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Should We Teach Microeconomics Before Macroeconomics? Evidence from An Australian University  Muni Perumal

Teaching Aghion and Howitt’s Model of Schumpeterian Growth to Graduate Students: A Diagrammatic Approach  Mario da Silva

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AUSTRALASIAN JOURNAL OF ECONOMICS EDUCATION

MISSION STATEMENT

The Australasian Journal of Economics Education is a peer-reviewed journal that publishes papers on all aspects of economics education. With a view to fostering scholarship in the teaching and learning of economics, it provides a forum for publishing high quality papers and seeks to bring the results to a widening audience. Given both the increasing diversity of the student clientele, and increasing calls for greater attention to the quality of tertiary teaching, this Journal seeks to foster debate on such issues as teaching techniques, innovations in the teaching of economics, student responses to such teaching, and the incentive systems which influence the academic teaching environment. The AJEE is interested in research involving both quantitative and qualitative analyses and also in interpretative analyses based on case studies. While the Journal is Australasian-focused, it encourages contributions from other countries in order to promote an international perspective on the issues that confront the economics discipline. AJEE aspires to:

1. Report research on the teaching of economics, and cultivate heightened interest in the teaching of economics and the scholarship of teaching.

Pedagogical issues will be a central feature, and will encompass work on the teaching of economics in diverse contexts, including large and small classes, undergraduate and postgraduate classes, distance learning, issues confronting foreign students on-shore and off-shore, and issues related to the teaching of fee-paying MBA and other postgraduate groups from diverse disciplinary backgrounds. Though economics is the prime focus, consideration will also be given to work on other subjects that have a demonstrated relevance for the teaching of economics.

Such issues will also involve evolutionary issues in the teaching of economics, in terms both of effective ways to teach evolving theory and of evolving technology with which to teach that theory (including on-line teaching).

Recognition will be given to the fact that economics as a discipline has not fared well in CEQ results (course experience questionnaire
results) since the reporting of those results began in Australia. Nor has economics teaching typically been well received in the USA or UK, according to survey evidence. In that context the relevance to teaching of changing administrative arrangements in universities will also be highlighted (eg in terms of contemporary quality assurance procedures and other government policy changes in Australia and New Zealand).

2. Report research on the nexus between teaching and research (including research on the diverse, changing and potentially conflicting incentives within the academic industry). Papers exploring the extent to which research and teaching activities are complementary or competitive will be welcomed.

3. Recognise the relevance of some more deep-seated implicit assumptions and issues of economic philosophy embedded in what is commonly taught, (as in Sen’s work on economics and ethics, for example). Inter alia, the question arises as to the way in which students respond to economics taught as a path to scientific certainty, as against economics taught as reflecting unsettled debate and vigorous controversy.

4. Recognise the place of history in the teaching of economics. Both HET and economic history tend to play a diminishing role in professional economics training, as emphasis on technique dominates. This a-historical approach to the teaching of economics has been criticised by many influential economists (including Joan Robinson, Leontief, Myrdal, Colander, and Robert Clower in his acerbic remarks about the value of much that is published in such prestigious journals as the AER). This line of criticism has been continued in the recent growth of heterodox economics associations in a number of countries (including one for Australia and New Zealand) and on the web through the Post Autistic Economics (PAE) newsletter. Historical and institutional factors will thus provide one focal interest.

5. Recognise interdisciplinary issues important to the presentation of economics in various contexts. On the one hand, economics students are not systematically exposed to the insights of other social sciences and the conformity or otherwise of their conclusions with those of economics. On the other hand, other disciplines within the social sciences and humanities (e.g. the Social Work profession) do not always include even an introduction to economics for their students, notwithstanding that economic issues are often very important
determinants of the environment within which they operate. More fundamentally, questions arise as to whether social science is more than the sum of its respective parts, and as to whether the roots of economics can be fully understood in isolation from the history not only of economics but also of politics and philosophy.

6. Establish a link to the teaching of economics in the secondary schools, given that tertiary enrolments in economics reflect fluctuating enrolments in economics in the secondary schools.

7. Encourage on-going surveys of student response to the teaching of economics across Australasian (and other) institutions, including response to experimental teaching and to differences between institutional approaches. (c.f. Colander and Klamer’s 1988 survey of economics students at USA ivy league institutions.)

8. Monitor trends in the teaching of economics both globally and in the Australian and New Zealand university systems (such as enrolments, staff-student ratios, international-domestic student ratios, offshore offerings etc), and the implications of those trends for various funding arrangements.

9. Promote a series of papers on specialised themes within the overall province of the teaching of economics e.g. on the teaching of Principles courses, the teaching of History of Economic Thought, the teaching of intermediate microeconomics and macroeconomics, the teaching of development economics, and likewise regarding teaching in such streams as Quantitative Methods, large first year classes, non-English speaking background students, the teaching of economics to non-economists, product differentiation in teaching economics, and professional education in economics in executive education programs outside conventional university contexts.

10. Monitor the measuring and rewarding of quality (economics) teaching within Australasian universities.
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SHOULD WE TEACH MICROECONOMICS BEFORE MACROECONOMICS? EVIDENCE FROM AN AUSTRALIAN UNIVERSITY*

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University of Canberra

ABSTRACT

Using a sample of 405 students from the Faculty of Business, Government and Law at the University of Canberra, this study compared the effects of how microeconomics and macroeconomics are sequenced on student grades for three different groups. The group using the macro/micro sequence improved their grade by around 1 to 27%, or an average of 7% in microeconomics, while both the micro/macro sequence group and those who studied micro and macro concurrently did not show any significant improvement in grades. These findings support the less conventional view that optimal sequencing in introductory economics requires macroeconomics to be taught before microeconomics. It was also found, consistent with other studies in Australia, that other major determinants of student performance in first year economics principles classes are UAI, age, major and mathematics background.

Keywords: micro-macro sequencing, determinants of grades, curriculum design.

JEL classifications: A22, B41

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1. INTRODUCTION
The issue of whether microeconomics should be taught before macroeconomics in first year economic principles courses has been a subject of much debate among economists for many years. No consensus yet exists about the optimal sequencing of economic principles courses (Lopus & Maxwell 1995; Terry & Galchus 2003). Most universities in Australia and abroad require students to do microeconomic principles prior to macroeconomics principles in their first year economics courses. Some universities, however, have taught macroeconomics first, while others do not specify the micro-macro sequence and allow students to enrol either in microeconomics or macroeconomics principles first. Very little research exists to assess whether student learning is enhanced by the order in which micro and macro principles are taught. The results of the few available studies provide conflicting conclusions about optimal sequencing (Terry & Galchus 2003). To my knowledge, no study of this kind has been undertaken in Australia.

The purpose of this paper is to provide empirical evidence from one Australian university on the effects of sequencing on students’ grades, and thus to add further evidence to the existing pile of mixed evidence on whether microeconomics is better taught before macroeconomics in first year principles courses. The findings relate to a sample of 405 students enrolled in first year economic principles courses between 2001 and 2004 in the Faculty of Business, Government and Law at the University of Canberra. As with several previous studies, the paper also investigates other determinants of student performance in first year economic principles courses.

The paper is structured as follows. The next section reviews the available studies and their findings. In Section 3, the data and model used in this study are discussed. The results are presented in Section 4, while the final section summarises the conclusions of the study.

2. PREVIOUS RESEARCH
One early study on whether sequencing affects student performance was Fizel & Johnson (1986). They found that “a micro/macro sequence of introductory economics will produce a better understanding of economics than would a macro/micro sequence” (p.94). On average, they found that micro/macro students did anywhere from 10 to 50 per cent better than macro/micro students.
A later study by Lopus & Maxwell (1995) found that students learn more in principles of microeconomics after taking a course in macroeconomics. However, students do not learn more in principles of macroeconomics after taking a course in microeconomics. As the authors note, “This implies that, ceteris paribus, principles of macroeconomics should be taught before principles of microeconomics for optimal student learning” (p. 336).

More recent research on this topic is by Terry & Galchus (2003). Using a sample of 870 students in the College of Business at the University of Arkansas at Little Rock, they looked at the question of whether, and to what extent, macro-micro course sequencing affects student performance in principles of economics. They found that optimal sequencing involves having students take the two principles of economics classes concurrently. They also found that grade point average (GPA), major, and to some extent, ethnicity and gender are also significantly related to performance in one or both of the principles of economics classes.

