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Abstract

Tournaments create incentives which motivate economic agents. Yet the effect of tournaments on agents’ behaviour critically depends on their ability to identify their incentives and react to them accordingly. We investigate here how agents react to changes in incentives during dynamic contests. We use a quasi-experimental situation occurring in real dynamic contests with large stakes. Using point by point ball tracking data in tennis matches, we isolate situations where balls bounce very close to the court’s lines, landing either in or out. We use the associated random variations in winning probability to estimate the causal effect of being ahead or behind in the dynamic contest. In line with predictions from contest theory, we find evidence of a momentum effect for male players. We do not find any significant effect for female players, suggesting gender differences in reaction to incentives in contests.

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Today, for many economists, economics is to a large extent a matter of incentives. *Laffont and Martimort. The theory of incentives, 2002.*

Understanding how agents react to incentives and finding good institutional designs that provide the right incentives are key research questions in economics. For this reason, tournaments have been the object of an extensive literature since Lazear and Rosen (1981). Whenever the effort level of agents is not fully observable, tournaments provide incentives to improve agents’ effort levels. They are de facto pervasive in society and economic organisations such as promotion tournaments, patent races, rent seeking, litigation, and political contests (see Konrad (2009) for a review of the literature).

Fifteen years ago, Prendergast (1999) observed a lack of empirical evidence on the effect of incentives on agents’ behaviour in tournaments. Since then, the empirical research on contests has flourished. A growing literature in laboratory experiments (for a survey of the large literature see: Dechenaux, Kovenock, and Sheremeta 2014) has provided some support to economic theory predictions. But it also suggests that many of the biases or non-conventional preferences found in behavioural economics limit the validity of standard economic predictions. Additionally, a substantial literature has used field data, in particular from sporting tournaments, to test economic theories. Many of these studies have found that well trained agents in competitive tournaments do follow standard predictions with agents reacting to the incentives in tournaments (Taylor and Trogdon 2002, Palacios-Huerta 2003, Brown 2011) and adopting strategic behaviour closer from economic predictions that what is typically found in laboratory studies (Chiappori, Levitt, and Groseclose 2002, Walker and Wooders 2001, Hsu, Huang, and Tang 2007, Klaassen and Magnus 2009, Malueg and Yates 2010, Abramitzky, Einav, Kolkowitz, and Mill 2012). However other results also indicate limitations whereby players do not react to the tournament incentives as would be expected due to non standard preferences (Romer 2006, Berger and Pope 2011, Pope and Schweitzer 2011) or psychological limitations (Paserman 2010, Apesteguia and Palacios-Huerta 2010, Genakos and Pagliero 2012).

The question whether agents do react optimally to incentives in tournaments is especially pertinent when contests take place over time and are dynamic in nature. In dynamic contests, the agents’ strategies have to take into account past and future actions of their opponents. Most real life tournaments take place over time and are de facto dynamic contests (Konrad 2009). The introduction of a time dimension
makes optimal strategies typically more computationally demanding than one period
contests (Klumpp and Polborn 2006, Konrad and Kovenock 2009). As a consequence,
dynamic contests provide a tough test of the validity of standard economic predictions
in regard to strategic behaviour. And it is a test which is worth doing; studying agents
strategic behaviour in dynamic contests is relevant to understand how contests and
tournaments work in practice as incentive schemes.

However, studying players’ behaviour and strategies in dynamic contests faces a
difficult challenge: past performances are typically correlated with unobserved char-
acteristics which influence future performances. This creates a potential endogeneity
bias which may explain the mixed evidence in the existing literature about whether
agents’ strategic behaviour reacts to incentives in dynamic contests as would be
predicted by theory (Ferrall and Smith 1999, Tong and Leung 2002, Malueg and

We address this difficult identification problem using a quasi-experimental setting.
We use a large data set on precise ball location during tennis matches between pro-
fessional tennis players. We exploit the fact that the probability for a player to win
a point varies discontinuously as a function of the location of the ball on the court.
The rules of tennis imply that balls landing just out of the court lines lead to a loss of
the point for the player who hit the ball. On the contrary, play continues if the ball
landed just on the inside of the court lines, giving the player a positive probability
to win the point. We use this discontinuity in the probability to win the point for
balls landing very close to court lines to implement a fuzzy regression discontinuity
design (Imbens and Lemieux 2008) in order to investigate how professional players’
performance in tennis games changes after winning or losing a point. To do so, we
extract a very small subset of points where the ball bounced within a few centimeters
of the court lines. In this setting we can investigate how agents adapt their behaviour
in a dynamic contest when their overall advantage relative to their opponent changes
for reasons which are not correlated with differences in players characteristics.

Tennis games are dynamic contests where points are sub-contests. Models of dy-
namic contests typically predict a “momentum effect” for the winner of a sub-contest
(also called “discouragement effect” for the losing player Konrad (2009)). We show
in this paper that this prediction extends to tennis games, such that players win-
ning points should theoretically increase their effort level in later points while losing
players should decrease theirs effort level, leading to a momentum effect. Noticeably,
this prediction goes against the dominant view regularly expressed by professional players that every point should be considered in isolation with the player making the maximum effort to win each of them. For instance, Novak Djokovic, ranked number one player in the world at the time, declared in 2013 after winning a match at the US Open “I was wishing [...] to be able to stay committed to play every point, to win every point, regardless of what’s the score.”

In contrast with this idea and in line with predictions from contest theory, we find a significant momentum effect for male players: players are 5 to 10 percentage points more likely to win the next point after winning a given point. We use the richness of our dataset to test further predictions such that the momentum effect should be larger when the scoreline is symmetric and towards the end of a game. We find that these predictions are indeed observed for professional male players. Looking at a smaller dataset for females we do not find evidence of a momentum effect. This result adds to a substantial body of research that has established the existence of gender differences in competition (see Niederle and Vesterlund (2011) for a review). It suggests that part of these differences may arise from differences in how males and females react to incentives in dynamic contests.

The remainder of the paper is as follows: Section 1 places the contribution of this paper in the existing literature, Section 2 presents our conceptual framework, Section 3 presents our identification strategy and our dataset, Section 4 presents the results, and Section 5 concludes.

1 Related literature

While a wide range of designs exist for dynamic contests, existing models typically suggest that if agents react appropriately to incentives during a dynamic contest, a “strategic momentum” should appear. The perfect equilibria of races modelled by Harris and Vickers (1985) and Harris and Vickers (1987) lead to the leader of the race making greater effort than the follower. It is also a result found by Ferrall and

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1It is a principle which can also be found as “strategy” advice given by coaches: “Focus on that skill to the extent that you are not thinking about the score or who is winning”, “Play every point and return every shot” (Bryant 2011, p. 73 and p. 90). Beyond tennis, it is common to hear this advice in other sports which are dynamic contests. The professional Canadian ice hockey player Guy Lafleur declared “Play every game as if it is your last one.”. The company Nike re-used this advice in a global advertising campaign at the start of the 2014 football world cup, inviting to “Play every game like it’s ‘The Last Game’”. 

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Smith (1999), Konrad and Kovenock (2009), Malueg and Yates (2010) and Page and Coates (2013) in the case of best-of-n contests and by Breitmoser, Tan, and Zizzo (2010) in the context of R&D races. On the other hand it has also been suggested that contestants follow a “hare-tortoise” heuristic where the “trailing contestant will exert more effort to catch up, whereas the leading contestant will slack off” (Tong and Leung 2002). Outside economics, the notion of “psychological momentum” has been proposed as reflecting that winning may help enhance contestants’ confidence with the consequence that “success breeds success” (Dorsey-Palmateer and Smith 2004, Vallerand, Colavecchio, and Pelletier 1988).

The existing empirical literature presents conflicting results. A large body of research in social psychology initiated by Gilovich, Vallone, and Tversky (1985) has argued that momentum does not exist in competitions, and that the layman perception of momentum (so called “hot hand” in their initial study on basketball players) is a cognitive illusion. This research has led to a substantial number of social psychology studies which have provided mixed empirical results with a tendency to suggest an absence of any momentum (Bar-Eli, Avugos, and Raab 2006). However, many of these studies use statistical tests with low power (Wardrop 1999).

On the contrary, studies in economics have aimed to test the results from game theoretical models suggesting the existence of momentum. Within the recent growth in laboratory experiments on contests, several studies have looked at this question in a controlled environment (Dechenaux, Kovenock, and Sheremeta 2014). The evidence here is also mixed: Tong and Leung (2002) and Fu, Ke, and Tan (2013) found support for a negative momentum effect such that trailing contestants expend more effort to catch up with leading contestants. On the contrary, Mago, Sheremeta, and Yates (2012) found experimental evidence consistent with the existence of a strategic momentum. The equilibrium strategies in dynamic contests are arguably complex and part of the mixed evidence may come from the unfamiliarity of participants with such strategic settings.

Looking at real contests in the field eliminates this concern. Several studies have looked at team competitions in sport contests. Ferrall and Smith (1999) estimated the parameters of a structural model of strategic behaviour in a best-of-7 contest using data from US championship series. They found an absence of momentum in these contests. Teams seem to play each game as well as they can. In an old study also using data from US Championship series, Simon (1971) actually found
a negative momentum effect, labelled as a “back to the wall” effect. Using a large
data set on professional basketball matches, Berger and Pope (2011) found that teams
just behind at half time are more likely to win the match, suggesting here again a
negative momentum effect.  

Ferrall and Smith (1999) pointed out that an individual competition like tennis
may however provide more leverage to study strategic incentives than team com-
petitions. The characteristics of tennis have indeed made this sport a workhorse
setting to study players’ behaviour in a strategic game in the field. It has been used
to study whether players can play mixed Nash equilibrium strategies (Walker and
Wooders 2001, Hsu, Huang, and Tang 2007), whether they optimise their chances of
winning when making risk-return trade-offs (Klaassen and Magnus 2009, Abramitzky,
Einav, Kolkowitz, and Mill 2012) and whether there are gender differences in perfor-
man ce in contests (Paserman 2010).

