TEACHING THE TWO-PERIOD CONSUMER CHOICE MODEL WITH EXCEL-SOLVER*

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ABSTRACT
This paper develops a tutorial exercise where students can solve and explore the basic two-period consumer choice model using Excel-Solver. It shows how the class exercise can be set up, how it can be used to teach comparative static analysis with an interactive diagram, and how borrowing constraints can be included. Pedagogical benefits of the approach are highlighted.

Keywords: Intertemporal consumer choice model, Excel-Solver, borrowing constraint.

JEL classifications: A22, D91, C65.

1. INTRODUCTION
Advances in information technology over the last couple of decades has been remarkable. The presence of personal computers both in domestic and professional environments, as well as their increased capacity of memory and speed of calculation, has substantially altered education capabilities. The use of computers in higher education allows part of the time that was previously devoted to the study of specific analytical procedures for solving particular problems to be saved. This time can be devoted to other aspects of the curriculum and teaching can focus on the interpretation of results rather than simply on their computation.

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In economics, students’ work in computer labs comes to largely replace traditional laboratories in other experimental disciplines. The basic function of applied sessions in these labs is to improve students’ skills of intuitive judgment without the inconvenience of long and tedious calculations. The power of lab sessions can be summarized using the words of an old Chinese proverb: I hear and I forget; I see and I remember; I do and I understand. Within the range of available resources, such as statistical packages, mathematics programs and more general software, spreadsheets have become one of the more widespread uses of personal computers in undergraduate Economics. In particular, Microsoft’s Excel Solver has been described as a user-friendly and flexible tool for economic optimization (MacDonald 1996). Since the introduction of computers in the classroom, several authors have developed Excel spreadsheets to solve economics problems (Houston 1997; Mixon & Tohamy 1999; Nævdal 2003; Holger 2004; Strulik 2004; and Gilbert & Oladi 2011).

2. BACKGROUND AND CONTEXT
As it is well known, the economist Irving Fisher developed a model that allows economists to analyze how rational, forward-looking consumers make intertemporal choices. According to the model, when people decide how much to consume and how much to save, they consider both the present and the future. The more consumption they enjoy today, the less they will be able to enjoy tomorrow. In making this tradeoff, a consumer must look ahead to the income they expect to receive in the future and to the consumption of goods they hope to be able to afford.

The two-period version of the model is generally taught at the undergraduate level. Some macroeconomics textbooks that include this model are Abel, Bernanke & Croushore (2010), Burda & Wyplosz (2009) and Mankiw (2011). In general, students learn how to solve this model analytically, which in turn requires them to use some mathematical optimization tools. In this paper we introduce a complementary tutorial exercise where students can solve the basic two-period consumer choice model using Excel-Solver. Moreover, in order to improve student understanding, we include an interactive Excel diagram alongside the Excel-Solver worksheet.

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1 The two-period consumer choice model is also taught in microeconomics at the intermediate level. See, for example, Varian (2006).
Our paper moves in the same direction as Barreto (2009) who also solves the optimal consumption choice model with Excel-Solver and provides some comparative static analysis to study the effects of a change in the interest rate on saving. Our paper, however, makes four main contributions. Firstly, we provide the instructions for solving the intertemporal consumption problem starting from an empty worksheet. Thus, once students know how to solve the benchmark model, they can modify it and include other extensions like introducing a different utility function or including taxes on saving. Secondly, we also include comparative static analysis of changes in present and future income and preferences for future consumption. The basic model assumes that the consumer can borrow as well as save, yet for many people with limited access to credit such borrowing is impossible. We thirdly, therefore, add a borrowing constraint to the problem and analyze its implications. Fourthly, we provide a worksheet that allows the student to visualize in a graph both the benchmark model and a modified parameterization of it.

The model can be taught to students with or without a knowledge of calculus. In the first case, instructors can focus the teaching session using the Excel diagram. In the second case, the exercise can provide substantial benefits by removing long and tedious calculations and by providing a visual representation of comparative static analysis. In both cases, the exercise allows students to see how changes in the parameters of the model such as income and the interest rate affect consumption, saving and the consumer’s lifetime utility. The tutorial can be done in about one hour. Finally, after completing the tutorial, students can use the worksheet to do comparative static analysis on their own, improving their learning of key concepts in a dynamic and interactive way.