As discussed above, these three studies resulted in three different conclusions regarding the best sequencing of micro and macro in first year economic principles courses. It was the inconclusive nature of these studies which prompted the investigation of this important question by using data collected at one Australian university. Economic principles courses taught at most Australian universities in first year serve two main groups – students taking economics principles as a terminating course in economics, and students who are prospective majors in economics. Informal feedback obtained from students at the University of Canberra suggests that students generally prefer the study of macroeconomics to that of microeconomics because the former helps them better understand what is going on in the economy on a daily basis as portrayed in the news and other media. One other compelling reason for teaching macroeconomics before microeconomics in Australian contexts concerns the composition of students enrolling in first year economics. The majority of such students are non-economics majors who often study no more than one or two compulsory economic principles courses offered in the first year of their undergraduate study. Their macroeconomics courses, dealing with such topics as unemployment, inflation, fiscal policy, and monetary policy, have relevance to everyday living and as such are more likely to engage students in their
study of economic principles. Further, as argued below, there is no *definitive* case for the superiority of requiring students to learn micro principles before embarking on macro principles.

About 40% of the highly ranked U.S. universities surveyed by Lopus & Maxwell (1995) did not specify an ordering for micro/macro principles, while another 40% required micro first. About 4% required macro to be done first and the remaining 16% did not split principles courses into micro and macro components. An equally important observation concerns the textbook sequencing of microeconomics and macroeconomics. In 1995, Lopus and Maxwell found the majority of introductory economics textbooks in the U.S. presented macroeconomics first followed by microeconomics. However, most of the current introductory economics textbooks in the US follow a micro-macro sequencing. In the case of Australia, information gathered from the respective websites of the 37 public universities as of September 2012, and from telephone clarification where the available information was unclear, indicates that currently about 43% of Australian universities follow the micro-macro sequence, 3% (1 only) requires macro to be done first, and another 14% did not specify an ordering for micro and macro, so implying that the subjects can be done in any sequence. In 35% (or 13) of the Australian universities, micro and macro principles are combined and taught as a one semester course under various names, such as Principles of Economics, Economics for Business, and Introduction to Economics. Two universities did not offer any majors program in microeconomics and macroeconomics. As to the textbooks, every major text on introductory economics in Australia, such as McTaggart *et al.* (2005) or Jackson *et al.* (2007), present microeconomics first followed by macroeconomics.

Every year, several thousand students enrol in first year economic principles courses in universities across Australia. Only a small fraction of these students go on to degrees or majors in economics. There must be some good reason as to why many students shy away from studying economics after their first year. According to Alauddin & Valadkhani (2003), one reason for the continuous decline in enrolment in economics is the inappropriateness of the product to an increasingly diverse clientele. Millmow (2000) points out that falling enrolment in economics were not unique to Australia and that similar trends have been observed in the United Kingdom and the United
States. Across the world, economists have noted students’ relative lack of interest in pursuing further study in economics. In particular, “the trend in the proportion of U.S. bachelor’s degrees awarded in economics has been negative since the 1950s, with a steep decline following a relative cyclical high in 1988” (Becker 2004, p.1). According to Becker (2001), one way to make economics more popular and reverse this declining trend is to make economics the “sexy” social science through more careful selection of content and utilising teaching methods which are more engaging. We have to convince our students that economics is not only a useful but also a fulfilling social science (Hoyt 2003). Sequencing of principles courses by teaching macroeconomics first followed by microeconomics may reduce some the declining enrolment in economics by teaching content which is more appropriate and relevant to the daily life of students, thereby keeping them motivated and building their interest in the subject.

One reason cited by many economists for micro/macro sequencing is the body of literature which emphasises that macroeconomics principles rest entirely on microeconomic foundations. This is evidenced by the numerous books and scholarly papers on this subject (see, for example, Weintraub 1979). This and similar contributions imply that prior knowledge of microeconomics is essential for learning macroeconomics (Lopus & Maxwell 1995). Alternatively, it has been argued that microeconomics has foundations in macroeconomics. Colander (1993, p.451) contends that “one must first determine the macro context within which that micro decision is made” (cf. Hahn 2003). DaSilva (2009, p.1) has argued also that “it is microeconomics that needs foundations, not macroeconomics”, and King (2012) has presented an extensive critique of the micro-foundations approach. These views suggest that there are good grounds for teaching macroeconomics principles first, especially in first year economics.

The appropriate sequence of microeconomics and macroeconomics is thus a contested issue. The objective of this paper is to cast further empirical light on the issue and, in particular, on how alternative sequences affect student learning. The following section outlines the empirical approach I took to meet this objective and describes the data used as the basis for analysis.
3. MODEL AND DATA
Data for this study were obtained from a sample of 405 students who studied microeconomics principles and macroeconomics principles in their first year, between 2001 and 2004 at the University of Canberra. Unlike many other Australian universities, there was no requirement that students should enrol in microeconomics principles (Microeconomics 1) before doing macroeconomics principles (Macroeconomics 1). The two units were offered in both semesters which made it possible in any semester for students to enrol in either Microeconomics 1 or Macroeconomics 1 or both concurrently. Semester 2, 2004 was the last time when first year economics was taught as the separate units of Microeconomics 1 and Macroeconomics 1. Beginning in Semester 1 2005, the University of Canberra revised its degree structure and offered a one semester unit called Introduction to Economics which combined microeconomics and macroeconomics principles in a single foundation unit in economics principles.

Prior to 2005, the first year principles courses were compulsory for all degree programs in economics, commerce, management, marketing and the social sciences, and were also offered as electives in other degree programs including science, information technology and engineering. Microeconomics 1 and Macroeconomics 1 were taught by the same academic staff members during the entire period (2001-04) which ruled out one source of variation in the data used for comparison. Records of both the letter grades and individual scores were obtained for the sample of students. Table 1 gives some characteristics of the students in the sample.

Of the 405 students, 183 students or 45.2% did micro first followed by macro, 163 students or 40.2% did macro first, and 59 or 14.6% did micro and macro concurrently in the same semester. There were roughly equal numbers of male and female students. Almost 80% of the students in the total sample were below 25 years of age, the minimum age being 18 years, the maximum 54 years and the average approximately 23 years. UAI scores were available for 342 students and the average UAI score for the group was 68.6. The majority of the students were domestic (89.1%), with the remaining 10.9% being full-fee paying international students. From Table 1 it is evident that the outcomes of the grading system used by the two lecturers teaching Microeconomics 1 and Macroeconomics 1 were roughly similar as
### Table 1: Characteristics of Students in the Sample

<table>
<thead>
<tr>
<th></th>
<th>Macro First</th>
<th>Macro/Micro Concurrent</th>
<th>Micro First</th>
<th>Total (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>163</td>
<td>59</td>
<td>183</td>
<td>405 (100.0)</td>
</tr>
<tr>
<td>Gender:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>71</td>
<td>33</td>
<td>93</td>
<td>197 (48.6)</td>
</tr>
<tr>
<td>Female</td>
<td>92</td>
<td>26</td>
<td>90</td>
<td>208 (52.4)</td>
</tr>
<tr>
<td>Fee Type:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic</td>
<td>148</td>
<td>53</td>
<td>160</td>
<td>361 (89.1)</td>
</tr>
<tr>
<td>International</td>
<td>15</td>
<td>6</td>
<td>23</td>
<td>44 (10.9)</td>
</tr>
<tr>
<td>Grade Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HD</td>
<td>4</td>
<td>2</td>
<td>9</td>
<td>15 (3.7)</td>
</tr>
<tr>
<td>D</td>
<td>18</td>
<td>8</td>
<td>23</td>
<td>49 (12.1)</td>
</tr>
<tr>
<td>CR</td>
<td>36</td>
<td>12</td>
<td>40</td>
<td>88 (21.7)</td>
</tr>
<tr>
<td>P</td>
<td>62</td>
<td>21</td>
<td>77</td>
<td>160 (39.5)</td>
</tr>
<tr>
<td>NX</td>
<td>41</td>
<td>16</td>
<td>34</td>
<td>91 (22.5)</td>
</tr>
<tr>
<td>Other</td>
<td>2</td>
<td></td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>Age when Doing Macro</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below 20</td>
<td>51</td>
<td>9</td>
<td>21</td>
<td>81 (20.2)</td>
</tr>
<tr>
<td>20-25</td>
<td>87</td>
<td>36</td>
<td>118</td>
<td>241 (59.5)</td>
</tr>
<tr>
<td>26-30</td>
<td>15</td>
<td>8</td>
<td>23</td>
<td>46 (11.4)</td>
</tr>
<tr>
<td>31-35</td>
<td>5</td>
<td>3</td>
<td>9</td>
<td>17 (4.2)</td>
</tr>
<tr>
<td>36-40</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>11 (2.7)</td>
</tr>
<tr>
<td>Above 40</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>9 (2.2)</td>
</tr>
<tr>
<td>Distribution (Macro)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HD</td>
<td>4</td>
<td>2</td>
<td>21</td>
<td>15 (3.7)</td>
</tr>
<tr>
<td>D</td>
<td>18</td>
<td>8</td>
<td>118</td>
<td>49 (12.1)</td>
</tr>
<tr>
<td>CR</td>
<td>36</td>
<td>12</td>
<td>23</td>
<td>88 (21.7)</td>
</tr>
<tr>
<td>P</td>
<td>62</td>
<td>21</td>
<td>9</td>
<td>160 (39.5)</td>
</tr>
<tr>
<td>NX</td>
<td>41</td>
<td>16</td>
<td>5</td>
<td>91 (22.5)</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>2 (0.5)</td>
</tr>
<tr>
<td>Distribution (Micro)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HD</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>15 (4.2)</td>
</tr>
<tr>
<td>D</td>
<td>19</td>
<td>8</td>
<td>24</td>
<td>51 (14.6)</td>
</tr>
<tr>
<td>CR</td>
<td>26</td>
<td>9</td>
<td>33</td>
<td>68 (19.5)</td>
</tr>
<tr>
<td>P</td>
<td>33</td>
<td>23</td>
<td>95</td>
<td>151 (43.5)</td>
</tr>
<tr>
<td>NX</td>
<td>12</td>
<td>13</td>
<td>11</td>
<td>36 (10.3)</td>
</tr>
<tr>
<td>Other</td>
<td>11</td>
<td>2</td>
<td>15</td>
<td>28 (8.0)</td>
</tr>
</tbody>
</table>