Previous studies have utilised tennis data to investigate the changes of players’
strategies in a dynamic contest. Looking at point by point data from the Wimbledon
tennis tournament, Klaassen and Magnus (2001) found the hypothesis that points
are iid should be marginally rejected in tennis, with the possibility of a momentum
effect in some cases between points. Malueg and Yates (2010) selected a sample of
268 tennis matches where players have identical betting odds, and found support of a
momentum effect between sets of tennis matches. A player winning a first set by two
games or less tends to win the second set more often than his opponent, even though
arguably they are very close in ability. Page and Coates (2013) adopted a similar
strategy, looking at matches where the first set ended up in a close tie-break. They
found the player winning the first set is significantly more likely to win the second
set.

The present study adds to this literature by looking at a quasi-experimental set-
ing within a large sample of tennis games. Our strategy addresses a key difficulty
when trying to estimate the effect of previous performance on later performance in
the field. Past and future performances are naturally correlated due to unobservable
characteristics which impact both of them. Even within a given dynamic contest,
characteristics such as the fitness and ability of the players may not be constant

\footnote{In the context of political races, Klumpp and Polborn (2006) found some evidence of “momentum” in the US primaries and stress that the importance given to the first primary in New-Hampshire could stem from the existence of such a momentum.}
over time (due to fatigue, injury, learning, or change in team composition for team contests). This hypothesis of constant underlying ability/skills of the contestants typically underlies field studies. The mixed evidence from existing studies using field data may partly come from the difficulty to eliminate the possibility that this hypothesis may not be fulfilled. Our identification strategy eliminates such a confounding factor in order to cleanly identify the causal effect of previous results on later performance (see Section 3).

2 Conceptual framework

Tennis is a dynamic contest composed of several sub-contest levels: players contest “points” to win “games”, which are needed to win “sets”. Here, we study the existence of a momentum between points within a game. A tennis game has a structure which is very similar to best-of-n multiple battle contests. It imposes an additional constraint whereby the winner needs to have won two more battles (“points”) than the other player. Figure 1 shows the structure of a tennis game.\(^3\) Starting from a state \((s_A, s_B)\) in the game where \(s_A\) and \(s_B\) are the scores of player A and B respectively, players move to state \((s_A + 1, s_B)\) if A wins the point and to \((s_A, s_B + 1)\) if B wins. This contest structure is studied in detail by Walker, Wooders, and Amir (2011) in the case where players’ strategies are costless. When players’ strategies involve choosing a costly investment (e.g. effort) as assumed in the literature on dynamic contests, a tennis game is a hybrid design between a best-of-n contest (which has a maximum of \(n\) sub-contests) and a “tug-of-war” (Konrad 2009) at the end of the game where players can move (potentially endlessly) back and forth between the states \((3,2), (2,2), (2,3)\).

Given such a contest structure, we first define the notion of momentum, formalising the concepts of “momentum” and “discouragement effect” which have been discussed in the literature on dynamic contests.

Definition 2.1. Let \((s_A, s_B)\) be the state of the game with \(s_A\) and \(s_B\) being the score of A and B respectively, and \(p_A\) and \(p_B\) being the players’ probabilities to win the point.

\(^3\)Note that the state \((2,2)\) contains both the \((30,30)\) scores and the \((40,40)\) scores or “deuce” as they are strategically equivalent. For such scores, players need to win two consecutive points to win the game.
Figure 1: A tennis game between two players A and B. A player wins if he/she wins at least 4 points and two points more than the other player.

(i) There is a momentum effect after state $(s_A, s_B)$ if $\mu_{s_A, s_B} = p_A(s_A + 1, s_B) - p_A(s_A, s_B + 1) > 0$.

(ii) A momentum effect is said to be larger after state $(s_A, s_B)$ than after state $(s'_A, s'_B)$ iff $\mu_{s_A, s_B} > \mu_{s'_A, s'_B}$.

Note that we define the momentum with the winning probabilities of player A, but they can symmetrically be defined with the winning probabilities of player B since $p_A(s_A, s_B) = 1 - p_B(s_A, s_B)$. Building on the results from Konrad and Kovenock (2005) and Konrad and Kovenock (2009), we show their finding of a momentum effect in “tug of war” and “best-of-n” contests generalises to this type of hybrid multi-battle contest.

Let’s consider two players, A and B, engaged in such a dynamic contest. We model each individual sub-contest (henceforth “point”) as an all-pay auction.\footnote{As pointed out by Konrad and Kovenock (2009), “the all-pay auction captures the notion that random external factors do not play a role in determining the outcome of the contest. The outcomes are random due to the endogenous uncertainty generated by the use of nondegenerate mixed strategies in equilibrium.” This absence of a lottery component in the contest makes it possible to derive closed form solutions for values and distributions at every state as well as transition probabilities between states of multi-battle contest. A full characterisation of equilibrium in multibattle contests with a lottery element in each state is still an open question and typically requires numerical meth-}
be the prizes of winning the whole contest (henceforth “game”) for players A and B respectively. We follow Konrad and Kovenock (2009) and allow the best-of-n contest part of the game to have intermediary prizes \( \Delta \), \( Z_B > \Delta \geq 0 \). In the context of a tennis game this allows for the possibility that players have a preference for winning points (Sheremeta 2010).

Given the presence of a tug-of-war element at the end, the game has a potentially infinite horizon. We assume that players maximise the expected discounted sum of per-period payoffs with \( 0 < \delta \leq 1 \) being a common discount factor. Each player expends his effort in each state \( j \), respectively \( a_j \) and \( b_j \). In case of equal effort, we follow Konrad and Kovenock (2005) and adopt a tie-breaking rule advantaging the player with the largest incentives. We can then establish the existence of a momentum in such a dynamic contest:

Proposition 2.2. If \( Z_A = Z_B \), there is a momentum effect in a tennis game with the following characteristics:

(i) There is a momentum effect after all the states in which the scoreline is equal or differs only by one point.

(ii) For a discounting factor that is not too small \( (\delta > 1/2) \), the momentum effect is larger after states with symmetric scorelines: \(((0,0),(1,1),(2,2))\) than after states with asymmetric scorelines.

(iii) For states with symmetric scorelines, the momentum effect is larger towards the end of the game: \( \mu_{2,2} > \mu_{1,1} > \mu_{0,0} \).

Proposition 2.2. (proof in the Appendix) not only says there should be a strategic momentum in a tennis game between players with similar incentives, it also indicates that the momentum should not be identical after every state. Specifically, it should be stronger after symmetric states and it should be larger at the end of the game, such as in the state \( (2,2) \), than at the start, such as in the state \( (0,0) \). These are testable predictions which are useful to assess whether a momentum exists, and if
so, whether it presents these patterns, which suggest momentum comes from players’
equilibrium strategies.

While players may have similar incentives at the start of the match, each game
will have a different value for each player depending on the scoreline in a match. It
is therefore meaningful to consider how the result of Proposition 2.2 extends to the
case of asymmetric incentives. Let $z_A(s_A, s_B)$ and $z_B(s_A, s_B)$ be the prizes to win
the point $(s_A, s_B)$ for players A and B respectively.

**Proposition 2.3.** If $Z_A > Z_B$:

(i) There is a momentum effect after all the states where $z_A(s_A, s_B) > \Delta$ and $z_B(s_A, s_B) > \Delta$

(ii) For $Z_B$ close enough from $Z_A$, these states are $(0,0), (0,1), (1,1), (1,2)$ and $(2,2)$.

Proposition 2.3. (proof in the Appendix) shows that a momentum also exists in
games where players have asymmetric incentives. In such games, (i) states that there
is a momentum effect after every point where both players have prizes higher than just
the intermediary prize $\Delta$. Such points are tipping points which are heavily contested.
The player winning these points benefits from an asymmetry in incentives in his/her favour while the losing player faces a disadvantageous asymmetry in incentives. For $Z_B$ not too far from $Z_A$ (ii) states that these points are those where the scoreline is symmetric or those where the players with the strongest incentives trail by one point. For low values of $Z_B$ relative to $Z_A$, symmetric scorelines stop being tipping points and the existence of a momentum effect is concentrated after states where A is trailing.

The variations in incentives across scorelines also suggest that some points are
likely to be more fiercely contested than others; in some both players will expend a
lot of effort to win, while others will be less contested. Specifically, we can make the
following prediction about the sum of effort expended by the players as a function of
their relative positions:

5Another reason to look at asymmetric incentives is the possibility for players to have different
costs of effort ($c_A, c_B$). In the all-pay auction framework, different effort costs can be represented by
different contest prizes by using normalised prizes: $Z_A' = Z_A/c_A$ and $Z_B' = Z_B/c_B$. One situation
where differences in effort costs is likely to arise in tennis is the service game where it is easier for
servers to win points than for receivers.
Proposition 2.4. (Sum of effort)

(i) If $Z_A = Z_B$, the sum of effort is highest for symmetric scorelines.

(ii) If $Z_A > Z_B$, the sum of effort is highest on tipping points where $z_A > \Delta$ and $z_B > \Delta$, whenever these sub-contest prizes are close enough.

Proposition 2.4. (proof in the Appendix) is intuitive, the sum of effort is highest on points where the asymmetry in the sub-contest prizes is the lowest. When $Z_A = Z_B$, the sub-contest prizes are equal for symmetric scoreline and that is where the effort is maximum. When $Z_A > Z_B$ the effort will be greater on tipping points where both players play for the grand contest, as long as the asymmetry in incentives is not too large in these states. For $Z_A$ and $Z_B$ close enough these states are (0,0), (0,1), (1,1), (1,2), (2,2). So that is where we can expect the sum of effort to be the largest and the points to be the most contested.

3 Identification strategy and Data

3.1 Identification strategy

A fundamental difficulty in the identification of a momentum is that a given state in a dynamic contest is reached as a consequence of the opposition of contestants whose characteristics are never fully observed. Any unobserved difference in ability will not only influence future performance, it will also have influenced past performance and therefore lead to different positions in the contest. This creates an endogeneity problem when trying to regress performance on relative positions in order to estimate a momentum effect in contests. Typically, the observation that a contestant leading in a competition has a higher level of performance than a trailing contestant does not prove the existence of a momentum as the leading contestant is most likely to have a higher level of ability.

