

An Introduction to Turbulent Mixing and Dispersion in Streams

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Aichi Forest Park, Japan (Photograph by H. CHANSON, Mar. 1999)

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Table of contents

	Page
Table of contents	II
Glossary	III
About the author	IX
1. Introduction	1-1
2. Turbulent shear flows	2-1
3. Diffusion : basic theory	3-1
4. Advective diffusion	4-1
5. Turbulent dispersion and mixing. (1) Vertical and transverse mixing	5-1
6. Turbulent dispersion and mixing. (2) Longitudinal dispersion	6-1
7. Turbulent jets and plumes	7-1
References	
Assignments	
Exercise solutions	
Assignment solutions	

Glossary

Académie des Sciences de Paris : The Académie des Sciences, Paris, is a scientific society, part of the Institut de France formed in 1795 during the French Revolution. The academy of sciences succeeded the Académie Royale des Sciences, founded in 1666 by Jean-Baptiste COLBERT.

Adiabatic : thermodynamic transformation occurring without loss nor gain of heat.

Advection : transport by an imposed current; movement of a mass of fluid which enhances change in temperature or in other physical or chemical properties of fluid.

Air : mixture of gases comprising the atmosphere of the Earth. The principal constituents are nitrogen (78.08%) and oxygen (20.95%). The remaining gases in the atmosphere include argon, carbon dioxide, water vapour, hydrogen, ozone, methane, carbon monoxide, helium, krypton ...

Air concentration : concentration of undissolved air defined as the volume of air per unit volume of air and water. It is also called the void fraction.

Analytical model : system of mathematical equations which are the algebraic solutions of the fundamental equations.

ARCHIMEDES : Greek mathematician and physicist. He lived between B.C. 290-280 and B.C. 212 (or 211). He spent most of his life in Syracuse (Sicily, Italy) where he played a major role in the defence of the city against the Romans. His treaty "On Floating Bodies" is the first-known work on hydrostatics, in which he outlined the concept of buoyancy.

ARISTOTLE : Greek philosopher and scientist (384-322 BC), student of Plato. His work "Meteorologica" is considered as the first comprehensive treatise on atmospheric and hydrological processes.

Avogadro number : number of elementary entities (i.e. molecules) in one mole of a substance : $6.0221367 \times 10^{23} \text{ mole}^{-1}$. Named after the Italian physicist Amedeo AVOGADRO.

BARRÉ de SAINT-VENANT : Adhémar Jean Claude BARRÉ de SAINT-VENANT (1797-1886), French engineer of the 'Corps des Ponts-et-Chaussées', developed the equation of motion of a fluid particle in terms of the shear and normal forces exerted on it.

BLASIUS : H. BLASIUS (1883-1970) was German scientist, student and collaborator of L. PRANDTL.

BOLTZMANN : Ludwig Eduard BOLTZMANN (1844-1906) was an Austrian physicist.

Boltzmann constant : ratio of the universal gas constant ($8.3143 \text{ K}\cdot\text{J}^{-1}\cdot\text{mole}^{-1}$) to the Avogadro number ($6.0221367 \times 10^{23} \text{ mole}^{-1}$). It equals : $1.380662 \times 10^{-23} \text{ J/K}$.

Boundary layer : flow region next to a solid boundary where the flow field is affected by the presence of the boundary and where friction plays an essential part. A boundary layer flow is characterised by a range of velocities across the boundary layer region from zero at the boundary to the free-stream velocity at the outer edge of the boundary layer.

BOUSSINESQ : Joseph Valentin BOUSSINESQ (1842-1929) was a French hydrodynamicist and Professor at the Sorbonne University (Paris). His treatise "Essai sur la théorie des eaux courantes" (1877) remains an outstanding contribution in hydraulics literature.

Buoyancy : tendency of a body to float, to rise or to drop when submerged in a fluid at rest. The physical law of buoyancy (or Archimedes' principle) was discovered by the Greek mathematician ARCHIMEDES. It

states that any body submerged in a fluid at rest is subjected to a vertical (or buoyant) force. The magnitude of the buoyant force is equal to the weight of the fluid displaced by the body.