Notes: HD = High distinction (85-100%); DI = Distinction (75-84%); CR = Credit (65-74%); P = Pass (64-50%); NX = Fail (Less than 49%); “Other” refers to incomplete grades.
indicated by the percentage distribution of letter grades for these two units. About 15.8% of the enrolled students were doing economics majors, including those enrolled in the Bachelor of Applied Economics and the Bachelor of Commerce in Banking and Finance. The remaining 342 students or about 84.2% were from non-economics majors, the majority of whom were doing the Bachelor of Commerce majoring in Accounting, followed by students majoring in marketing, advertising and communication studies.

The aim of the study was to measure the impact of micro/macro course sequencing on students’ ability to learn economic principles. Student marks or grades were used as a proxy for their underlying knowledge of economics. A paired value $t$-test was used to determine whether studying microeconomics principles first will improve students’ ability to understand macroeconomics principles (as indicated by their grades in macro principles). It is hypothesised that micro/macro sequencing does not matter, so that doing microeconomics first will not improve the grade in macroeconomics, and vice versa. The null hypothesis was that there was no significant difference in the mean marks of the three groups of students in the study. In the first group, students followed the macro/micro sequence, while in the second group, students studied micro and macro principles concurrently in one semester. The third group of students followed traditional micro/macro sequencing, doing macro principles in the following or later semesters.

Ordinary Least Squares (OLS) regression models were used to determine the main factors influencing the performance of students. Following Mallik & Varua (2008), Nolan & Ahmadi-Esfahani (2007) and others, it was hypothesised that UAI (University Admission Index), Gender, Age, Mathematics background (College or HSC), Economics Major/Non-Economics Major, and Attendance (part-time versus full time study) were significant variables in explaining overall performances in macro and micro principles courses. The dependent variable was the overall mark (score) obtained by students in Microeconomics 1 or Macroeconomics 1. Several dummy variables were used to determine the effects of categorical predictors. The following regression model was thus estimated for the three groups in the study:

$$ GRADE_i = \beta_0 + \beta_1 UAI_i + \beta_2 GENDER_i + \beta_3 AGE_i + \beta_4 MATHS_i + \beta_5 MAJOR_i + \beta_6 ATTEND_i + \varepsilon_i $$ (1)
where $\text{GRADE}_i$ represents the overall mark (score) in percentiles of the $i$th students in Microeconomics 1 or Macroeconomics 1; $\text{UAI}_i$ represents the University Admission Index Score obtained from University admission records; $\text{GENDER}_i$ is a dummy variable for gender, 1 for male and 0 for female; $\text{AGE}_i$ is the age in years when the $i$th student enrolled in the principles course; $\text{MATHS}_i$ is a dummy variable which takes a value of 1 for students who have done college level Mathematics and 0 for students who have not done any Mathematics in College or in the HSC examination; $\text{MAJOR}_i$ is a dummy variable which takes a value of 1 for students who were doing commerce related types of degree courses which require further studies in economics including economics major and 0 for non-commerce degree programs; and $\text{ATTEND}_i$ is a dummy variable for enrolment type which takes a value of 1 for fulltime study and 0 for part-time study.

4. RESULTS AND DISCUSSION
From the original sample of 405 students, the number of paired samples obtained for the three groups are shown in column 4 of Table 2. It is of interest to note that of the 183 students that did micro first (Table 1), only 112 subsequently enrolled in macroeconomics, so permitting paired samples for the t-test. The use of a paired t-test is appropriate in this instance as the pair-differences in the mean marks are continuous dependent variables, and the characteristics of the population (the micro and macro groups) are likely to be similar as they constitute the first year cohort of students enrolled in first year economics. Table 2 also shows the results of the t-test to determine whether student learning is affected by sequencing. In particular, do students learn more in macroeconomics after doing a course in microeconomics, or do they learn more in microeconomics after undertaking a course in macroeconomics? The test was significant at the 5% level only for the group which took macroeconomics first. The null hypothesis that there is no difference in the mean marks for this group is rejected, indicating that doing macroeconomics first did enhance the learning of microeconomic principles. In fact, students generally did better in microeconomics by around 1 to 27%, or an average of 7% after completing macroeconomics first. For the group which followed the traditional micro/macro sequence, the test was not significant, implying that studying microeconomics first did not
Table 2: Results of the t-test for Paired Samples

<table>
<thead>
<tr>
<th>Groups</th>
<th>Mean Marks in Macro</th>
<th>Mean Marks in Micro</th>
<th>Sample (N)</th>
<th>t value</th>
<th>Sig (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macro First</td>
<td>60.03</td>
<td>62.32</td>
<td>93</td>
<td>-2.038*</td>
<td>0.044</td>
</tr>
<tr>
<td>Concurrent</td>
<td>58.65</td>
<td>58.93</td>
<td>41</td>
<td>-0.170</td>
<td>0.866</td>
</tr>
<tr>
<td>Micro First</td>
<td>59.00</td>
<td>59.24</td>
<td>112</td>
<td>-0.270</td>
<td>0.878</td>
</tr>
</tbody>
</table>

Note: * indicates significance at the 5% level.

improve the subsequent learning of macroeconomic principles (using grades as a proxy for students’ knowledge). The test was also not significant for the group which did micro and macro concurrently. Somewhat surprisingly for the latter two groups, the mean scores in microeconomics and macroeconomics were almost identical.

These results are consistent with the findings of Lopus and Maxwell (1995) who used U.S. data. Table 3 shows regression results using the combined student mark in macroeconomics and microeconomics as the dependent variable. The most important predictor of students’ scores in microeconomics and macroeconomics is the UAI. The UAI coefficient was significant at the 1 percent level for Group 1, and at the 5% level for Group 2. Other things being equal, students who previously studied economics in their final year (Year 12) at high school could be expected to perform better in first year economics, and to have done so irrespective of whether micro or macro was chosen first. In this study, the number of students doing economics after having completed economics in Year 12 was negligible. With regard to macroeconomics, other factors which determine students’ grades are age and mathematics background, but not gender, mode of attendance or economics major which do not seem to influence performance. On the other hand, with regard to microeconomics, only UAI and MAJOR seem to predict student results.