If the contestants’ abilities were fixed over time, the endogeneity problem could be addressed using a fixed effect estimator to control for the contestants’ unobserved characteristics. However, in most cases the hypothesis of time invariant ability is unrealistic. It is not just the case over large time spans (days/months), but also within a given competition taking place in a short amount of time (few hours). In the
example of a tennis match, a player may get worse if he gets a minor injury (strain, blisters) or gets tired from the physical effort (muscle soreness, cramping), he/she may get better if he/she learns how to use the weaknesses of his opponent. Assuming wrongly that the contestants’ ability is fixed will bias the estimated effect of previous performance on current performance. For example, if a contestant’s ability is affected by random shocks over time and follows a moving average, assuming a fixed ability will induce a positive serial auto-correlation in the errors relative to the average ability. This, in turn, will create an illusion of “momentum”, with above average performance more likely to be followed by above average performance.

We propose here a new empirical strategy which solves the identification problem in the estimation of a momentum effect. To identify the effect of agents’ position in a dynamic contest on their current performance, one would ideally have an experimental setting where agents are randomly allocated to different possible states in the contest. To approximate this ideal in the field, we look for a quasi-experimental situation. In sport matches, scorelines often evolve differently around some threshold of performance. We use this to look at situations where contestants with very similar performance end up in a different relative position to each other (ahead vs. behind). Analysing point by point tennis data, we exploit a discontinuity in the probability of winning a point depending on the ball location on the court. When the ball is in, the player who hits the ball has a positive probability of winning the point. When the ball is out, the rules state that the player loses the point. Under the identification assumption that, for balls hit close enough to the line, there is no difference in average ability between players putting the ball slightly in and those putting the ball slightly out, the in/out position of the ball provides an exogenous variation in the probability to win the current point. We can use this variation to estimate the causal effect of winning the current point on winning the next point, using a fuzzy regression discontinuity design (FRD) (Hahn, Todd, and Van der Klaauw 2001, Imbens and Lemieux 2008).

Formally, to study the causal effect of a binary variable $y_1$ on variable $y_2$, the sharp RD design approach exploits a discontinuity in the values of the variable $y_1$ around a threshold $c$ of a forcing variable $d$ to study the causal effect of $y_1$ on $y_2$. The fuzzy RD design extends this approach to situations where there is a discontinuity in the expectation $E(y_1|d)$ around $c$. In our setting, the forcing variable $d$ of the regression discontinuity is the relative distance of the ball bounce from the court’s lines. The
value of $d$ is positive if the ball is inside the court and negative if the ball lands outside the court. Our goal is to estimate the causal effect of winning the present point (binary variable $y_1$) on the chance to win the next point (binary variable $y_2$). Around the threshold $d = 0$, tennis rules imply a jump in the probability to win the current point for the player who hit the ball. This jump can be used to estimate the effect of winning the present point on the probability to win the next point, using the Wald estimator:

$$
\tau_{FRD} = \frac{\lim_{d \downarrow 0} E(\Delta y_2 | d) - \lim_{d \uparrow 0} E(\Delta y_2 | d)}{\lim_{d \downarrow 0} E(\Delta y_1 | d) - \lim_{d \uparrow 0} E(\Delta y_1 | d)} = \frac{\tau_{SRD,y_2}}{\tau_{SRD,y_1}}
$$

(1)

As the effect of the ball being in and out is not fully deterministic, but instead produces a change in the probability to win the point, the fuzzy regression design estimator rescales the exogenous variation in the probability to win the next point to provide the full causal effect of winning one point.

### 3.2 Data

Our dataset corresponds to the official Hawk Eye data, for all the matches played at the international professional level where this technology was used between March 2005 and March 2009. Hawk-Eye is a computer system used in tennis and other sports to record the trajectory of the ball. Most of the matches are either from Grand Slam and ATP (Association of Tennis Professionals) tournaments or lower level ITF (International Tennis Federation) tournaments: Challengers, Futures and Satellite. In addition, some matches are from diverse cups like the Davis Cup or the Olympic games. Overall the dataset contains 3,163 different single matches.

For each point we know the position of every bounce recorded by the Hawk-Eye, as well as which player is serving, the current score, and the winner of the point. The Hawk-Eye estimates very precisely the location of ball bounces with a mean prediction error of 0.36 cm.\(^6\) In total, we are observing the location of 1,515,077 ball bounces (332,330 points) for male players and 774,760 (164,487 points) for female players. While bounce data is automatically recorded by the tracking system, the scores, the identity of the players and the name of the server in each game are manually entered. This leads to some discrepancies due to data entry errors. We excluded

\(^6\)As advertised by the Hawk-Eye Innovations website: [http://www.hawkeyeinnovations.co.uk/page/sports-officiating/tennis.](http://www.hawkeyeinnovations.co.uk/page/sports-officiating/tennis)
every game where we observed some discrepancy in at least one point (32.27% of the ball bounces).\textsuperscript{7} We also excluded serves in our analysis. Serves are highly rehearsed shots where players have a high degree of control on the ball’s position and speed which may allow good players to place the balls close to the lines (20.23% of the bounces are serves). Furthermore, we include neither game points, as their outcome can lead to the end of the game (28.82% of the bounces), nor points from tie-breaks, where the scoring rule differs (2.8% of the data). We otherwise include all points, including points where players “challenged” the ruling of the line judge about the position of the ball. In such cases, the challenge of the player may lead to the point being replayed. As the challenge is motivated by the visual perception from the player about the location of the ball’s bounce mark, as well as by their strategic incentives in the match (Abramitzky, Einav, Kolkowitz, and Mill 2012), excluding them could lead to a selection bias around the court lines.\textsuperscript{8}

Table 3.2 presents a description of the type of matches included in the dataset. Matches from both genders and from a wide range of competitions are included. As the Hawk-Eye system is usually restricted to the main tournaments, the dataset contains a large proportion of matches from top tournaments (i.e. Grand Slams). Within tournaments, matches are more likely to feature top players as the system is used on the main courts and is often absent from minor courts. This aspect implies that the matches contained in the dataset are more likely to feature the best male and female players over that period. Any strategic effect to be found is therefore unlikely to be due to a lack of experience from the players.

For each bounce, the dataset records the location of a bounce mark which is an oval shape made of 51 different dots. We use this bounce mark to compute the exact distance between the ball impact and the court’s lines.\textsuperscript{10} We then use this distance as a running variable in a regression discontinuity as the probability to win the point

\textsuperscript{7}Similarly to Abramitzky, Einav, Kolkowitz, and Mill (2012)’s study on players’ challenge calls, we are not concerned by a potential selection issue due to these errors in the recording of the tennis points’ data. They find the reasons for lost observations unlikely to be systematically related with the players’ strategic incentives in the point. Like them, we computed our estimations on less restrictive samples when possible and never found noticeable differences.

\textsuperscript{8}Challenge points can lead to the point being awarded to one player or to be replayed if the umpire’s initial call is overturned. We actually do not observe in the dataset which points are challenged. We included all the replayed points to ensure that we do not exclude challenged points (0.8% of the data).

\textsuperscript{10}A detailed explanation about how this distance is computed is included in the Supplementary Material.
<table>
<thead>
<tr>
<th>Event Features</th>
<th>Female</th>
<th>Male</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
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<td>300</td>
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<tr>
<td>Hard</td>
<td>913</td>
<td>1,249</td>
<td>2,162</td>
</tr>
<tr>
<td>Best of 3</td>
<td>1,169</td>
<td>1,400</td>
<td>2,569</td>
</tr>
<tr>
<td>Best of 5</td>
<td>-</td>
<td>594</td>
<td>594</td>
</tr>
<tr>
<td>Davis Cup (Fed Cup)</td>
<td>8</td>
<td>18</td>
<td>26</td>
</tr>
<tr>
<td>Grand Slam</td>
<td>453</td>
<td>525</td>
<td>978</td>
</tr>
<tr>
<td>Olympics</td>
<td>19</td>
<td>16</td>
<td>35</td>
</tr>
<tr>
<td>ATP (Premier)</td>
<td>659</td>
<td>100</td>
<td>759</td>
</tr>
<tr>
<td>International</td>
<td>-</td>
<td>473</td>
<td>473</td>
</tr>
<tr>
<td>Master</td>
<td>-</td>
<td>826</td>
<td>826</td>
</tr>
<tr>
<td>Hopman Cup</td>
<td>30</td>
<td>36</td>
<td>66</td>
</tr>
<tr>
<td>Total</td>
<td>1,169</td>
<td>1,994</td>
<td>3,163</td>
</tr>
</tbody>
</table>

Table 1: Break-down of the matches included in the dataset.

Changes markedly around the line; within tennis rules the player hitting the ball has a positive probability to win the point if the ball is just in but has null probability to win the point if the ball is just out.

Figure 2 shows the selected bounces for our preferred window of 4 cm around the court’s line.\(^{11}\) It represents a tennis court with each bounce mark in our dataset represented by its dot which is closest to the court lines. It is this dot which is relevant to measure its relative distance \(d\) to the court lines. Figure 2 shows graphically that only a very small subset of observations is used in the estimation. After cleaning the dataset we have 780, 548 (406, 885) bounces for male (female), and we only use 4,939 (2,744) of them in the estimation, that is 0.67% (0.63%) of the total number.

For a large sub-sample of the matches where this information was available we collected the ATP ranking of the players as well as the odds given by the bookmakers prior to the match. We were able to get this information for 94% of male players’ matches and 68% of the female players’ matches.\(^{12}\) In this sub-sample there are

\(^{11}\)We explain this choice in Section 4.1.