3. THE TWO-PERIOD MODEL

The model is taken from the fifth edition of Mankiw’s macroeconomics textbook (2003, chapter 16). The intertemporal choice model includes the consumer constraints, his preferences, and how these constraints and preferences together determine his choices about intertemporal consumption and saving. It is assumed that the consumer lives only two periods. He is young in period one and old in the second period. The consumer earns income $Y_1$ and consumes $C_1$ in period one, and earns income $Y_2$ and consumes $C_2$ in period two. Moreover, the consumer has the opportunity to borrow or save in the first period to accomplish his
Teaching the Two-Period Consumer Model

consumption purposes. Thus, consumption in one of the periods can be either greater or less than income in that period. In the first period, consumption equals income minus saving, that is:

\[ C_1 = Y_1 - S \]  

where \( S \) is saving. In the second period, consumption equals the second-period income plus the accumulated saving, including the interest earned on that saving. That is:

\[ C_2 = Y_2 + (1 + r)S \]  

where \( r \) is the real interest rate. Note that the variable \( S \) can represent either saving or borrowing and that the equations hold in both cases. If first-period consumption is less than first-period income, the consumer is saving, and \( S \) is greater than zero. If first-period consumption exceeds first-period income, the consumer is borrowing, and \( S \) is less than zero. For simplicity, we assume that the interest rate for borrowing is the same as the interest rate for saving.

Isolating \( S \) in constraint (1) and substituting it into (2) gives the following intertemporal budget constraint:

\[ C_1 + C_2/(1 + r) = Y_1 + Y_2/(1 + r) \]  

This implies that the present discounted value of consumption, that is the sum of today’s and tomorrow’s consumption valued in terms of goods today, must equal the present discounted value of the income earned.

The consumer’s lifetime utility regarding consumption in the two periods can be represented by the following equation,

\[ U(C_1, C_2) = \ln(C_1) + \beta \ln(C_2) \]  

where \( \beta \) is between zero and one and measures the consumer’s degree of impatience for consumption during the first period. When it is close to one, utility derived from a unit of consumption in period 2 is almost equal to the utility derived from a unit in period one, and therefore the degree of impatience is low. On the contrary, when it is close to zero, the utility derived from \( C_1 \) has more weight, which implies that the consumer is more impatient.

After defining the consumer’s intertemporal budget constraint and utility, we can consider the decision about how much he should
consume. The consumer would like to end up with the best possible combination of consumption in the two periods—that is, on the highest possible level of utility. To do that the consumer must choose the two levels of consumption that maximize his utility subject to the intertemporal budget constraint:

$$\max_{C_1, C_2} U(C_1, C_2) = \ln(C_1) + \beta \ln(C_2)$$

subject to

$$C_1 + C_2/(1 + r) = Y_1 + Y_2/(1 + r)$$

In general, introductory macroeconomics text books discuss the optimal condition for this problem diagrammatically as shown in Figure 1. This indicates that the optimal allocation of consumption (C₁, C₂) lies on the budget constraint at the point where it just touches the highest possible indifference curve.

Figure 1: The Consumer’s Intertemporal Consumption Problem

In more advanced undergraduate macroeconomics courses, students also learn to solve this problem using analytical tools such as the Lagrange function:
\[ L(C_1, C_2, \lambda) = \ln(C_1) + \beta \ln(C_2) \]
\[-\lambda[C_1 + C_2/(1+r) - Y_1 + Y_2/(1+r)] \quad (6)\]

where the first order conditions for a maximum are:

\[ \frac{\partial L}{\partial C_1} = 1/C_1 - \lambda \quad (7) \]
\[ \frac{\partial L}{\partial C_2} = \beta /C_2 - \lambda/(1 + r) \quad (8) \]
\[ \frac{\partial L}{\partial \lambda} = -C_1 - C_2/(1 + r) + Y_1 + Y_2/(1 + r) \quad (9) \]

Finally, using conditions (7) to (9), the Euler condition for optimality can be obtained as follows:

\[ C_2/\beta C_1 = 1 + r \quad (10) \]

This shows that the consumer chooses consumption in the two periods so that the marginal rate of substitution (or the slope of the indifference curve in Figure 1) equals the marginal rate of substitution plus the real interest rate (the slope of the budget line in the same figure).