The significance of age and mathematics background in determining marks in Macroeconomics 1 was as expected. The Macroeconomics 1 unit taught at the University of Canberra required a fair amount of mathematical analysis involving the calculation of equilibrium GDP, derivation of the various multiplier formulae and
the application of multiplier analysis. On the other hand, the Microeconomics 1 unit contained barely any mathematical applications apart from the estimation of simple elasticities. Age as a factor in explaining variation in the marks for macroeconomics is also not unexpected, for understanding the topics covered (which ranged across inflation, fiscal and monetary policy, interest rates, exchange rates and so on) can be expected to improve with maturity of mind. The variable MAJOR was not significant in determining the grades in Macroeconomics 1, but was significant at the 10% level for Microeconomics 1. This suggests that students who performed well in microeconomics came from commerce-related degree programs which required further studies in economics. The negative sign for the AGE coefficient in Model 2 seems to indicate that younger students tend to perform better than older students in microeconomics, while AGE is a positive factor in the case of macroeconomics. In particular, the mean age of students in the macro first sample was 23.2 years, while it was

Table 3: OLS Estimates of Grade Determinants in Macro- and Microeconomics Principles Courses

<table>
<thead>
<tr>
<th>Variable</th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Macro First</td>
<td>Micro First</td>
</tr>
<tr>
<td>Constant</td>
<td>-16.353 (-1.119)</td>
<td>21.499 (1.472)</td>
</tr>
<tr>
<td>UAI</td>
<td>0.600 (4.096)*</td>
<td>0.465 (2.562)**</td>
</tr>
<tr>
<td>GENDER</td>
<td>1.851 (0.734)</td>
<td>2.769 (1.096)</td>
</tr>
<tr>
<td>AGE</td>
<td>0.922* (3.005)</td>
<td>-0.196 (-1.001)</td>
</tr>
<tr>
<td>MATHS</td>
<td>5.992 (2.001)**</td>
<td>0.809 (0.302)</td>
</tr>
<tr>
<td>MAJOR</td>
<td>-1.644 (-0.600)</td>
<td>4.667 (1.889)***</td>
</tr>
<tr>
<td>ATTEND</td>
<td>2.164 (0.734)</td>
<td>1.450 (0.529)</td>
</tr>
<tr>
<td>R</td>
<td>0.432</td>
<td>0.430</td>
</tr>
<tr>
<td>R²</td>
<td>0.186</td>
<td>0.185</td>
</tr>
<tr>
<td>F value</td>
<td>4.579*</td>
<td>2.009***</td>
</tr>
</tbody>
</table>

Note: *, ** and *** imply significance at 1%, 5% and 10% levels respectively.
22.1 years for those who did micro first. The difference in the mean age between the two groups was significant at the 5% level.

These findings concerning the ‘non-sequencing’ variables are generally consistent with other studies undertaken in Australia and elsewhere. Malik & Varua (2008) found gender was not an important factor in predicting student’s performance in Introduction to Economic Methods at the University of Western Sydney. Nolan & Ahmadi-Esfahani (2007) showed that UAI is a good predictor of students’ performance in undergraduate agricultural economics at the University of Sydney. Elzinga & Melaugh (2009), using more than 35,000 students from the University of Virginia in the US, found that SAT scores in mathematics are the best predictor of success in the principles course.

5. CONCLUSIONS

This study has examined the question of whether we should follow the traditional path taken in most Australian universities and elsewhere, and teach microeconomics before macroeconomics in first year principles courses. On the basis of this study, which used data from one Australian university and grades as a proxy for students’ knowledge, it was found that students studying in a macro-micro sequence significantly improved their understanding of microeconomic principles. On average, students who studied macro principles first, improved their score in micro principles by between 1 to 27%, or an average of 7%. This finding is consistent with the study by Lopus & Maxwell (1995) who also found that students who studied macroeconomics first improved their performance in microeconomics rather than the other way round. By contrast, students who followed the traditional micro/macro sequence and those who studied micro and macro concurrently did not show any improvement in their grade in macroeconomics. The results of this study thus support the view that optimal sequencing involves having students take macro principles first followed by micro principles. The policy implication for first year undergraduate courses is then to adopt the macro-micro sequence, which is contrary to the practice in most Australian universities.

The suggestion that we should teach macro first may appear counter-intuitive; as most economists believe that teaching micro first is theoretically or “intellectually” better. It may well be that at more advanced levels (intermediate and graduate) the transfer of economic
knowledge is best served by offering micro first (as noted by Lopus & Maxwell 1995, p.348). However, at the first year level (and in Australia at least), a good case can be made for teaching macro principles first followed by micro principles which helps explain the empirical findings of this study. Given that the majority of students enrolled in first year economics principles courses are from non-economics majors, there is a need to keep these students motivated and engaged, and also to encourage them to take further courses of study in economics (or even to switch majors). In Australia, the daily news about economic issues is most often dominated by a range of macroeconomic issues (such as inflation, GDP, interest rates, unemployment, the global financial crisis) rather than microeconomic ones (such as trade-offs or the marginal costs of firms). What students are absorbing on a daily basis from the media is thus likely to seem to them to be more relevant to their economic studies, which gives what they are learning in their macroeconomics lectures and tutorials greater significance. Becker (2004, p.2) makes a similar point in a slightly different way – “many of the things of interest to students and things that they see and hear in the popular media do not lend themselves to simple textbook supply and demand analysis”. The empirical outcomes of this study support these good qualitative reasons for adopting macro-micro sequencing in first year economics (whether this is taught in the form of separate courses or a single macro-micro course).

REFERENCES


TEACHING AGHION AND HOWITT’S MODEL OF SCHUMPETERIAN GROWTH TO GRADUATE STUDENTS: A DIAGRAMMATIC APPROACH*

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ABSTRACT

A mainstay of undergraduate courses on economic growth is an exposition of Robert Solow’s famous 1956 model. Solow’s famous diagram is an important vehicle for teaching this model in both undergraduate and early graduate courses on growth. A desire to extend Solow’s framework and to explain the determination of technical progress motivated the development of endogenous growth theory inaugurated by the seminal contribution of Paul Romer (1986). This eventually led to the Schumpeterian model of Aghion and Howit (1992) which is sufficiently complicated that it tends to be omitted from even introductory graduate courses. This paper presents a slightly simplified treatment of the Aghion-Howitt model that might be used to complement and extend treatments of growth theory in textbooks such as Jones (1998) and Romer (2012) for introductory graduate courses. The centerpiece of this treatment is a four-quadrant diagram that makes the key ideas of the model more accessible. The paper argues that diagrams and figures can be powerful pedagogical tools for teaching complex models, essentially rephrasing an old dictum as ‘one picture is worth a thousand equations’.

KEYWORDS: Endogenous growth, graduate teaching, innovation, creative destruction.

JEL classifications: A23, O31

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1. INTRODUCTION

A mainstay of undergraduate courses on economic growth is an exposition of Robert Solow’s famous 1956 model. In the Solow model, the long-run growth rate of output per worker is determined by technological progress but this variable is treated as being exogenous and is left un-modeled. A desire to extend this framework and to explain how this important driver of economic growth is determined, motivated the development of endogenous growth theory inaugurated by the seminal contribution of Paul Romer (1986).

This development occurred in three phases. In the first, so-called AK models hypothesized that high rates of growth depend upon thrift, some of which finances a higher rate of technological progress, resulting in higher growth (see Romer 1987; and Rebelo 1991). No explicit distinction was made, however, between technological progress and capital accumulation in these models. In the second phase, “innovation-based” models posited that innovation causes productivity growth by creating new varieties of intermediate goods. Here innovations do not necessarily generate better intermediate products, just more of them, and the increased use of these goods associated with their greater supply and variety leads to higher growth (see, for example, Romer 1990). In the third phase, innovation-based theory took a Schumpeterian approach. Aghion and Howitt (1992, 1998), for example, developed a model in which a version of Schumpeter’s process of creative destruction generates vertical innovations that drive the development of technological knowledge, increasing productivity, and fueling economic growth. Innovations in this model are the result of deliberate investment in research processes but because newly developed intermediate goods render existing ones obsolete, the expectation of future spending on research acts as a brake on current research spending, and firms must balance the costs and benefits of this spending.

The complexity of Aghion and Howitt’s original paper means that it tends to be excluded from undergraduate and early graduate treatments.

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1 While Robert Solow is most often remembered for his contribution to the development of modern growth theory, it should be remembered that Trevor Swan (1956) independently published a model with virtually the same structure just a few months after Solow’s paper came out. See Spencer and Dimand (2010) for a brief discussion of the relationship between these contributions.
of economic growth. Jones (1998 pp.20-45; 88-113), for example, discusses the Solow model and outlines a simplified version of Romer’s (1990) model but provides only passing reference to the Aghion-Howitt model. Romer (2012 pp.6-45; 102-145), a text used in many introductory graduate courses, follows a similar pattern although more advanced mathematics are employed.

This paper presents a slightly simplified treatment of the Aghion-Howitt model that might be used to complement and extend treatments of growth theory in textbooks such as Jones (1998) and Romer (2012) for introductory graduate courses. The centerpiece of this treatment is a four-quadrant diagram that makes the key ideas of the model more accessible. The paper argues that diagrams and figures can be powerful pedagogical tools for teaching complex models, essentially rephrasing an old dictum as ‘one picture is worth a thousand equations’.

The paper is organized as follows. Section 2 provides the educational justification for developing the four-quadrant diagram. Section 3 outlines a simplified version of Aghion and Howitt’s (1992) model of Schumpeterian growth. Section 4 presents and explains the diagram, and Section 5 contains some brief concluding remarks.