\(^{12}\)We used the freely available data from http://www.tennis-data.co.uk/alldata.php and used the odds from the bookmaker http://www.bet365.com.au/en/ which are available in the dataset.
Figure 2: Tennis court lines with the bounce marks present in our overall dataset (light grey) and those used for our regression discontinuity design (black) for male players.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Female</th>
<th>Male</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 10 vs Top 10</td>
<td>97</td>
<td>173</td>
<td>270</td>
</tr>
<tr>
<td>Top 10 vs Non-Top 10</td>
<td>432</td>
<td>949</td>
<td>1,381</td>
</tr>
<tr>
<td>Non-Top 10 vs Non-Top 10</td>
<td>283</td>
<td>794</td>
<td>1,077</td>
</tr>
<tr>
<td>Total</td>
<td>798</td>
<td>1,882</td>
<td>2,680</td>
</tr>
</tbody>
</table>

Table 2: Distribution of players’ ranking across matches, only for the sub-sample for which the players’ ranking is available.

263 male players and 181 female players. Table 2 shows the distribution of players’ ranking across matches.

4 Results

4.1 Momentum: main effect

Figure 3 shows the probability of winning the current point depending on how far the ball bounces away from the court’s line. When the ball bounces inside the court, the probability of winning the point is positive, whilst when the ball is outside the court
the probability is close to zero. Note that the probability of winning the point is not zero when the ball is just out of the court. This is due to the fact that line judges make mistakes and rule “in” some balls which are just “out” of the court but close to the line. Symmetrically, some balls which are just “in” are sometimes ruled “out” by line judges and this leads to a drop in probability to win the point. Such mistakes are well documented and stem from perceptual errors coming with the challenging task of judging the landing position of a small and fast moving ball (Whitney, Wurnitsch, Hontiveros, and Louie 2008). For balls right on the line there is a high probability of mistakes each way.

Figure 3: Probability of winning a point as a function of the distance (in meters) from which the ball hits the court’s line for male players. Estimation by local linear regression. In order to capture the possible non linearities of the regression function around the threshold we use a triangular kernel and a bandwidth of 0.5 cm.

As a consequence, Figure 3 does not show a marked discontinuity in zero. Umpires’ mistakes may have a detrimental effect for our analysis: it blunts the discontinuity in zero which weakens the power of the fuzzy regression discontinuity estimation. To address this issue we implement a donut regression design whereby the observations in a small neighborhood of the threshold are excluded (Barreca, Lindo, and Waddell 2015, Barreca, Guldi, Lindo, and Waddell 2011, Lindo, Siminski, and Yerokhin 2013, Hansen 2015). This allows us to keep observations very close to the line while not including the balls right on the line. Balls right on the line carry too many umpire’s mistakes about the ball position. Their exact position relative to the line (ie in/out)

---

13 Note that when the ball gets close to the line the probability of winning the point tends to increase due to the fact that the ball is harder to play back than when it is further away in the court. However, for balls very close to the line there is a drop induced by umpires’ mistakes.
is therefore a weak predictor of the player probability of winning the current point. We chose a donut size of 1 cm for the results presented in the main body of the paper.\textsuperscript{14} We also use as a non-parametric estimator of the regression function a kernel regression with rectangle kernels.\textsuperscript{15}

To ensure that our identification assumption is respected, we focus on the points where the ball fell very close to the line. Our preferred definition of “very close” which we use in the text to comment on results is a window of 4 cm or less from the court’s line. To give an order of comparison, tennis balls are designed with a diameter of approximately 6.7 cm. When they hit the court at high speed they make an oval shaped bounce mark. The biggest distance between antipodal points on such oval bounce marks is on average 9 cm in our dataset. As a comparison, the court’s lines are 5 cm wide. A distance of 4 cm is therefore roughly equal to half the size of the mark made by the bounce of the ball on the court and it is smaller than the width of the court line. The choice of 4 cm has two additional motivations. First, it is a small bandwidth, which ensures that the identification assumption is valid (see section 4.2 for our tests of validity our identification assumption). Second, when considering all the balls landing within 30 cm from the line, it is the MSE-optimal bandwidth using Calonico, Cattaneo, and Titiunik (2014)’s approach.

In any case, given that regression discontinuity estimates are often sensitive to bandwidth choices, we follow the recommendation from Imbens and Kalyanaraman (2012) to present results with different bandwidths. Our results are in graphical form presenting point estimates and confidence intervals for a wide range of bandwidths for the distance to the line: from 2 cm to 6 cm.

\textsuperscript{14}All the results for donut sizes of 0 cm and 0.5 cm are included in the Supplementary Material.

\textsuperscript{15}This specification presents the interest to estimate $\tau_{SRD,y_1}$ and $\tau_{SRD,y_2}$ as local averages from point located very close from the line, rather than to extrapolate their values in zero where $\tau_{SRD,y_1}$ tends toward zero. When the derivative of the regression function is not zero, kernel regression estimators have a boundary bias for too large a bandwidth around the threshold. Given our large number of observations, we are able to use bandwidths which are very small around the threshold which mitigates this concern. Furthermore, in our case, the conditional expectation to win the next point is flat around the threshold. Our estimation of the raw effect $\tau_{SRD,y_2}$ of placing the ball just in rather than just out are therefore unbiased. Given the non-linearity of the regression function depicted in Figure 3, a boundary bias may affect our estimation of $\tau_{SRD,y_1}$ such that our estimate of the effect of a full point $\tau_{FRD}$ may be slightly dampened towards 0. As a consequence our results about the effect of a full point are actually conservative.
Figure 4: Raw effect of the ball position on the probability to win the next point for male players (left) and female players (right) with the 5% level confidence interval. The top panels present the winning probability around the threshold estimated by kernel regression with a rectangle kernel of bandwidth 4 cm and a donut of 1 cm. The jump in zero is the estimation of the effect $\tau_{SRD,y}^2$. The bottom panels present the values of this effect estimated for bandwidths ranging from 2 cm to 6 cm. The point on the curve represents the estimate of the jump visible in the upper panel.

Figure 4 shows the discontinuity $\tau_{SRD,y}^2$ around the threshold 0. This estimator does not reflect the effect of winning a whole point. Rather, it reflects the effect of the jump in probability to win a point which is observed in Figure 3 around the court’s lines. The difference is significant and equal to 4.16% ($p=0.013$, $N=3,588$) for males. It is not significant for female players with an estimate of 0.76% ($p=0.739$, $N=1,953$). The lower panels present the point estimate of $\tau_{SRD,y}^2$ for different bandwidths. Our estimates are significant at 5% for a wide range of bandwidths. This suggests that putting a ball just “in” rather than just “out” has a positive effect on the probability of winning the next point for male players. The existence of a significant difference is
important here. As it indicates there is a relationship between an exogenous variation in the probability to win a point and the chance to win the next point as predicted by the momentum effect. The existence of a significant effect \( \tau_{SRD,y} \) is therefore the primary indication that there is a momentum effect.

The fuzzy regression discontinuity estimator (1) rescales the difference \( \tau_{SRD,y} \) to estimate the full effect of winning one point on the chances of winning the next point, \( \tau_{FRD} \). Looking at male players, this effect is equal to 7.21 percentage points (\( p=0.012, N=3,588 \)) for balls landing within 4 cm from the court lines. A male player who wins a point is more likely to win later points and this effect is not trivial. Our data therefore points to a clear evidence of a momentum effect for male players.

Figure 5 presents the results for bandwidths between 2 cm and 6 cm. The point estimate is most of the time significant and located between 5 and 10 percentage points. Conversely, looking at female players, there is no bandwidth for which the effect is significant.\(^{16}\) In the light of the literature on gender differences in behaviour in competitive environments (Niederle and Vesterlund 2011), our result suggests that men and women may react differently to incentives in a dynamic contest.\(^{17}\)

![Figure 5: Effect of winning a point on the probability of winning the next with the 5% level confidence interval for different bandwidths using a local Wald estimator for male players (left) and female players (right).](image)

\(^{16}\)While the point estimates are very close from zero, they present more variations than males’ estimates when changing the size of the donut (see Supplementary Material for different donut choices). However, in no specification, do we find effects close in magnitude to the one for male player (nor are they significant).

\(^{17}\)Results for other donuts size are included in the Supplementary Material.
4.2 Robustness checks

Our identification assumption is that players who put the ball very close to the line are of similar ability on average whether the ball landed just in or just out. A possible confounding explanation of the existence of a momentum effect for male players would be that the players putting the ball slightly in have a higher ability than players putting the ball slightly out. To test whether our identification assumption is respected we run a series of robustness checks.

First, we test for the existence of a discontinuity in the density of the running variable (Imbens and Lemieux 2008). A discontinuity in the density of the balls’ distance to the court lines would naturally arise if good players were able to precisely aim on the inside of the court lines. We use the test proposed by McCrary (2008) to control for such a possibility. This test consists of running a local linear regression in the values of a thinly binned histogram on each side of the threshold and to estimate the discontinuity at the threshold. Figure 6 shows an absence of manipulation of the running variable. In this figure the bandwidth is set to 4 cm and the binwidth is chosen optimally by the algorithm \((b = 0.0017)\). The point estimate is \(0.053\) \((p = 0.33, N = 6,536)\). In practice it means that good male players are not able to put their ball inside significantly more often than outside when they hit a ball close to the line.

![Figure 6](image.png)

**Figure 6:** McCrary test of a break in the density of the ball bounces around the threshold for a bandwidth of 4 cm for male players.

\(^{18}\)In this tests we did not include a donut. Furthermore, in the previous estimations we excluded the game-points as the server and receiver are switching at the end of the game. This game-points are included in this test.
Second, following the recommendation from Imbens and Lemieux (2008), we test for the existence of discontinuities in other covariates which could have an influence on the result. We use the information we have for a large sub-sample of observations on players’ professional rankings (Association of Tennis Professionals rankings) and about their ex-ante winning odds for the match. Using these variables, it is possible to test whether players putting the ball very close to the line but inside the court tend to have on average better ranking and better betting odds than those putting the ball just outside.

Following Klaassen and Magnus (2001) we do not directly use the ATP ranking. The quality difference between two top ranked players (e.g. ranked 1 and 2) is more pronounced than between two lower ranked players (e.g. ranked 100 and 101). Hence, we use a smoother measure of ranking proposed by Klaassen and Magnus (2001) by transforming the ATP ranking of each player into a variable $R$ as follows:

$$R = 8 - \log_2(RANK_{ATP}).$$

The betting odds give the equivalent winning probabilities $p$ estimated ex-ante by the betting market. Numerous studies have found they are very good predictors of the winning probability (Williams 2009).