Buoyant jet : submerged jet discharging a fluid lighter or heavier than the mainstream flow. If the jet's initial momentum is negligible, it is called a *buoyant plume*.

Clean-air turbulence : turbulence experienced by aircraft at high-altitude above the atmospheric boundary layer. It is a form of Kelvin-Helmholtz instability occurring when a destabilising pressure gradient of the fluid become large relative to the stabilising pressure gradient.

Convection : transport (usually) in the direction normal to the flow direction induced by hydrostatic instability : e.g. flow pas a heated plate.

Darcy-Weisbach friction factor : dimensionless parameter characterising the friction loss in a flow. It is named after the Frenchman H.P.G. DARCY and the German J. WEISBACH.

Density-stratified flows : flow field affected by density stratification caused by temperature variations in lakes, estuaries and oceans. There is a strong feedback process : i.e., mixing is affected by density stratification, which depends in turn upon mixing.

Diffusion : the process whereby particles of liquids, gases or solids intermingle as the result of their spontaneous movement caused by thermal agitation and in dissolved substances move from a region of higher concentration to one of lower concentration. The term turbulent diffusion is used to describe the spreading of particles caused by turbulent agitation.

Diffusion coefficient : quantity of a substance that in diffusing from one region to another passes through each unit of cross-section per unit of time when the volume concentration is unity. The units of the diffusion coefficient are m^2/s .

Diffusivity : another name for the diffusion coefficient.

Dimensional analysis : organisation technique used to reduce the complexity of a study, by expressing the relevant parameters in terms of numerical magnitude and associated units, and grouping them into dimensionless numbers. The use of dimensionless numbers increases the generality of the results.

Dispersion : longitudinal scattering of particles by the combined effects of shear and diffusion.

Drag reduction : reduction of the skin friction resistance in fluids in motion.

Ecole Nationale Supérieure des Ponts et Chaussées, Paris : French civil engineering school founded in 1747.

The direct translation is : 'National School of Bridge and Road Engineering'. Among the directors there were the famous hydraulicians A. CHEZY and G. de PRONY. Other famous professors included B.F. de BELIDOR, J.B.C. BELANGER, J.A.C. BRESSE, G.G. CORIOLIS and L.M.H. NAVIER.

Eddy viscosity : another name for the momentum exchange coefficient. It is also called 'eddy coefficient' by SCHLICHTING (1979). (See Momentum exchange coefficient)

FICK : Adolf Eugen FICK was a 19-th century German physiologist who developed the diffusion equation for neutral particle (FICK 1855).

FOURIER : Jean Baptiste Joseph FOURIER (1768-1830) was a French mathematician and physicist known for his development of the Fourier series. In 1794 he was offered a professorship of mathematics at the Ecole Normale in Paris and was later appointed at the Ecole Polytechnique. In 1798 he joined the expedition to Egypt lead by (then) General Napoléon BONAPARTE. His research in mathematical

physics culminated with the classical study "Théorie Analytique de la Chaleur" (FOURIER 1822) in which he enunciated his theory of heat conduction.

Gas transfer : process by which gas is transferred into or out of solution : i.e., dissolution or desorption respectively.

Hydraulic diameter : is defined as the equivalent pipe diameter : i.e., four times the cross-section area divided by the wetted perimeter. The concept was first expressed by the Frenchman P.L.G. du BUAT (BUAT 1779).

Ideal fluid : frictionless and incompressible fluid. An ideal fluid has zero viscosity : i.e., it cannot sustain shear stress at any point.

Inlet : (1) upstream opening of a culvert, pipe or channel; (2) a tidal inlet is a narrow water passage between peninsulas or islands.

Intake : any structure in a reservoir through which water can be drawn into a waterway or pipe. By extension, upstream end of a channel.

Interface : surface forming a common boundary of two phases (e.g. gas-liquid interface) or two fluids.

KARMAN : Theodore von KARMAN (or von KÁRMÁN) (1881-1963) was a Hungarian fluid dynamicist and aerodynamicist who worked in Germany (1906 to 1929) and later in USA. He was a student of Ludwig PRANDTL in Germany. He gave his name to the vortex shedding behind a cylinder (Karman vortex street).