2. USING DIAGRAMS TO TEACH ECONOMICS

Ramsden (2003, p.86) argues that effective learning, and teaching that facilitates such learning, is best conceptualized as a change in the way learners understand the world around them. The ‘world around them’ includes the concepts and methods central to the students’ field of learning. Such learning results when students focus on thoroughly engaging with course material and constitutes what Ramsden (2003, pp.46-47) calls a deep approach to learning which generates high-quality, well-structured learning outcomes. Surface approaches, on the other hand, encourage students to focus on activities such as merely reproducing facts and leads at best to an ability to retain unrelated details.

Some ways of addressing complex relationships do not, therefore, help students connect with the key ideas in an area of study, according to Ramsden’s perspective. Many students would have difficulty, for example, engaging with the exposition of a complex model in algebraic or mathematical form. For Ramsden (2003, p.88-89), a key feature of
effective teaching is the capacity to explain complex material plainly so that students can better access its underlying structure.

Economics instructors often try to achieve this goal by using curves and diagrams. Textbooks, in particular, make extensive use of diagrams to represent mathematical functions and economic relationships because some students have a stronger visual sense than a sense of numbers so that, for them, thinking about complex relationships is more readily facilitated by seeing them in spatial terms than it is by thinking about them abstractly using systems of equations. Solow’s famous diagram is a case in point. Being able to picture a region where saving exceeds breakeven investment so that the capital-labour ratio increases through time, and another region where the opposite is true, makes the idea of an equilibrium capital–labour ratio much more intuitively accessible (see, for example, Romer 2012, p.16). Even if it is important for students to understand the mathematics of a formal economic model, approaching the model by way of a diagrammatic structure first may pave the way for understanding the mathematics later.

Blaug & Lloyd (2010, p.5) point out that figures and diagrams have also been used in economics for high level analytical research functions such as discovering results and demonstrating proofs, as well as for pedagogy. Geometry is sometimes better suited than algebra to highlighting the essential features of a complex model characterized by a large number of variables and relationships. A well designed diagram can bring out the central features of a model, show interactions between relationships, and convey information contained in two or more curves that cannot be easily seen using equations or words. In economics ‘one picture is worth a thousand words’, but perhaps we should rephrase this old dictum as ‘one picture is worth a thousand equations’.

We use this pedagogical perspective to develop a geometric approach for illustrating the dynamic general equilibrium of the challenging Aghion and Howitt (1992) Schumpeterian growth model. We bring together in one diagram the key variables and functional relationships of the model so that we may keep track of complex interactions among sectors and show how these interact so as to be simultaneously in equilibrium. We hope this will be of use to graduate students and professors in thinking about how they might make a complex model more accessible to their students. The following section outlines the
The precise structure of the Howitt and Aghion model we have in mind for exposition in diagrammatic form as explained in the section after that.

3. AGHION AND HOWITT’S MODEL

Aghion and Howitt’s (1992) model is technically complex and a first step towards generating the kind of diagram that will make the model more accessible to graduate students is to develop a simplified version of the model. A useful guide in this respect that we will follow is the approach taken by Jones (1998 pp.101-111) in outlining a simplified version of Romer’s (1990) model of endogenous growth. Jones’ approach makes a number of assumptions in addition to those made by Romer with the aim of significantly reducing the complexity of the model while still retaining the fundamental principles and results of Romer’s analysis. In this way students are able to follow the basic economic reasoning of the Romer model and to understand its basic results without having to struggle with the difficulty of Romer’s original paper. We take a parallel approach with respect to Aghion and Howitt (1992).

The fundamental idea of the Aghion and Howitt (1992) model is that entrepreneurs invest in research and development processes that enhance the productivity of intermediate (capital) goods used in the production of final goods. Such productivity-enhancing innovations drive the pace of technical progress and are thus a key source of economic growth. But new technologies produced by the research and development industry also render previous technologies obsolete so that those investing in this industry run the risk that the profitability of their innovations may be relatively short lived. Since they are profit maximisers, the amount of investment that underlies the discovery of innovations is the result of a business decision and is thus endogenous to the economy’s internal workings. We may thus think of research and development as generating a series of innovations that are reflected in the particular characteristics of the intermediate good. The \( t^{th} \) innovation generates the \( t^{th} \) intermediate good which replaces the \((t-1)^{st}\) intermediate good. The number \( t \) therefore represents the number of innovations that have occurred to date.

Like Aghion and Howitt (1992, p.327) we will assume that the economy has four sectors: a final goods sector; an intermediate (capital) goods sector; a research and development sector; and a labour market.
Each sector is described in turn before the character of the economy’s general equilibrium is described.

(a) The Final Goods Sector
The economy produces a single final (consumption) good under conditions of perfect competition, using one input – a single intermediate good. Production of the final good is governed by a Cobb-Douglas-type production function of the following form:

\[ y = Ax^\alpha \]  

where: \( y \) is output of the consumption good; \( x \) is the amount of the intermediate good used to produce the final good; \( A \) is a productivity parameter that reflects the current quality of the intermediate good; and \( \alpha \) is a coefficient that lies between zero and one.

Recall that the final good is produced under perfect competition, using the intermediate product as the only input. If we take the price of the intermediate good as given, the profit maximizing choice of this input will be determined where the marginal revenue product of the intermediate good is equal to its unit price. Using the final good as numéraire, so that its unit price is equal to one, the following expression gives the condition for the profit maximizing choice of intermediate good use in the production of final goods:

\[ \frac{1}{\alpha - 1} = \frac{\partial y}{\partial x} = A x_t^{\alpha - 1} \]  

where: \( p_t \) is the price of the \( t^{th} \) intermediate good. This expression may also be taken as the inverse demand curve for intermediate goods.

(b) The Intermediate Goods Sector
The intermediate good is produced using labour according to a simple one-for-one technology. That is, each unit of the intermediate good produced requires the input of one unit of labour. Thus \( x \) is also the amount of labour currently employed in production of the intermediate good:

\[ x_t = L_t \]  

where: \( L_t \) denotes the amount of labour used to produce the \( t^{th} \) intermediate good.
The intermediate sector is also assumed to be a monopoly industry. Firms that manufacture state-of-the-art intermediate goods enjoy monopoly power and earn positive profits by virtue of possessing patent rights over the production of particular intermediate goods which have been purchased from firms in the research and development sector. However since new technology is constantly being discovered in the research and development sector, patented technology is constantly being superseded in the intermediate goods sector. We will assume that firms in the intermediate goods sector purchase only one patent so that when that patent is superseded, they leave the industry to be replaced by a new firm that has purchased the latest patent technology. The identity of the monopoly firm producing in the intermediate goods sector is thus constantly changing.

Behaviour of the intermediate sector monopolist is also driven by maximizing profit which may be written as follows:

$$\pi_t = p_t x_t - w_t L_t$$

(4)

where: $\pi_t$ denotes profit from the $t^{th}$ innovation; and $w_t$ is the unit real wage paid to labour. Substituting for the price of intermediate goods from equation (2) and labour used to produce intermediate goods from equation (3), differentiating the resulting equation with respect to the choice of output for the intermediate good and setting equal to zero yields the first order condition for a maximum profit. If we also define the productivity-adjusted wage rate by the ratio of the real wage to the economy’s productivity level: $\omega_t \equiv w_t / A_t$, then the following equation is obtained after some rearranging:

$$\omega_t = \alpha^2 x_t^{\alpha - 1}$$

(5)

We may refer to this equation as: $\omega_t = \tilde{\omega}(x_t)$. Solving equation (5) for the optimal level of intermediate good production yields:

$$x_t^* = \left(\frac{\alpha^2}{\omega_t}\right)^{\frac{1}{1-\alpha}}$$

(6)

and this expression may also be referred to as: $x_t = \tilde{x}(\omega_t)$. This function is also the demand for labour in the intermediate goods industry and it
implies that this demand is a decreasing function of the productivity-adjusted wage rate, $\omega_t$.²

Profit in the intermediate goods sector can be expressed as a function of the quality of the intermediate product, $A_t$, and the productivity adjusted wage. Assume that the intermediate goods firm practices markup pricing, with the price-elasticity of demand facing the firm being constant. This elasticity, $\varepsilon$, is $1/(\alpha - 1)$ and the monopolist’s price is given by $p_t = w_t/(1 + \varepsilon^2) = w_t/\alpha = A_t(1/\alpha)\omega_t$.³ Now, substituting for $p_t$ in equation (4) and collecting terms yields a new expression for the profit function:

$$\pi_t = A_t\left(\frac{1}{\alpha} - 1\right)\omega_t x_t, \tag{7}$$

We may also substitute for $x_t$ from equation (6) to get:

$$\pi_t = A_t \frac{1 - \alpha}{\alpha} \left(\frac{\alpha^2}{\omega_t^\alpha}\right)^{\frac{1}{1-\alpha}} \tag{8}$$

which we may abbreviate as $\pi_t = A_t \tilde{\pi}(\omega_t)$ and where $\pi_t$ is a decreasing function of $\omega_t$.