Figure 7: Differences in measure of ranking (left) and ex-ante probability of winning the match (right) between male players putting a ball just inside or just outside the court when considering windows between 2 cm and 6 cm around the court’s lines.

Figure 7 shows the estimate of the difference in ranking measure $R$ and ex-ante winning probability $p$ around the threshold $d = 0$ for a wide range of bandwidths. The
point estimate for a bandwidth of 4 cm is 0.52 ($p = 0.921, N = 6,188$) for differences in rankings and 0.23% ($p = 0.74, N = 6,139$) for the ex-ante probability to win the match. The absence of discontinuity between the ability of players who hit balls that are in when close to the line, and those who hit the ball out when close to the line, further supports our identification hypothesis.

However, the most convincing robustness test is our third test. If players putting the ball just inside the court were better than those putting the ball just outside, then we should observe differences in results not only in later points but also in previous points. On the contrary, the momentum effect can occur only for future points. We can therefore run a placebo regression, where hitting the ball in or out is used as a predictor of the previous point. Figure 8 displays the results for the different bandwidths. For a bandwidth of 4 cm the point estimate is with 0.75% ($p=0.629, N=4,147$) very small which clearly shows that our assumption holds for such a bandwidth. For a bandwidth of 4 cm, the point estimate of the placebo regression is almost zero. Overall, these results suggest that balls landing close to the line offer an ideal quasi-experimental setting as they provide variations in winning probabilities which are not correlated with players’ abilities.

Figure 8: Placebo test of the effect of winning a point on the probability of winning the previous one with its 5% level confidence interval for different bandwidths using a kernel regression and a rectangle kernel (left) and a local Wald estimator (right) for male players.

Since the server and receiver are switching at the end of each game the game-points were not included in the previous estimations. Similarly the first point of the games are not taken into account in this computations.

Robustness checks for female players and other donut’s size are included in the Supplementary
4.3 Secondary results

4.3.1 Momentum as a function of the relative position of players in the contest

Propositions 2.2. and 2.3. predict that the strategic momentum should be higher for symmetric scorelines whenever there is not too large of an asymmetry in incentives between players. Table 3 shows the effect of winning a point on the next one depending on the scoreline. As predicted, the momentum effect changes with the score for male players. The effect is much stronger when the scoreline is symmetric at (1,1) and (2,2). When comparing all the symmetric scorelines vs asymmetric scorelines, there is a significant difference for male players: the momentum effect is 10.57% (p=0.011, N=1,747) for symmetric scorelines and only 3.83% (p=0.338, N=1,841) for non-symmetric scorelines. Moreover, as predicted by Proposition 2.2., the effect is highest for points at the end of the game. The momentum effect is 14.09% (p=0.044, N=561) for scorelines of (2,2) and 20.41% (p=0.017, N=394) for scorelines of (1,1) versus only 2.46% (p=0.706, N=792) for scorelines of (0,0).

No such patterns can be found for females. The momentum effect is not significant for any scoreline, and the point estimate is actually smaller for symmetric (-0.25%, p=0.96, N=1,010) than for asymmetric scorelines (3.18%, p=0.564, N=943). Furthermore, there is no indication of a higher level of momentum for later points in the game, with the momentum in (2,2) being almost zero (2.2%, p=0.786, N=395).

These results by scoreline support the suggestion that male players react to the changes of incentives during a dynamic contest in a way consistent with contest theory. The momentum effect is observed for states where the contest is most balanced and is higher towards the end of the game. On the contrary, we do not observe such patterns for female players. This adds to the initial absence of a significant overall momentum effect for female players and it may point to gender differences in how players react to incentives in dynamic contests. Proposition 2.3. predicts that when players have different levels of ability or incentives, the momentum effect may also be present in states where the advantaged player is trailing in the game.

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21Results for different bandwidths and donut sizes can be found in the Supplementary Material.

22We also investigated how players’ sum of effort varies for different scorelines. Using the duration of the point (measured with the number of bounces) as a proxy for the sum of effort, we found players expend more effort on points with symmetric scoreline as would be predicted by our model. We included these results in Section: 4.3.2.
Table 3: Wald estimator (in percent) of the effect of winning a point on the probability of winning the next depending on the scoreline, from the point of view of the player ahead in the game (bandwidth of 4 cm and a donut of 1 cm). The scores 30-30 and 40-40 are merged in (2,2), since they are strategically equivalent in the game of tennis (see Figure 1). P-values and sample size are indicated below each coefficient. * indicates significance at the 5% level.

To test this prediction, we estimated the momentum effect where one player has an advantage over the other one in terms of skills or incentives. We considered four situations of inequalities in skills and incentives between players. First, we look at the effect of asymmetries in professional ranking (Association of Tennis Professionals (ATP) for male players, Women’s Tennis Association (WTA) for female players) which are indicative of different levels of achievements in the last 12 months. Second, we compared servers and receivers. Serving creates a substantial difference in the chance
### Male players

<table>
<thead>
<tr>
<th></th>
<th>Player better ranked</th>
<th>Player serving</th>
<th>Player ahead in set</th>
<th>Player ahead in match</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahead in the game</td>
<td>−0.06</td>
<td>4.08</td>
<td>14.93</td>
<td>−2.58</td>
</tr>
<tr>
<td></td>
<td>(0.992)</td>
<td>(0.371)</td>
<td>(0.346)</td>
<td>(0.738)</td>
</tr>
<tr>
<td></td>
<td>898</td>
<td>1,213</td>
<td>104</td>
<td>452</td>
</tr>
<tr>
<td>Symmetric scoreline</td>
<td>10.98**</td>
<td>9.82*</td>
<td>5.13</td>
<td>12.18*</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.014)</td>
<td>(0.602)</td>
<td>(0.030)</td>
</tr>
<tr>
<td></td>
<td>1,646</td>
<td>1,747</td>
<td>234</td>
<td>786</td>
</tr>
<tr>
<td>Trailing in the game</td>
<td>4.70</td>
<td>3.15</td>
<td>27.96</td>
<td>5.65</td>
</tr>
<tr>
<td></td>
<td>(0.405)</td>
<td>(0.649)</td>
<td>(0.104)</td>
<td>(0.549)</td>
</tr>
<tr>
<td></td>
<td>837</td>
<td>628</td>
<td>116</td>
<td>383</td>
</tr>
<tr>
<td>All</td>
<td>6.78*</td>
<td>6.71*</td>
<td>12.68</td>
<td>7.10</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.015)</td>
<td>(0.091)</td>
<td>(0.084)</td>
</tr>
<tr>
<td></td>
<td>3,381</td>
<td>3,588</td>
<td>454</td>
<td>1,621</td>
</tr>
</tbody>
</table>

### Female players

<table>
<thead>
<tr>
<th></th>
<th>Player better ranked</th>
<th>Player serving</th>
<th>Player ahead in set</th>
<th>Player ahead in match</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahead in the game</td>
<td>5.53</td>
<td>1.56</td>
<td>−5.01</td>
<td>6.33</td>
</tr>
<tr>
<td></td>
<td>(0.496)</td>
<td>(0.819)</td>
<td>(0.796)</td>
<td>(0.623)</td>
</tr>
<tr>
<td></td>
<td>367</td>
<td>556</td>
<td>55</td>
<td>160</td>
</tr>
<tr>
<td>Symmetric scoreline</td>
<td>−1.59</td>
<td>−1.23</td>
<td>10.26</td>
<td>−5.72</td>
</tr>
<tr>
<td></td>
<td>(0.779)</td>
<td>(0.805)</td>
<td>(0.528)</td>
<td>(0.531)</td>
</tr>
<tr>
<td></td>
<td>742</td>
<td>1010</td>
<td>126</td>
<td>313</td>
</tr>
<tr>
<td>Trailing in the game</td>
<td>3.48</td>
<td>3.20</td>
<td>−1.90</td>
<td>−20.85</td>
</tr>
<tr>
<td></td>
<td>(0.724)</td>
<td>(0.724)</td>
<td>(0.892)</td>
<td>(0.081)</td>
</tr>
<tr>
<td></td>
<td>320</td>
<td>387</td>
<td>141</td>
<td>126</td>
</tr>
<tr>
<td>All</td>
<td>1.52</td>
<td>0.51</td>
<td>2.49</td>
<td>−6.79</td>
</tr>
<tr>
<td></td>
<td>(0.717)</td>
<td>(0.890)</td>
<td>(0.792)</td>
<td>(0.289)</td>
</tr>
<tr>
<td></td>
<td>1,429</td>
<td>1,953</td>
<td>322</td>
<td>599</td>
</tr>
</tbody>
</table>

Table 4: Wald estimator (in percent) of the effect of winning a point on the probability of winning the next depending on the scoreline in the game from the point of view of the player who is best ranked (first column). Using the ATP rankings for male players and in the WTA rankings for female players, the player serving (second column). The player ahead in the set by at least one break (third column). The player ahead in the match by at least one set (fourth column). Bandwidth is 4 cm and donut 1 cm. P-values and sample size indicated below each coefficient. * significance 5%, ** at 1%.
to win the point and the game, so it is a natural asymmetry between the players. Third, we considered situations where one player is ahead in the set by at least one break.\textsuperscript{23} Such a lead puts him/her in a very good position to win the set. Finally, we considered situations where one player is ahead in the match by one set or more. In all these situations, the advantaged player should find it easier or have a greater incentive to win the game. As a consequence, the momentum could appear for states where this advantaged player is trailing. Table 4 shows such estimates for these four situations: when the player considered is ahead in the game, when the scoreline is symmetric, or when the player is trailing in the game. For symmetric scorelines, the momentum effect is almost always significant for males. For asymmetric scorelines, the point estimates tend to be higher for male players when the advantaged players is trailing, in line with theoretical predictions. Though, they are not significant. Like for all previous results, we do not find evidence of a momentum effect for female players.