4. OPTIMIZATION WITH EXCEL-SOLVER

With the help of Excel-Solver we can introduce a complementary classroom exercise that solves (5) for the best combination of consumption in the two periods that the consumer can afford. To do this, we use Solver’s Generalized Reduced Gradient Nonlinear Optimization Method (GRG Nonlinear). We provide instructors and students with an Excel worksheet that will be progressively modified as the exercise develops. This worksheet contains the basic set-up for doing comparative statics and interactive graphical analysis and is shown in Figure 2. First, it is necessary to find the initial solution of the two-period model which we do using Table 1 as shown in Figure 2. We can then undertake some comparative static analysis which is shown in Table 2 which appears in Figure 3.

(a) Initial Solution

Table 1, shown in Figure 2, sets up the initial optimization problem of the representative consumer. Rows 5 to 8 in column C include standard parameter values that we introduce. The rate of time preference, \( \beta \), is
0.85; the interest rate is 0.25; income \( Y_1 \) is 1000 and \( Y_2 \) is 2000. To make the analysis simple, we assume that all variables are real. That is, they are adjusted for inflation. We introduce the initial values to the present and future consumption in rows 11 to 12 of column C (we set them at 500 units but this is arbitrary). The utility function is introduced in cell C21 while the intertemporal budget constraint and the value of saving are included in rows 15 and 18 of column C, respectively.

Now we are ready to use Solver. Choose Solver from the Data menu in Excel 2010.\(^2\) The Solver Parameters window will open. Set the Objective Cell C21 to the location of the objective function value which is the utility function in our case, select Max, and set the Changing Variable Cells C11 and C12 to the locations of the decision variables \( C_1 \) and \( C_2 \) in Table 1. Now we need to introduce the constraint (3). Go to the Subject to Constraints box and select Add. The Add Constraint window will appear. In this window, we tell the solver that cell C15 must be equal to 0. Then select OK since there are no more constraints to add. You will return to the Solver Parameters window as shown in Figure 2. Finally, introduce the value for saving in Cell C18.

At this point, we have defined all the necessary components to solve the model. In the Solver Parameters window click Solve. A window will appear telling us that Solver has found a solution. Select Keep Solver Solution and click OK. We just solved the consumer optimization problem as shown in Table 1 of Figure 3. As you can see, the consumer has maximized his utility by consuming 1405 units when he is young and 1493 units during his old age. Since his income is lower than his consumption during the first period, the consumer borrows 405 units. Also notice that the value of the cell C15 is equal to zero, implying that the consumer satisfies the intertemporal budget constraint. There is also an Excel graph that shows the initial solution. Both, the initial budget constraint and the initial utility curve appear in continuous lines.

Comparative Static Analysis

Now, we can perform some comparative static analysis by modifying the parameters of the model. To do this, we first copy the values of the parameters as well as the initial solution of \( C_1 \) and \( C_2 \) in Table 2. We

also need to copy the utility function, the intertemporal restriction and the expression of saving from Table 1 to Table 2 as it appears in Figure 4.

*Higher Present Income, \( Y_1 \), and Future Income, \( Y_2 \)*

Now we can start by modifying present income \( Y_1 \). In this case, we need to include the new value of the present income parameter in the
corresponding cell, in this case F6, and call the solver again. Set the Objective Cell F26 to the location of the objective function value, and set the Changing Variable Cells F11 and F12 to the locations of the decision variables $C_1$ and $C_2$ in Table 2. Go to the Subject to Constraints box and select Change. The Change Constraint window will appear. In this window, we tell the solver that cell F15 must be equal to 0. Then select OK since there are no more constraints to add. You will return to the Solver Parameters window as shown in Figure 5.

In Figure 5 we can see how the increase in first period income ($Y_1$) from 1000 to 1500 increases the consumption in the first period from 1405 units in Table 1 to 1676 in Table 2, while the consumption in the second period is increased from 1493 to 1780, respectively. As a result, the level of utility increases from 13.460 to 13.786 units. The Excel graph shows both the final budget constraint and utility function in dashed lines that can be compared with the initial solution shown in continuous lines. Notice that, in this case, the individual borrows 176 instead of 405 and still pays an interest rate, $r$, of 25%. Thus, a higher current income increases saving or, equivalently, reduces borrowing.

Similar comparative static analysis can be done by modifying the rest of the model’s parameters and comparing the new solution with the initial one that appears in Table 1. For example the effect of a higher future income on consumption and saving is shown in Figure 6. In this case, we first restore $Y_1$ to its initial value 1000 in Cell F6 and then increase $Y_2$ from 2000 to 2500 in cell F7. Similarly to the case of an increase in the current income, a higher future income increases both
present and future consumption. In this case, $C_1$ increases from 1405 to 1522 while $C_2$ increases from 1493 to 1723, which implies an increase in the utility level from 13.460 to 13.725 units. Current consumption increases because, given the interest rate, $r$, is 0.25, the individual borrows more money (622 instead of 405 in the initial solution). Thus, higher future income decreases saving or equivalently, increases borrowing.