(c) The Research and Development Sector

Improvements in the productivity, and thus in the quality, of the intermediate good are the result of technological innovations, and the role of research and development firms is to make investments aimed at creating these innovations. Aghion & Howitt (1992, p.329) model innovations as occurring with a Poisson arrival rate of $\lambda \cdot \phi(z,s)$ where $z$ and $s$ are types of labour used in the research and development process. We simplify this arrival rate to $\lambda n$, where $\lambda$ is a probability parameter and $n$ is the amount of labour used in the research and development industry. This means that increasing the amount of labour used in research and

² A different interpretation of result (5) is that labour in the intermediate goods industry is paid its (productivity-adjusted) marginal-revenue product. It is not the current wage rate that really matters for rational decision making in the intermediate sector but the wage rate relative to the economy’s productivity

³ Alternatively substitute equation (6) into equation (2) to give $p_t = (1/\alpha)w_t$ or $p_t = A_t(1/\alpha)\omega_t$. 

development can increase the rate at which innovations arrive and $\lambda$ is a kind of measure of the industry’s own productivity. When an innovation does occur, it enhances the productivity of intermediate goods in the production of final goods by a factor of $\gamma > 1$. The productivity of the new “state-of-the-art” intermediate good is thus given by:

$$A_t = \gamma A_{t-1}$$

(9)

where $t$ again indexes the $t^{th}$ innovation rather than time.$^4$

The research and development sector is assumed to be competitive. This competition can be likened to the race for a patent. Because the market delivers a profit to the monopoly situation in the production of intermediate goods, there is competition for the patents that create this monopoly situation. Research and development firms try to be the next firm to innovate, obtain a patent and then sell the rights to produce the intermediate good under this patent. Whenever a new innovation occurs, the rights conferred under the new patent make those under the previous one obsolete, and monopoly profits associated with producing the old intermediate good evaporate. A successful innovator today therefore becomes the monopolist tomorrow, or in our set up, sells the patent to the firm that becomes the monopolist tomorrow in the production of intermediate goods.

To determine the amount of research and development that should be undertaken by firms in this sector will equate the marginal cost of a unit of research labour with the expected marginal benefit. Marginal cost is simply the wage, $w_t$. The expected marginal benefit arises from raising the probability of innovation by $\lambda$ for each additional worker employed. If we denote $V_{t+1}$ as the value of the next patent, research firms should thus hire labour for the research and development process up to the point where the following condition holds:

$$w_t = \lambda V_{t+1}$$

(10)

If the value of the new patent is invested for a unit time interval (say one day) at the risk free rate, $r$, the return will be $rV_{t+1}$. If this amount is

---

$^4$ A time period can be thought of in the model as the interval between two successive innovations, the symbol $\Delta_t$ being used to denote the interval starting with the $t^{th}$ innovation and ending just before the $(t+1)^{st}$ innovation.
alternatively used to purchase the patent, the return in the unit time interval period will be the monopoly profits associated with the \( t+1^{st} \) innovation plus the expected rate of capital appreciation (or loss) that results from the change in value of the patent.\(^5\) This can be written as \( \pi_{t+1} + dV^e_{t+1} \). The second part of this expression is related to the probability of a new innovation being discovered that makes the patent worthless. This probability is \( \lambda n_{t+1} \) and generates an expected capital loss of \( [\lambda n_{t+1}0 + (1 - \lambda n_{t+1})V_{t+1}] - V_{t+1} \). Expanding this expression allows the expression for the return from investing in the patent to be written as \( \pi_{t+1} - \lambda n_{t+1}V_{t+1} \). In equilibrium, the returns from investing in the risk free rate and the patent must be the same. This equilibrium, or no arbitrage, condition may be written as:

\[
rV_{t+1} = \pi_{t+1} - \lambda n_{t+1}V_{t+1}.
\]

Solving this expression for \( V_{t+1} \) yields:

\[
V_{t+1} = \pi_{t+1}/(r + \lambda n_{t+1})
\]

where the numerator reflects monopoly profit and the denominator can be seen as the “effective” interest rate, that is, the discount rate adjusted by the obsolescence rate of the knowledge underlying the monopoly profit. This equation, therefore, gives the value of the patent associated with the \((t + 1)^{st}\) innovation.

In the previous sub-section we represented monopoly profit in the intermediate goods sector associated with \( t^{th} \) innovation as a linear function of the productivity level \( A_t \):

\[
\pi_{t+1} = A_t \gamma \tilde{\pi}(\omega_{t+1})
\]

Substituting for \( \pi_{t+1} \) in the research arbitrage equation (10) above, the factor \( \gamma \) shows on the right-hand side of the equation. Then, dividing through by \( A_t \), the research arbitrage equation can now be re-expressed as:

\[
\omega_t = \lambda \frac{\gamma \tilde{\pi}(\omega_{t+1})}{r + \lambda n_{t+1}}
\]

Thus the allocation of labour between research and the production of the intermediate good is a function of the productivity-adjusted wage rate \( \omega_t \).

\(^5\) Jones (1998, pp.106-107) has a good description of this method.
The concept of arbitrage also proves to be useful in determining what fraction of the population works in each sector.

The equilibrium level of labour employed in the research sector is neither determined by the current wage rate, nor affected by the actual change of the productivity parameter stemming from an innovation. The first-order condition for the maximization problem of research firms suggests that the marginal benefit of an additional unit of research labour is higher, the higher the productivity increase in the economy. But this advantage is just offset if the marginal cost (the wage rate) of an additional unit of labour is also higher because of the higher productivity. Thus the optimal solution remains the same as before any increase in productivity.

(d) The Labour Market

The labour market is also assumed to be competitive and frictionless. We assume that there are \( N \) individuals in the labour market, each of whom has one unit of labour to supply inelastically (that is, no matter what the wage rate). Equation (14) is a labour market clearing equation, which states that for each innovation total labour supply \( N \) is equal to employment in the intermediate goods industry, \( x_t \), plus employment in the research and development industry, \( n_t \). Thus the economy faces a resource constraint defined in labour units.

\[
N = x_t + n_t
\]  

(14)

Substituting for \( x_t \) from equation (6) this becomes:

\[
N = \bar{x}(\omega_t) + n_t
\]  

(15)

In equilibrium, all workers must be paid the same wage. The price of labour will, therefore, adjust so equation (15) is satisfied and the productivity-adjusted wage rate \( \omega_t \) is a function of the labour market clearing condition.

(e) Equilibrium Research and Growth

General equilibrium in the aggregate economy requires that both conditions (13) and (15) are simultaneously satisfied. We are now in a position to find the equilibrium value of research employment. By
substituting for $\omega_t$ in the research arbitrage equation (13) using the marginal-revenue function (5), dividing through by $\lambda$, and using the labour market clearing equation (15) to substitute for $x_t$, we may obtain the following:

$$\frac{\alpha^2 (N - n_t)^{\alpha-1}}{\lambda} = \frac{\gamma \alpha (1 - \alpha) (N - n_{t+1})^\alpha}{r + \lambda n_{t+1}}$$

(16)

The left-hand side of equation (16) is an increasing function of $n_t$ and can be interpreted as the “marginal cost of research” $c(n_t)$. The right-hand side is a decreasing function of $n_{t+1}$ and can be interpreted as the “marginal benefit of research” $b(n_{t+1})$. Thus equilibrium in the economy requires $c(n_t) = b(n_{t+1})$.

Individual researchers, being small relative to the economy as a whole, take the wage rate as given. They do not recognize that the wage increases as more labour enters the research sector. In other words, the many small research firms are price takers in the labour market. The research arbitrage equation (13) above illustrates this assumption. In the economy as a whole, however, the wage rate earned by labour in the research sector during the development of a particular innovation is an increasing function of the amount of research during the period of time over which that innovation is being developed. While the wage will change by only a negligible amount in response to the research efforts of a single researcher, it clearly varies with aggregate research effort. In the model, the productivity-adjusted wage rate $\omega_t$ is increasing in $n_t$. To see this, note from equation (5) that the marginal-revenue product of labour in the manufacturing sector is decreasing in manufacturing employment $x_t$, and from equation (15) that $x_t$ equals the residual supply of manufacturing labour $N - n_t$. The implication of this result of the model is that the marginal cost of research $c(n_t)$ is increasing and the marginal benefit of research $b(n_{t+1})$ is decreasing.