Within our conceptual framework, these results may suggest the asymmetries in skills and incentives we are considering here are not pronounced enough for a momentum to appear when the advantaged player is trailing. Hence, the momentum is still strongest for symmetric scorelines.

### 4.3.2 Effect on effort expenditure

As in most dynamic contest models, in our model the strategic momentum emerges as a consequence of players’ decisions in terms of resource/effort allocation. As tennis is an interactive game, it is hard to isolate individual effort decisions. How much a player runs or how hard he/she hits the ball is influenced by the shots’ speed and location chosen by the opponent. However, Proposition 2.4. predicts that the sum of efforts itself should vary as a function of the scoreline such that it is greatest when the players’ rewards to win the point are closer to each other.

\textsuperscript{23}The leading player has won a game where the other player served.
Table 5: Wald estimator (in percent) of the effect of winning a point on the number of bounces in the next point depending on the scoreline (with a bandwidth of 4 cm and a donut of 1 cm) from the point of view of the server. The scores 30-30 and 40-40 are merged in (2,2), since they are strategically equivalent in the game of tennis (see Figure 1). P-values and sample size are indicated below each coefficient. * indicates significance at the 5% level.

<table>
<thead>
<tr>
<th></th>
<th>Male players</th>
<th></th>
<th>Female players</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2,0)</td>
<td>(2,1)</td>
<td>(2,2)</td>
<td>(2,0)</td>
</tr>
<tr>
<td></td>
<td>-2.27**</td>
<td>-1.59</td>
<td>-1.15</td>
<td>-1.87</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.055)</td>
<td>(0.203)</td>
<td>(0.163)</td>
</tr>
<tr>
<td></td>
<td>(307)</td>
<td>(374)</td>
<td>(561)</td>
<td>(122)</td>
</tr>
<tr>
<td></td>
<td>(1,0)</td>
<td>(1,1)</td>
<td>(1,2)</td>
<td>(1,0)</td>
</tr>
<tr>
<td></td>
<td>-1.75*</td>
<td>-1.30</td>
<td>0.98</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.188)</td>
<td>(0.494)</td>
<td>(0.815)</td>
</tr>
<tr>
<td></td>
<td>(532)</td>
<td>(394)</td>
<td>(227)</td>
<td>(254)</td>
</tr>
<tr>
<td></td>
<td>(0,0)</td>
<td>(0,1)</td>
<td>(0,2)</td>
<td>(1,0)</td>
</tr>
<tr>
<td></td>
<td>1.05</td>
<td>-2.04</td>
<td>-1.11</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.091)</td>
<td>(0.586)</td>
<td>(0.878)</td>
</tr>
<tr>
<td></td>
<td>(792)</td>
<td>(288)</td>
<td>(113)</td>
<td>(395)</td>
</tr>
<tr>
<td>Server leading</td>
<td>-1.85**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1,213)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symmetric scorelines</td>
<td>-0.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.617)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1,747)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Receiver leading</td>
<td>-0.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.343)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(628)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the case of a tennis point, the duration of the point is likely to be a good indication of the sum of effort expended on that point. It is reasonable to assume that points are going to last longer when no player gives up. The main driving force of the momentum effect is the discouragement effect for the player lagging behind. One can expect that this discouragement leads players to give up quicker during points,
leading these points to be shorter in duration. Therefore, we investigate how the duration of points varies for different scorelines to provide complementing evidence for the pattern of momentum.

In order to test Proposition 2.4, we extracted the number of bounces in each point. We then use our identification strategy to assess whether there is a negative effect on players’ effort when the scoreline moves away from a symmetric score. We focus on points where the ball gets close to the line. To control for the length of the point where the ball landed close to the line, we define $n^*_t$ the number of bounces at the time the ball lands close to the line ($n^*_t > 1$, since we do not include the serves). Let’s define $n_{t+1}$ as the total amount of bounces in the next point. We are interested in $\tau_{FRD,n} = E(n_{t+1} - n^*_t | \text{ball in}) - E(n_{t+1} - n^*_t | \text{ball out})$, the effect in the change in number of bounces caused by winning the point.

Table 5 shows $\tau_{FRD,n}$ for each scoreline from the point of view of the server (using a bandwidth of 4 cm). It shows that part of the decrease away from the symmetric scoreline is causal, with the number of bounces being significantly lower when the server moves to (2,0) compared to a move to (1,1) and when he moves to (3,0) compared to (2,1). The points’ duration therefore decrease when the server takes a substantial advantage in the game. This suggests that the sum of effort expended in a point varies as a function of the players’ relative positions in the way predicted by Proposition 2.4.

### 4.4 Alternative explanations

#### 4.4.1 Learning

Models of contests typically assumes that players’ strength are common knowledge. This is obviously a simplifying assumption and one may assume that in many contest situations, players do not know exactly the strength of their opponent, and sometimes not even fully their own. It is easy to foresee how a momentum effect could arise from a mechanism of learning during the contest. From past victories, a player could learn that he/she is stronger that the opponent. Difference in strength can easily be conceived as leading to differences in incentives as a weaker opponent either faces

---

24 This is likely to be the case on average. Obviously, on specific points, one player making a greater effort may lead to him/her winning the point in fewer shots.

25 Note that $n^*_t$ is a proxy for the effort expended in the point, but it is not the total number of bounces in the point.
higher costs of effort or is required to expend more effort to win. As a consequence, one could expect a momentum effect to arise from players learning about their differences in strength from their past opposition.

Whilst this line of argument is intuitive, optimal strategies in dynamic contests with asymmetric information are much more complex than this simple description. It is certainly for this reason that there is only limited work within such a framework. A notable exception is the study of Münster (2009) who models a repeated contest with players who are ignorant of the ability of their opponent. Noticably, the main result of the model is not the apparition of a momentum effect but the decision from high ability contestants to put in low effort in early rounds in order to deceive their opponent into believing that they have a low ability.

Notwithstanding this result, we investigate whether learning could be an explanation of the momentum effect we observe. To do so, we compare how the momentum effect evolves during the match. If it is due to learning, it should arguably be larger at the beginning of the match where players are gauging their respective strengths. On the contrary, one would expect players to have a better idea of their own and their opponent’s strength towards the end of the match.

Table 6 shows the momentum depending on which set players are in. The splitting of the sample makes the estimated momentum effect quite imprecise and none of them are significant at the conventional level. It is however noticeable that the momentum effect is almost constant over the first three sets for male players. This pattern does not suggest that the momentum effect we measure is driven by learning. Note that the

<table>
<thead>
<tr>
<th>Set</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.57</td>
<td>7.01</td>
<td>7.04</td>
<td>9.99</td>
<td>-15.62</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.111)</td>
<td>(0.265)</td>
<td>(0.451)</td>
<td>(0.558)</td>
</tr>
<tr>
<td></td>
<td>1,369</td>
<td>1,342</td>
<td>649</td>
<td>157</td>
<td>71</td>
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</tbody>
</table>

<table>
<thead>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>9.21</td>
<td>-3.44</td>
<td>-7.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.114)</td>
<td>(0.542)</td>
<td>(0.438)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>867</td>
<td>821</td>
<td>265</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Effect of winning a point on the probability of winning the next depending on the set for a bandwidth of 4 cm and a donut of 1 cm.
characteristics of our setting are unlikely to make learning the preferred explanation. As we focus on points won almost by chance, rational players should not infer much information about their relative strength from these points.\footnote{Obviously, players may still make such inferences from these points, if players process information about their own ability in a biased manner (Mobius, Niederle, Niehaus, and Rosenblat 2011).}

### 4.4.2 Psychological momentum

The momentum predicted by our model of dynamic contests comes from the strategic response of players to an asymmetry in incentives between the players as a function of their relative position in the contest. It is also sometimes suggested that a momentum can arise for psychological reasons (Mago, Sheremeta, and Yates 2012). There is no formal model as such of psychological momentum. In many cases, it is fully compatible with the economic framework with a psychological momentum arising from relative position in the contest. However, the psychological momentum is often presented as being path dependent. In our framework, strategies in a given state \((s_A, s_B)\) are fully determined by the payoffs in that state. A path dependent momentum would suggest that the past sequence of wins and losses preceding a given state can influence the player’s winning chances in that state.

To investigate whether the momentum we observe is path dependent, we compared the momentum we observe for different paths taken by players to reach the considered scoreline. Specifically let’s consider the scores \((1, 1)\), \((2, 2)\) and \((2, 1)\). In each of these scores each player has at least won and lost one point. Focusing on such situations we can test whether the momentum observed for the given scoreline differs depending on whether the winning player won or lost the precedent point. For instance, is the momentum larger if player A wins at \((1, 1)\), reaching \((2, 1)\) when he is on a streak, coming back from \((0, 1)\) or when player B has just caught back with him after a \((1, 0)\)? Table 7 compares the momentum effect for each of these scorelines depending on whether the winning player had won or lost the precedent point. For men, the momentum after scorelines of \((1, 1)\) is almost identical in each situation. For \((2,2)\) the two point estimates of the momentum are larger when the last point was won but both coefficients are well within one standard error from each others. For \((2,1)\) scorelines, there is no significant momentum effect, whether the previous point was won or lost. For women, there is no significant momentum whether the previous was won or lost, and even the point estimate of the effect is greater after a previous point
lost for the scorelines (2,2).