**Higher Interest Rate, $r$**

Figure 7 shows that an increase in the interest rate from 25% to 50% reduces the consumption in the first period from 1405 units in Table 1 to 1261 units in Table 2, while the consumption in the second period is increased from 1493 to 1608, respectively. Since the consumer is a borrower and borrowing money becomes more expensive, the consumer reduces it, which in turn reduces $C_1$. As a result, the level of utility falls from 13.460 to 13.415 units because the consumer is less inclined to defer consumption to his old age.

**Lower Preferences for Future Consumption, $\beta$**

Figure 8 shows what happens when the individual reduces his preferences for future consumption. In this case, only the shape of the indifference curve is changed while the budget constraint remains unchanged. As expected, in the new optimal point, future consumption is reduced from 1493 to 1083 while $C_1$ increases from 1405 to 1733 financed with an extra borrowing of 328 monetary units. Total utility falls from 13.460 to 10.952.
Figure 6: The Effects of an Increase in Second Period Income

(b) The Borrowing Constraint

Until now we have assumed that the consumer can borrow as well as save. The ability to borrow allows current consumption to exceed current income. When the consumer borrows, he consumes some of his future income today. Yet for many people such borrowing is impossible. For example, an unemployed person wishing to buy a car would probably be unable to finance this consumption with a bank loan. Let’s, therefore, solve the model under a situation where the consumer cannot borrow. The inability to borrow prevents current consumption
Figure 8: Effects of an Increase in Preference for Current Consumption

from exceeding current income. A constraint on borrowing can therefore be expressed as:

\[ S = Y_1 - C_1 \geq 0 \]  

(11)

This inequality states that consumption for the consumer must be less than or equal to his income (period one). This additional constraint on the consumer is called a borrowing constraint or, sometimes, a liquidity constraint. We next introduce this constraint into the optimization problem (5):

\[
\max_{C_1, C_2} U(C_1, C_2) = \ln(C_1) + \beta \ln(C_2) \\
\text{subject to} \\
C_1 + C_2/(1 + r) = Y_1 + Y_2/(1 + r) \quad \text{and} \\
S = Y_1 - C_1 \geq 0
\]  

(12)

Solving the model analytically under the scenario of borrowing constraint is not straightforward as in (5). In this case, more advanced methods may be required to solve this problem. Fortunately, Excel-Solver can solve the problem by introducing a new restriction to the optimal problem.

In excel, we say “Yes” to the borrowing constraint condition in cell F20 and introduce this new restriction in cell F23 of Table 2. Then, we add this constraint in Excel-Solver by Clicking on the Subject to
Figure 9: Setting Up the Borrowing Constraint Problem

Figure 10: Solving the Borrowing Constraint Problem

Constraints box and select Add. The Add Constraint window will appear. In this window, we tell the solver that cell F23 must be higher than or equal to 0 and select OK (there are no more constraints to add). It will return to the Solver Parameters as in Figure 9. In the Solver Parameters window click Solve. A window will appear telling us that Solver has found a solution. Select Keep Solver Solution and click OK.
We have solved the consumer optimization problem under a *borrowing constraint* as shown in Figure 10. Since the borrowing constraint is binding in this case, first-period consumption equals first-period income. The level of utility is reduced from 13.415 to 13.369 units because the consumer would like to borrow to consume more in the first period (as Table 1 shows) but he is not able to do it. Finally, the *Excel* graph shows that there is a corner solution. Notice that, the final budget constraint (dashed line) indicates that $C_1$ cannot be higher than 1000 units.

### 5. CONCLUSION

The accessibility and flexibility of *Excel* spreadsheets gives economics instructors a great tool to complement the traditional analysis of economic problems. With this tool, applications that avoid long and tedious calculations can be easily designed to enhance meaningful learning. In this paper we have shown how to solve the basic two-period consumer choice model using a spreadsheet and *Excel-Solver* and illustrated the problem using an example from Mankiw’s macroeconomics textbook. While completing the proposed exercise, students can explore the main features of the model and learn key concepts in a more dynamic and interactive way.

### REFERENCES