Equilibrium condition (16) determines the amount of current research as a decreasing function of the amount of expected future research: $n_t = \psi(n_{t+1})$. It is demonstrated in the Appendix that the functional relationship $\psi$ in the economy is given by:
The fact that the functional relationship \( \psi \) between research employment \textit{during the development of two consecutive innovations} is strictly decreasing suggests that, in equilibrium, the allocation of labour between research and manufacturing is likely to change over time. However, rather than taking a general approach to finding the equilibrium, we will look for an equilibrium where the amount of labour employed in research is constant.

In a steady-state (or stationary) equilibrium the allocation of labour between research and manufacturing remains constant. In a steady state the expected growth rate of final output (or consumption good) in the economy is also constant over time. The economy has a unique steady-state equilibrium for a given set of parameter values. To determine the economy’s growth rate, we need to determine the equilibrium value of research employment \( \hat{n} \). Using the fact that the amount of labour employed in research remains constant in a steady-state, it is simplest to leave out the subscript \( t \).

Condition (17) will pin down the equilibrium value of research employment. That is, solving \( n = \psi(n) \) for the steady-state amount of research labour yields the expression:\(^6\)

\[
\hat{n} = \frac{\lambda \gamma \frac{1-\alpha}{\alpha} N - r}{\lambda \left(1 + \gamma \frac{1-\alpha}{\alpha}\right)}
\]  

(18)

One can show that this level of research will produce an economy’s average growth rate equal to:

\[
g = \lambda \hat{n} \ln \gamma
\]

(19)

\(^6\) Proof of this solution is shown in the Appendix.
The equilibrium expected rate of growth in steady-state is determined by the characteristics of the economic environment as described by the parameters \( \lambda, \gamma, a, r, \) and \( N \) of the model. In particular, parameters \( \lambda \) and \( \gamma \) affect growth at least in part by determining the amount of labour employed in research according to result (18).

The economy’s expected growth rate (19) equals the probability of innovation \( \hat{n} \) times the size of innovation \( \ln \gamma \). How does final-good production in the economy evolve over time in a steady-state? Economic growth results from innovations that raise the productivity parameter \( A_t \). The economy’s growth rate is the proportional growth rate of the productivity parameter \( A_t \). To see this, note first that final good output is proportional to the productivity of the intermediate good according to equations (1) and (18), the latter of which implies the equilibrium amount of manufacturing labour. That is, \( y_t = A_t(N - \hat{n})^a \). This result, together with the fact that \( A_{t+1} = \gamma A_t \), implies that \( y_{t+1} = \gamma y_t \). Now suppose that \( t \) innovations have occurred up to the present date \( \tau \). Consider the unit-time interval between \( \tau \) and \( \tau + 1 \). Then, with probability \( \hat{n} \), research firms will succeed in discovering innovation number \( t + 1 \), and consequently \( g(\tau) = \ln y_{\tau+1} - \ln y_\tau = \ln \gamma = \ln A_{\tau+1} - \ln A(\tau) \). With probability \( 1 - \hat{n} \), research firms will fail to innovate, and consequently \( g(\tau) = 0 \). Thus, the economy’s expected growth rate between \( \tau \) and \( \tau + 1 \) is equal to \( \hat{n} \ln \gamma + (1 - \hat{n}) \cdot 0 = \hat{n} \ln \gamma \).

This outlines the complete theoretical structure of the simplified Aghion and Howitt Model. We can now outline a diagrammatic representation of this model which we think will make its interpretation and learning by students easier.

4. A DIAGRAM TO ILLUSTRATE THE MODEL

The model outlined in the previous section can be viewed as being made up four key variables and four functional relationships. The four key variables are: the volume of final goods produced, \( y_t \); the volume of intermediate goods produced, \( x_t \); the productivity-adjusted wage, \( \omega_t \); and the amount of labour currently employed in the research and development sector, \( n_t \). A fifth variable is, however, also relevant which is the amount of labour employed in research and development in the next period, \( n_{t+1} \). Earlier in the paper we suggested that a time period can be thought of as the interval between two successive innovations, the
symbol \( \Delta_t \) being used to denote the interval starting with the \( t^{th} \) innovation and ending just before the \((t + 1)^{st}\) innovation. It is important to keep in mind this conception of time in the model. The key variables are determined jointly by the functional relationships shown in expressions (1), (6), (13), and (15) which are reproduced below.

\[
y = Ax^\alpha \tag{1}
\]

\[
x_t = \left( \frac{\alpha^2 / \omega_t}{1-\alpha} \right)^{\frac{1}{1-\alpha}} \tag{6}
\]

\[
\omega_t = \lambda \frac{\gamma \pi (\omega_{t+1})}{r + \lambda n_{t+1}} \tag{13}
\]

\[
n_t = N - \tilde{x}(\omega_t) \tag{15}
\]

In developing our four quadrant diagram, we will put aside equation (1) and output in the final goods sector (which will ultimately be the variable of interest) and return to these aspects of the model later. For the moment, our attention will focus on equations (6), (13) and (15) and the four endogenous variables within these equations: \( x_t, \omega_t, n_t \) and \( n_{t+1} \). Equation (6) represents the demand for labour in the intermediate goods sector as a function of the productivity-adjusted wage. This relationship is shown in the third quadrant (bottom right) of Figure 1 where employment \( x_t \) is a decreasing function of the productivity-adjusted wage rate \( \omega_t \). Equation (15) is the equilibrium condition for the overall labour market and this is shown in quadrant IV of Figure 1 as a straight line with slope minus one in \((x_t, n_t)\) space. Because the labour market is competitive, all transactions in the labour market occur at the same market price. With aggregate labour supply equal to \( N \), the productivity-adjusted wage rate \( \omega_t \) will, therefore, be determined by equation (15).

Equation (13) represents the inverse relationship between \( n_{t+1} \) and \( \omega_t \) and is depicted by curve A in the second quadrant of Figure 1. This research arbitrage condition indicates the incentive for firms in the research and development sector to invent a new intermediate good. Innovations arrive randomly but the probability of innovations depends positively on the amount of research labour employed \( n_t \). Research firms will thus choose the amount of labour so that the marginal benefit of
employing labour, which is decreasing in \( n_t \) under the general case because the incremental probability of innovation is a decreasing function of \( n_t \), is equal to the marginal cost which is increasing in \( n_t \) because the productivity-adjusted wage rate is increasing in \( n_t \). Hence, the arbitrage equation implies the equilibrium research amount \( n_t \). The research arbitrage curve (A) is thus decreasing in the future amount of labour employed because the value of making the next innovation is decreasing in the future amount of research. The value of making the next innovation is decreasing in the probability of innovation \( \lambda n_{t+1} \) (under the linear research technology case), where \( \lambda \) is a parameter indicating the productivity of the research sector. Consequently the current wage decreases as well. The research arbitrage condition (A) governs the dynamics of the economy over its successive innovations. The optimal investments in innovations are determined by this condition and govern the dynamics of the economy. Observe that the negative slope of the curve corresponding to (A) reflects the sum of two elements, the influence of a creative destruction effect, and the impact of two general

**Figure 1: Aghion & Howitt’s Model of Innovation and Growth**
equilibrium effects, with implications for the slope of the functional relationship between research in two successive periods in the first quadrant. The general equilibrium effects of wages on profits created by current research and of the level of manufacturing employment on wages are added to the following analysis.

A higher level of research \( n_{t+1} \) tomorrow implies two things. The first is a higher rate of creative destruction next period, that is a higher probability of innovation shortening the expected lifetime of the monopoly to be enjoyed by the next innovator. The second is higher future wages \( \omega_{t+1} \), lessening the stream of profits to be appropriated by the next innovator. The second effect is indicated by pairs of boldfaced arrows \( \rightarrow \) in \((x_t, n_t)\) and \((x_t, \omega_t)\) spaces, with current period subscript \( t \) in the axes of both spaces now being replaced by future period subscript \( t + 1 \). This will lower the discounted expected payoff of the \((t + 1)^{th}\) innovation and, hence, will discourage the amount of research today. Note that the second effect above is the general equilibrium effect of future research on the profits created by current research referred to above. In turn, a lower expected value of an hour in research will imply a lower current wage \( \omega_t \), hence explaining the negative slope of the curve corresponding to (A).

Equations (6), (13) and (15) thus constitute the foundation on which Figure 1 is constructed. But it will be observed that this subsystem of three equations contains four endogenous variables. The structure of this system, however implies a relationship between \( n_t \) and \( n_{t+1} \) which is essentially a forward-looking difference equation of the form \( n_t = \psi(n_{t+1}) \), and this equation is important for the dynamic behavior of the economy and for its convergence to an equilibrium growth path.