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Male players</th>
<th></th>
<th></th>
<th>Female players</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Player 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Win t-1 Lose t-1 All</td>
<td>Win t-1 Lose t-1 All</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,1)</td>
<td>21.15 20.05 20.41</td>
<td>(1,1)</td>
<td>1.72 -4.01 -0.74</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-val</td>
<td>(0.085) (0.09) (0.017)</td>
<td>p-val</td>
<td>(0.904) (0.799) (0.945)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>12.27 11.84 8.55</td>
<td>SE</td>
<td>14.17 15.78 10.67</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>(199) (195) (394)</td>
<td>N</td>
<td>(105) (115) (220)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,2)</td>
<td>17.82 10.41 14.10*</td>
<td>(2,2)</td>
<td>-6.29 11.54 2.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-val</td>
<td>(0.072) (0.299) (0.044)</td>
<td>p-val</td>
<td>(0.544) (0.352) (0.786)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>9.90 10.03 7.01</td>
<td>SE</td>
<td>10.36 12.40 8.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>(292) (269) (561)</td>
<td>N</td>
<td>(197) (198) (395)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,1)</td>
<td>5.61 -11.04 0.02</td>
<td>(2,1)</td>
<td>20.10 6.16 14.95</td>
<td></td>
<td></td>
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<tr>
<td>p-val</td>
<td>(0.506) (0.359) (0.998)</td>
<td>p-val</td>
<td>(0.112) (0.700) (0.131)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>8.43 12.04 6.92</td>
<td>SE</td>
<td>12.64 15.99 9.89</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>(385) (216) (601)</td>
<td>N</td>
<td>(205) (115) (320)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Wald estimator (in percent) of the effect of winning a point on the probability of winning the next, depending on the path taken by the players to reach the scores (1,1), (2,2) and (2,1) (with a bandwidth of 4 cm and a donut of 1 cm).

Overall these results do not suggest that our main momentum effect is driven by path dependence such that the momentum effect depends on the path the players took to reach a specific scoreline. The pattern of momentum we observe (such as the fact that it is greater for symmetric scorelines) suggests that an economic model predicting players strategies from the evolution of their incentives in the dynamic contest is enough to explain the observed behaviour, without the need for additional psychological hypotheses.\textsuperscript{27}

\textsuperscript{27}This absence of evidence of path dependence in our dataset echoes recent experimental finding of no such psychological momentum in an experimental contest (Mago, Sheremeta, and Yates 2012).
5 Conclusion

Tournaments are institutional designs which have attracted a lot of attention for their ability to motivate agents to expend effort. However, this effect on agents’ behaviour critically depends on their ability to identify and react to their incentives during the competition. Most tournaments taking place in the field are dynamic contests as they take place over time. Agents have to adjust their strategies depending on the evolution of their relative place in the contest, which affects their expected final rewards. It is not necessarily trivial for agents to figure out the optimal way to respond to the evolution of their standing in a dynamic contest.

For this reason, studying agents’ reactions to their relative positions in contests brings valuable insights into how tournaments work in practice as institutions shaping agents’ behaviour. In this context, our study sheds new light on how well trained agents behave in a dynamic contest. Using a large and precise dataset tracking the play of the ball in professional tennis matches, we have investigated whether players’ reactions to incentive changes in a tennis contest can be predicted by economic theory. We find evidence of a momentum effect for male players; winning a point in a game increases their chance to win the next one. Importantly, we are able to test predictions regarding specific patterns of this momentum effect as a function of the state in the dynamic contest. Our findings are in line with the theory which predicts the momentum effect to be stronger when players are in a close contest and when they are nearing the final states of the contest. These results suggest the observed momentum effect is a consequence of variations in incentives which occur in a dynamic contest.

However, we do not find evidence of such an effect for female players. This points to a possible gender difference in the momentum effect. Previous research on gender differences in behaviour in competitive environments has established men tend to have a greater preference for competitive environments (Niederle and Vesterlund 2007), and they also tend to perform better than females in mixed tournaments (Gneezy, Niederle, and Rustichini 2003). Our results suggest men may adopt more efficient strategies in dynamic contests by modulating effort as a function of the importance of the stakes present at any given moment in time. As a large part of gender inequalities arise in competitive settings such as promotion tournaments, this possible gender difference in strategic behaviour would be worth investigating further.

Note that the evidence we present in favour of a strategic momentum does not
necessarily mean male players’ behaviour is optimal. There is widespread empirical evidence of overdispersion in contest whereby contenders invest more resources (time, effort, money) than what equilibrium strategies would require. In the case of a dynamic contest, such an overdispersion can appear if players continue to expend a lot of effort even when they no longer have a good chance to win the contest. The quote from Novak Djokovic in the introduction may suggest that, while male professional players react to incentive changes in a tennis game, they may fail to do so optimally by expending too much effort on minor points.

As tennis is an individual sport, agency or team production arguments are unlikely to be an issue, making it an ideal setting to test contest theories (Abramitzky, Einav, Kolkowitz, and Mill 2012). However, in many tournaments teams of individuals oppose each other. Building on our results, it would be valuable to investigate how teams react to variations of incentives in dynamic contests. In comparison to competing individuals, teams decision making presents more challenges. First, in many cases players incentives do not align perfectly with the team’s goal, even when a clear optimal team strategy exists (Gauriot and Page 2015). Second, even when such incentive conflicts do not exist, optimal strategies may be harder to achieve if players need to coordinate when several equilibrium strategies exist. Conversely, teams may have a greater ability to use either the knowledge of their best members or the “wisdom of the crowd” to approximate equilibrium strategies.

Finally, the well established evidence of individuals’ concerns for relative positions transforms many non-agonistic situations into “contest for status”. This potentially extends the relevance of contest theory beyond formal contests within and between economic organisations. Contests for status may play a significant role both in the area of consumption choice (Hopkins and Kornienko 2004, Kuhn, Kooreman, Soetevent, and Kapteyn 2011) and for working decisions within organisations (Moldovanu, Sela, and Shi 2007, Besley and Ghatak 2008). Such decisions take place over time with agents adapting their consumption and work choices as a function of their relative standing. Understanding how agents react to changes in relative position during dynamic contests may have a wide range of applications in economics.
References


Fu, Q., C. Ke, and F. Tan (2013): “” Success breeds success" or" Pride goes before a fall”? Teams and Individuals in Best-of-Three Contests,” Discussion paper, No. tax/mpg-rps-2013-06.


A Appendix: Proofs

Proposition 2.2. To prove Proposition 2.2., we start with the tug-of-war part of the tennis game and use the results from Konrad and Kovenock (2005). The tug-of-war part of the tennis game has a possible infinite horizon. However Konrad and Kovenock show that each point is played as an individual all-pay auction where the players’ strategies are purely determined by one of the three states they are in. There is a unique Markov perfect equilibrium in such a game characterised by the existence of “tipping states” where both contestants invest substantial amounts of effort to win. For other states around these tipping states, an asymmetry in incentives creates an advantage for one of the players. Relative to Konrad and Kovenock, we introduce the possibility of a positive and intermediary prize $\Delta \geq 0$. The size of this prize has no effect on equilibrium continuation values in each state as the intermediate prizes are netted out from a higher effort level of contestants. The proof of existence and uniqueness of a Markov perfect equilibrium in the tug-of-war relies only on states’ continuation values and it can therefore be extended to the case with a positive intermediary prize. The magnitude of $\Delta$ does impact the equilibrium distribution of efforts and the respective winning probabilities in each individual points (we describe this effect below). When $Z_A = Z_B = Z$, the continuation values at the equilibrium are given by Proposition 1 from Konrad and Kovenock (2005): $v_A(3, 2) = \delta Z$, $v_B(3, 2) = 0$; $v_A(2, 2) = 0$, $v_B(2, 2) = 0$; $v_A(2, 3) = 0$, $v_B(2, 3) = \delta Z$. The state $(2,2)$ is the only tipping state.

By plugging these values as end values of the game of tennis, one can use the method of Konrad and Kovenock (2009) to solve the rest of the game. Relative to their model of a multibattle contest we introduce a discount factor $\delta < 1$. This
changes the continuation values and winning probabilities without changing their results qualitatively. By backward induction the continuation values are well defined and uniquely determine point prizes and equilibrium strategies at each point. With the previous result of existence and uniqueness of a Markov perfect equilibrium in the tug-of-war part of the tennis game, there is therefore a unique Markov perfect equilibrium in the whole game. At any point, the prizes for the players to win the point are defined as:

\[
\begin{align*}
  z_A(i,j) &= \delta v_A(i+1,j) - \delta v_A(i,j+1) + \Delta \\
  z_B(i,j) &= \delta v_B(i,j+1) - \delta v_B(i+1,j) + \Delta.
\end{align*}
\]

(2)

And the continuation values are:

\[
\begin{align*}
  v_A(i,j) &= \max(0, z_A(i,j) - z_B(i,j)) + \delta v_A(i,j+1) \\
  v_B(i,j) &= \max(0, z_B(i,j) - z_A(i,j)) + \delta v_B(i+1,j).
\end{align*}
\]

(3)

At each point the outcome is determined by an all-pay auction. Assume that \( z_A(i,j) \geq z_B(i,j) \) (the result is symmetric if \( z_B(i,j) \geq z_A(i,j) \)). Following Hillman and Riley (1989), the players’ winning probabilities at each point are:

\[
\begin{align*}
  p_A(i,j) &= 1 - \frac{z_B(i,j)}{2z_A(i,j)} \\
  p_B(i,j) &= \frac{z_B(i,j)}{2z_A(i,j)}.
\end{align*}
\]

(4)

By backward induction, equations (2), (3) and (4) uniquely determine prizes, continuation values and winning probabilities for every point of the game. From the winning probabilities, the momentum effects after each point are found to be:

\[
\begin{align*}
  \mu_{2,2} &= 1 - \frac{\Delta}{\delta Z + \Delta} \\
  \mu_{1,1} &= 1 - \frac{\Delta}{\delta^2 Z + \Delta} \\
  \mu_{0,0} &= 1 - \frac{\Delta}{\delta^3 Z + \Delta} \\
  \mu_{1,0} &= \mu_{0,1} = 1/2 - \frac{\Delta}{2Z(\delta^2 - \delta^3) + \Delta} \\
  \mu_{2,1} &= \mu_{1,2} = 1/2 - \frac{\Delta}{2Z(\delta - \delta^2) + \Delta} \\
  \mu_{2,0} &= \mu_{0,2} = \frac{\Delta}{2(\delta^2 Z + \Delta)} - \frac{\Delta}{2Z(\delta - \delta^2) + \Delta}.
\end{align*}
\]
Figure 9: Momentum effects are larger for symmetric scorelines for $\delta$ large enough.

