{xi} + VA_{yi} \).
2.7. Notation

Subscript ‘$A$’ denotes autarky. Subscript ‘$t$’ refers to an ad-valorem subsidy. Equilibrium solutions are denoted by a circumflex (\(^\hat{*}\)). The social optimum is denoted by a tilde (\(^\sim\)).

$L_i$ Population/labour supply in country $i$.
$\bar{Y}_i$ Per-capita endowment of good $Y$ in country $i$.
$s_{hki}$ Ownership share of consumer $h$ in firm $k$ in country $i$.
$\sigma$ Substitutability between $X$ type goods.
$\beta_i$ Elasticity of R&D expenditure with respect to $u_{ki}$ in country $i$.
$u_{oi}$ Initial conditions in country $i$.
$\delta_i$ Ad-valorem subsidy on R&D in country $i$.
$V_i$ Utility function in country $i$.
$x_{ki}$ Production of firm $k = 1, \ldots, n_i$ in country $i$.
$x_k$ Production of firm $k$ in either country.
$x_i = (x_{1i}, \ldots, x_{ni})$ Production vector in country $i$.
$x = (x_1, x_2)$ Worldwide production vector.
$u_{ki}$ Quality of product $k$ in country $i$.
$u_k$ Quality of product $k$ in either country.
$u_i = (u_{1i}, \ldots, u_{ni})$ Quality vector in country $i$.
$u = (u_1, u_2)$ Worldwide quality vector.
$p_{ki}$ Price of good $k$ in country $i$.
$p_k$ Price of good $k$ in either country.
$n_i$ Number of firms in country $i$.
$N = n_1 + n_2$ Worldwide number of firms.
$w_i$ Wage rate in country $i$.
$Y_i$ Per-capita consumption of good $Y$ in country $i$.
$\pi_{ki} = p_{ki}x_{ki}$ Per-capita gross profits of firm $k$ in country $i$.
$\pi_{ki}(u)$ Stage 3 solved out payoff of firm $k$ in country $i$.
$\Pi_{ki}$ Net profits of firm $k$ in country $i$.
$F(u_{ki}, w_i) = w_i f(u_{ki})$ R&D expenditure of firm $k$ in country $i$.
$f(u_{ki}) = (u_{ki}/u_{oi})^{\beta_i}$ labour requirement to achieve $u_{ki}$.
$TB_i = TB_{xi} + TB_{yi}$ Trade balance in country $i$.
$TB_{xi}$ Trade balance for type $X$ goods in country $i$.
$TB_{yi}$ Trade balance for good $Y$ in country $i$.
$T_i$ Per capita lump sum tax on consumers.
REFERENCES


..., “International Trade in General Oligopolistic Equilibrium,” mimeo, University College Dublin and Oxford University (2003).


### Table I

**General Equilibrium Outcomes**

<table>
<thead>
<tr>
<th>Free Trade</th>
<th>Autarky</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \widehat{n}_i = \frac{(\beta - 2)(2 - \sigma)}{\sigma} + 1 )</td>
<td>( \widehat{n}_A = \frac{(\beta - 2)(2 - \sigma)}{2\sigma} + 1 )</td>
</tr>
<tr>
<td>( \widehat{u} = u_{oi} \left( \frac{L_i}{\widehat{n}_i} \right)^{1/\beta} )</td>
<td>( \widehat{u}<em>{i,A} = u</em>{oi} \left( \frac{L_i}{n_A} \right)^{1/\beta} )</td>
</tr>
<tr>
<td>( \widehat{w}<em>i = \frac{L_i + L_A}{L_i} \frac{u</em>{oi}}{2} \left( \frac{L_i}{\widehat{n}_i} \right)^{2/\beta} \left( \frac{\widehat{n}_i^{1 - 2/\beta}}{\beta \left(1 - \frac{\sigma}{2}\right) + \sigma} \right) )</td>
<td>( \widehat{w}<em>{i,A} = \frac{u</em>{oi}^2}{2} \left( \frac{L_i}{\widehat{n}_A} \right)^{2/\beta} \left( \frac{\widehat{n}_A^{1 - 2/\beta}}{\beta \left(1 - \frac{\sigma}{2}\right) + \sigma} \right) )</td>
</tr>
</tbody>
</table>

### Table II

**Social Optimum (First Best)**

<table>
<thead>
<tr>
<th>Free Trade</th>
<th>Autarky</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{n}_i = \frac{(\beta - 2)(1 - 2\sigma)}{\sigma} )</td>
<td>( \tilde{n}_A = \frac{(\beta - 2)(1 - 2\sigma)}{4\sigma} )</td>
</tr>
<tr>
<td>( \tilde{u} = u_{oi} \left( \frac{L_i}{\tilde{n}_i} \right)^{1/\beta} )</td>
<td>( \tilde{u}<em>{i,A} = u</em>{oi} \left( \frac{L_i}{n_A} \right)^{1/\beta} )</td>
</tr>
<tr>
<td>( \tilde{w}<em>i = \frac{L_i + L_A}{L_i} \frac{u</em>{oi}}{2} \left( \frac{L_i}{\tilde{n}_i} \right)^{2/\beta} \left( \frac{\tilde{n}_i^{1 - 2/\beta}}{\beta \left(1 - \frac{\sigma}{2}\right) + \sigma} \right) )</td>
<td>( \tilde{w}<em>{i,A} = \frac{u</em>{oi}^2}{2} \left( \frac{L_i}{\tilde{n}_A} \right)^{2/\beta} \left( \frac{\tilde{n}_A^{1 - 2/\beta}}{\beta \left(1 - \frac{\sigma}{2}\right) + \sigma} \right) )</td>
</tr>
</tbody>
</table>
Figure I
Optimal Reply Schedules for the Open Economy ($\beta_i > \beta_j$, $\hat{u}_i < \hat{u}_j$)
Figure II
Effects of Changes in Initial Conditions
Figure III
Welfare Gains and Losses from Trade
Figure IV
Wage Equalization Locus: \( \tilde{w}_i = \hat{w}_i \) and \( \tilde{w}_{iA} = \hat{w}_{iA} \)

\[
\tilde{w}_i > \hat{w}_i, \quad \tilde{w}_{iA} > \hat{w}_{iA}
\]

\[
\tilde{w}_i = \hat{w}_i, \quad \tilde{w}_{iA} = \hat{w}_{iA}
\]

\[
\tilde{w}_i < \hat{w}_i, \quad \tilde{w}_{iA} < \hat{w}_{iA}
\]
Figure V
The Effects of Changing Population Size ($u_{oi} < u_{oj}$)
Figure VI
Indifference Loci: Effect of Changes in $\beta$