In the previous section of the paper we derived this implicit relation in the form of expression (17) which is also reproduced below.

\[
n_t = N - \left[ \frac{\lambda y}{\alpha} \frac{1 - \alpha}{\alpha} (N - n_{t+1})^\alpha \right]^{\frac{1}{\alpha - 1}} \frac{1}{r + \lambda n_{t+1}} \tag{17}
\]

But we can also obtain this relation diagrammatically in Figure 1. From the representation of relationships in the second through fourth
quadrants, we can derive the $\psi(n_{t+1})$ curve in the first quadrant. We begin with a particular value for the level of labour employed in future research and development, $n_{t+1,0}$, in quadrant II, and trace the corresponding values of the variable sequence $\omega_t$, $x_t$, $n_t$ through the relationships in quadrants II, III, and IV. This is shown by the rectangle $B_3B_2B_1B$ and ultimately gives a value of $n_{t,0}$ for labour currently employed in research and development in panel IV. We can plot the resulting $(n_t, n_{t+1})$ combination as point B in quadrant I. If we continue to choose values for $n_{t+1}$ and find their corresponding $n_t$ values, and plot the resulting combinations in quadrant I, we will define the $\psi(n_{t+1})$ curve in that quadrant.

This is the curve of the relationship in equation (17) and its negative relationship between labour employed in current and future research tells us that a higher amount of future research will discourage current research. A higher amount of future research will decrease the marginal benefit of a unit of research labour (through the influence of a creative destruction effect and the impact of a general equilibrium effect), which will imply a lower current wage (in order to reestablish the equality between the marginal cost of research and the marginal benefit of research taken from the research arbitrage equation). The graphical depiction of the response of current wages to changes in future research could be shown as downward movements along the curve $(A)$ in the second quadrant of Figure 1 above. Given that workers contribute less than before to productivity growth and profits if employed in research rather than in manufacturing, the demand for labour will tend to increase in manufacturing, inducing a greater fraction of labour to move away from research. This in turn will reduce the value of the marginal product of labour in manufacturing and will push down current wages. Those employment changes could be represented by downward movements along the demand manufacturing labour curve $\bar{x}(n_t)$ and the residual supply research labour curve $n_t = N - x_t$ (taken from the labour market-clearing equation (L)), respectively in the third and fourth quadrants.

Determination of equilibrium in Figure 1 now hinges on a solution to the forward-looking difference equation $n_t = \psi(n_{t+1})$ represented by the downward sloping curve in quadrant I. If we begin arbitrarily at point B, the current level of research is $n_0$. Moving horizontally to the right towards the $45^\circ$ line in the first quadrant and then vertically downward,
we register $n_j$, which is the horizontal coordinate of point C on the $\psi(n_{t+1})$ curve and the value of $n$ that we could expect to be chosen next period. This will eventually form a sequence $\{n_0, n_1, n_2, \ldots\}$ constructed from the clockwise spiral starting at $n_0$ converging to the point where the $\psi(n_{t+1})$ curve intersects with the 45°-line. The resulting choice of labour allocated to research and development is thus stable through time and is the same value as that given by expression (18) in the previous section of the paper.

Economically, we can think about this result in terms of agent maximization and perfect foresight. Aghion and Howitt’s version of the Schumpeterian model abstracts from elements such as uncertainty about future research efforts and uses these modern tools of agent maximization and perfect foresight. Under perfect foresight, research firms make no mistakes while choosing the amount of labour employed in research each period and correctly anticipate or foresee any increase or decrease in research next period. In this manner the model does not rely on agents’ expectations that are repeatedly and systematically fooled along the solution path of the economy. Research firms form expectations about the path of research efforts and these expectations are fulfilled by the chosen amounts of labour employed in innovation. In this sense any perfect foresight equilibrium is a self-fulfilling equilibrium. A perfect foresight equilibrium (PFE) is defined as a sequence $\{n_t\}_0^\infty$ satisfying $n_t = \psi(n_{t+1})$ for all $t \geq 0$. In quadrant I of Figure 1, the sequence $\{n_0, n_1, n_2, \ldots\}$ that we constructed above from the clockwise spiral starting at $n_0$ constitutes a PFE.

Now that the equilibrium values for $n_t$ and $n_{t+1}$ have been determined, we may substitute the value for $n_{t+1}$ into the function in quadrant II to find the equilibrium value for $\omega_t$, and the value for $n_t$ into the function in quadrant IV to find the equilibrium value for $x_t$, which will be consistent with the values for these variables from the function in quadrant III. Economically both of these variables will be determined simultaneously by the dynamic process which establishes the values of $n_t$ and $n_{t+1}$ in the first place. We may then substitute the value for $x_t$, into equation (1) for the output of final commodities and determine the growth rate by the process outlined at the end Section 3.

While we have presented the development of our diagram by first outlining the mathematics of the simplified Aghion and Howitt model
and then describing the diagram, we have done this for the sake of exposition to professors. Pedagogically, we would approach exposition of the model very differently. For presentation to graduate students, we would outline the assumptions of the model and explain the economics of equations (1), (6), (13) and (15) which we think would be relatively straightforward. We would then plot each of these functions in quadrants II through IV without anything appearing in quadrant I. We would then derive the function in quadrant I diagrammatically, insert the 45° line and explain the intuition of the convergent dynamic process that establishes the equilibrium values for \( n_t \) and \( n_{t+1} \). We would then back out the equilibrium values for \( \omega_t \) and \( n_t \) as outlined above and describe the implications for growth using equation (1) and the brief discussion of growth at the end of Section 3, largely as one does in the Solow model once the equilibrium value for the capital-labour ratio is determined. Only then would we engage in the full mathematical treatment of the model and review the core economics of what the model represents.

5. CONCLUSION
This paper has argued that the geometric method is very effective in getting key concepts across to students and providing clear explanations of complex ideas in economic modeling. It can thus be a powerful and flexible pedagogical tool even for teaching graduate students. We have used these principles to develop a diagram which illustrates the key features of Aghion and Howitt’s (1992) model of endogenous growth through creative destruction. We outline a simplified version of the Aghion and Howitt model, trace through the mathematics of the model, present out four quadrant diagram and then explain briefly how this material would be presented to graduate students. Now we can only hope that we have also had the ability to make the Aghion and Howitt material genuinely interesting, so that students find it a pleasure to learn.

REFERENCES
APPENDIX

In this appendix we derive results (17) and (18).

Proof of result (17):

Recall the economy’s equilibrium condition (16):

\[ \frac{\alpha^2 (N - n_t)^{a-1}}{\lambda} = \frac{\gamma \alpha (1-\alpha)(N-n_{t+1})^a}{r + \lambda n_{t+1}}. \]

Then, multiplying through by \( \lambda/\alpha^2 \), we obtain:

\[ (N - n_t)^{a-1} = \frac{\lambda \gamma \alpha^{a-1}(1-\alpha)(N-n_{t+1})^a}{r + \lambda n_{t+1}}. \]
Now raising both sides of this equation to the power $1/(\alpha - 1)$, we obtain:

$$N - n_r = \left( \frac{\lambda \gamma \alpha^{-1} (1 - \alpha)(N - n_{r+1})^\alpha}{r + \lambda n_{r+1}} \right)^{\frac{1}{\alpha-1}}$$

and therefore result (17) holds:

$$n_r = N - \left( \frac{\lambda \gamma \frac{1-\alpha}{\alpha} (N - n_{r+1})^\alpha}{r + \lambda n_{r+1}} \right)^{\frac{1}{\alpha-1}}.$$

**Proof of result (18):**

Now, the starting point is equation (17). Specifically, we solve $n = \psi(n)$ for the steady-state amount of research labour. From (17), after successive algebraic manipulations, we obtain:

$$-(N - n) = \left( \frac{\lambda \gamma \frac{1-\alpha}{\alpha} (N - n)^\alpha}{r + \lambda n} \right)^{\frac{1}{\alpha-1}}$$

or:

$$N - n = \left( \frac{\lambda \gamma \frac{1-\alpha}{\alpha} (N - n)^\alpha}{r + \lambda n} \right)^{\frac{1}{\alpha-1}}$$

followed by:

$$(N - n)^{\alpha-1} = \frac{\lambda \gamma \frac{1-\alpha}{\alpha} (N - n)^\alpha}{r + \lambda n}$$

and then by $r + \lambda n = \lambda \gamma \frac{1-\alpha}{\alpha} (N - n)$. Now, collecting terms, we obtain

$$n(\lambda + \lambda \gamma \frac{1-\alpha}{\alpha}) = \lambda \gamma \frac{1-\alpha}{\alpha} N - r.$$ Result (18) immediately follows from here:

$$\hat{n} = \frac{\lambda \gamma \frac{1-\alpha}{\alpha} N - r}{\lambda \left(1 + \gamma \frac{1-\alpha}{\alpha}\right)}.$$