For $\delta \in [0, 1]$, the momentum effects $\mu_{2,2}$, $\mu_{1,1}$, $\mu_{0,0}$, $\mu_{1,0}$, $\mu_{2,1}$ are positive. The momentum effect $\mu_{2,0}$ is the only one not to always be positive. $\mu_{2,0} > 0$ iff $\delta < 1/2$. This proves point (i) of Proposition 2.2.

It is also the case that $\mu_{2,2} > \mu_{1,1} > \mu_{0,0}$ which proves point (iii) of Proposition 2.2.

In addition $\mu_{2,1} > \mu_{1,0}$. Given the previous results, the momentum effect for symmetric scorelines will be larger than for the scorelines with one point difference whenever $\mu_{0,0} > \mu_{2,1}$. Let’s rewrite $\Delta = \lambda Z$. Solving the inequality $\mu_{0,0} - \mu_{2,1}$ for $(\delta, \lambda) \in [0, 1]^2$ gives for each $\lambda$ a $\delta^*(\lambda)$ such that $\mu_{0,0} > \mu_{2,1}$ iff $\delta > \delta^*(\lambda)$ and $\mu_{0,0} \leq \mu_{2,1}$ otherwise. The Figure 9 shows the plot of the function $\delta^*(\lambda)$. For $\delta > 1/2$ the momentum effect is always larger for symmetric scorelines. This proves the point (ii) of Proposition 2.2.

Proposition 2.3. We start by proving (i). Let $\Sigma(k)$ be the set of possible scoreline after $k$ points have been played in the game $\Sigma(k) = \{(s_A, s_B) : s_A > 0, s_B > 0, s_A + s_B = k\}$. We say that a state $(i_k, k - i_k)$ of $\Sigma(k)$ is a tipping state if and

---

28We use the same notation as Konrad and Kovenock (2009) however, for simplicity in our framework, we define here $k$ as the number of individual contests already played while they defined as the number of contests left to play.
only if \( z_A(i_k, k - i_k) > \Delta \) and \( z_B(i_k, k - i_k) > \Delta \).

At these tipping states, a momentum effect appears in the next state between winners and losers. Given that \( z_A(i_k, k - i_k) > \Delta \) and \( z_B(i_k, k - i_k) > \Delta \), it means that \( v_A(i_k + 1, k - i_k) > v_A(i_k, k - i_k + 1) \) and \( v_B(i_k, k - i_k + 1) > v_B(i_k + 1, k - i_k) \). Furthermore, by definition of continuation values, we have either \( v_A(i_k + 1, k - i_k) > 0 \) and \( v_B(i_k + 1, k - i_k) = 0 \) or \( v_A(i_k + 1, k - i_k) = 0 \) and \( v_B(i_k + 1, k - i_k) > 0 \).

The same applies for state \((i_k, k - i_k + 1)\). As a consequence \( v_A(i_k + 1, k - i_k) \) and \( v_A(i_k, k - i_k) \) cannot be jointly positive as it would contradict the inequality \( v_B(i_k, k - i_k + 1) > v_B(i_k + 1, k - i_k) \). They cannot be jointly equal to zero either as it would contradict the inequality \( v_A(i_k + 1, k - i_k) > v_A(i_k, k - i_k + 1) \). Therefore it can only be the case that \( v_A(i_k + 1, k - i_k) > 0 \), \( v_B(i_k + 1, k - i_k) = 0 \), \( v_B(i_k, k - i_k + 1) > 0 \), \( v_A(i_k, k - i_k + 1) = 0 \) which implies \( z_A(i_k + 1, k - i_k) > z_B(i_k + 1, k - i_k) \) and \( z_B(i_k, k - i_k + 1) > z_A(i_k, k - i_k + 1) \). By definition of the equilibrium winning probabilities, this implies \( \mu_{(i_k, k - i_k)} = p_A(i_k + 1, k - i_k) - p_A(i_k, k - i_k + 1) \) and \( \delta \).

We now prove \((i)\). Let’s consider the situation where \( Z_B < Z_A \), with \( Z_B \) close enough from \( Z_A \) such that \( Z_B > \frac{\delta + \delta^2 - \delta^3}{\delta^2 + \delta^3 + \delta^4} Z_A \). This condition implies \( \delta Z_B > \delta^3 Z_A \). From, Konrad and Kovenock (2005)’s Proposition 1 gives the equilibrium continuation values, in that case in \((3, 2), (2, 2)\) and \((2, 3)\).

\[
\begin{align*}
\left\{ \begin{array}{l}
v_A(4, 2) = Z_A \\
v_A(3, 2) = \delta Z_A \\
v_A(2, 2) = \frac{1}{1 - \delta^2} (\delta^2 Z_A - \delta^3 Z_A) \\
v_A(2, 3) = v_A(2, 4) = 0.
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{l}
v_B(4, 2) = v_B(3, 2) = v_B(2, 2) = 0 \\
v_B(2, 3) = \frac{1}{1 - \delta^2} (\delta Z_B - \delta^3 Z_A) \\
v_B(2, 4) = Z_B.
\end{array} \right.
\end{align*}
\]

From these continuation values we can compute the prize in the states of \( \Sigma(k), (1,3), (2,2), (3,1) \).

---

29 Using the notation of Konrad and Kovenock (2005)’s m=4 and \( j_0 = 3 \).
\[
\begin{aligned}
  z_A(3,1) &= \delta v_A(4,1) - \delta v_A(3,2) + \Delta = \delta Z_A - \delta^2 Z_A + \Delta > \Delta \\
  z_B(3,1) &= \delta v_B(3,2) - \delta v_B(4,1) + \Delta = \Delta.
\end{aligned}
\]
\[
\begin{aligned}
  z_A(2,2) &= \delta v_A(3,2) - \delta v_A(2,3) + \Delta = \delta^2 Z_A + \Delta > \Delta \\
  z_B(2,2) &= \delta v_B(2,3) - \delta v_B(3,2) + \Delta = \delta Z_B - \delta^3 Z_A + \Delta > \Delta.
\end{aligned}
\]
\[
\begin{aligned}
  z_A(1,3) &= \delta v_A(2,3) - \delta v_A(1,4) + \Delta = \Delta \\
  z_B(1,3) &= \delta v_B(1,4) - \delta v_B(2,3) + \Delta = \delta Z_B - \delta Z_A + \Delta > \Delta.
\end{aligned}
\]

In that case, there is therefore a unique tipping state \((i_4, 4-i_4)\) in \(\Sigma(4)\) such that \(z_A(i_4, 4-i_4) > \Delta\) and \(z_B(i_4, 4-i_4) > \Delta\). This state is \((2,2)\) with \(i_4 = 2\).

Using these prizes and continuation values, one can calculate the prizes for states in \(\Sigma(3)\). Doing so, one finds a unique tipping state in \((1,2)\) with prizes: \(z_A(1,2) = \delta \delta Z_A - \delta^2 Z_B + \Delta\) and \(z_B(1,2) = \delta^2 Z_B + \Delta\). Pursuing the backward induction process, one finds that for \(Z_B > \frac{\delta Z_A}{\delta^3 - \delta^3 + 1}\), then \((1,1)\) is a tipping state in \(\Sigma(2)\). This inequality is respected for the initial condition we have imposed on \(Z_B\). From \((1,1)\), by backward induction, \(Z_A > Z_B\) implies that \((0,1)\) is a tipping state in \(\Sigma(1)\). Finally, for \(Z_B\) close enough from \(Z_A\), with \(Z_B > \frac{\delta \delta Z_A - \delta^3 Z_B}{\delta^3 - \delta^3 + 1}\) \(Z_A\) (the condition we posed initially) one finds by backward induction that \((0,0)\) is also a tipping state. Figure 10 shows the range of \(\lambda = Z_B/Z_A\) satisfying this condition for different values of \(\delta\). When \(Z_B\) is close enough from \(Z_A\) such that this condition is respected, the states \((2,2), (1,2), (1,1), (0,1)\) and \((0,0)\) are tipping states. This proves the point \((\text{ii})\) of the Proposition.

Note that for lower values of \(Z_B\) relative to \(Z_A\), tipping points (and the associated momentum) would still exist in the game, they would simply tend to move to more asymmetric states where player A is trailing player B.

\textbf{Proposition 2.4.} Following from Proposition 2.2.’s proof, by backward induction, equations (2), (3) and (4) uniquely determine prizes, continuation values, and winning probabilities for every point of the game. In addition, from Hillman and Riley (1989) the agents expected sum of effort in any sub-contest \((s_A, S_B)\) is \(E(s_A, s_B) = \)

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\( \frac{1}{2} z_B \left( 1 + \frac{\eta}{z_A} \right) \). From this, we get for \( Z_A = Z_B \):

\[
\begin{align*}
E(2, 2) &= \delta^2 Z_A + \Delta \\
E(1, 1) &= \delta^3 Z_A + \Delta \\
E(0, 0) &= \delta^4 Z_A + \Delta
\end{align*}
\]

Therefore we have \( E(2, 2) > E(1, 1) > E(0, 0) > \Delta > E(s_A, s_B) \), for any asymmetric scoreline \( (s_A, s_B) \).

For \( Z_B < Z_A \), in non tipping points, the sum of effort is inferior to \( \Delta \) similarly to the case above. Let’s consider a tipping point \( (s_A, s_B) \). Suppose, that \( z_A > z_B \) with \( z_A = \eta z_B \) and \( \eta \in [0, 1] \). Let’s also write \( z_B = x + \Delta \) where \( x \) is the part of part of the sub-contest prize which stems from the grand contest incentives. We have \( E(s_A, s_B) = \frac{1}{2} (x + \Delta) (1 + \frac{1}{\eta}) \). For \( z_B \) close enough from \( z_A \), that is for \( \eta \) close enough from 1, we have \( x^{\frac{n+1}{\eta-1}} > \Delta \) which entails \( E(s_A, s_B) > \Delta \) and the effort at this tipping state is higher than at any non tipping state. The demonstration is identical in the symmetric case where \( z_A < z_B \).