Karman constant (or von Karman constant) : 'universal' constant of proportionality between the Prandtl mixing length and the distance from the boundary. Experimental results indicate that $K = 0.40$.

Laminar flow : is characterised by fluid particles moving along smooth paths in laminas or layers, with one layer gliding smoothly over an adjacent layer. Laminar flows are governed by Newton's law of viscosity which relates the shear stress to the rate of angular deformation : $\tau = \mu * \partial v / \partial y$.

Left bank : bank on the left-hand side of an observer when looking downstream.

Mixing : process by which contaminants combine into a more or less uniform whole by diffusion or dispersion.

Mixing length : The mixing length theory is a turbulence theory developed by L. PRANDTL, first formulated in 1925 (PRANDTL 1925). PRANDTL assumed that the mixing length is the characteristic distance travelled by a particle of fluid before its momentum is changed by the new environment.

Mole : mass numerically equal in grams to the relative mass of a substance (i.e. 12 g for Carbon-12). The number of molecules in one mole of gas is $6.0221367 \text{ E}+23$ (i.e. Avogadro number).

Momentum exchange coefficient : In turbulent flows the apparent kinematic viscosity (or kinematic eddy viscosity) is analogous to the kinematic viscosity in laminar flows. It is called the momentum exchange coefficient, the eddy viscosity or the eddy coefficient. The momentum exchange coefficient is proportional to the shear stress divided by the strain rate. It was first introduced by the Frenchman J.V. BOUSSINESQ (1877,1896).

Normal depth : uniform equilibrium open channel flow depth.

Outlet : (1) downstream opening of a pipe, culvert or canal; (2) artificial or natural escape channel.

PRANDTL : Ludwig PRANDTL (1875-1953) was a German physicist and aerodynamicist who introduced the concept of boundary layer (PRANDTL 1904) and developed the turbulent 'mixing length' theory. He was Professor at the University of Göttingen.

REYNOLDS : Osborne REYNOLDS (1842-1912) was a British physicist and mathematician who expressed first the 'Reynolds number' (REYNOLDS 1883) and later the Reynolds stress (i.e. the turbulent shear stress).

Reynolds number : dimensionless number proportional to the ratio of the inertial force over the viscous force.

RICHARDSON : Lewis Fry RICHARDSON (1881-1953) was a British meteorologist who pioneered mathematical weather forecasting. It is believed that he took interest in the dispersion of smoke from shell explosion while he was an ambulance driver on the World War I battle front, leading to his classical publications (RICHARDSON 1922,1926).

Richardson number : dimensionless number characterising density-stratification, commonly used to predict the occurrence of fluid turbulence and the destruction of density currents in water or air. A common definition is :

$$Ri = \frac{g}{\rho} * \frac{\partial \rho / \partial y}{(\partial V / \partial y)^2}$$

Right bank : bank on the right-hand side of an observer when looking downstream.

Secondary current : is a flow generated at right angles to the primary current. It is a direct result of the Reynolds stresses and exists in any non-circular conduits (LIGGETT 1994, pp. 256-259). In natural rivers, they are significant at bends, and between a flood plain and the main channel.

Separation : In a boundary layer, a deceleration of fluid particles leading to a reversed flow within the boundary layer is called a separation. The decelerated fluid particles are forced outwards and the boundary layer is separated from the wall. At the point of separation, the velocity gradient normal to the wall is zero :

$$\left(\frac{\partial v_x}{\partial y} \right)_{y=0} = 0$$

Separation point : in a boundary layer, intersection of the solid boundary with the streamline dividing the separation zone and the deflected outer flow. The separation point is a stagnation point.

Shear flow : The term shear flow characterises a flow with a velocity gradient in a direction normal to the mean flow direction : e.g., in a boundary layer flow along a flat plate, the velocity is zero at the boundary and equals the free-stream velocity away from the plate. In a shear flow, momentum (i.e. per unit volume : $\rho * V$) is transferred from the region of high velocity to that of low-velocity. The fluid tends to resist the shear associated with the transfer of momentum.

Shear stress : In a shear flow, the shear stress is proportional to the rate of transfer of momentum. In laminar flows, Newton's law of viscosity states :

$$\tau = \mu * \frac{\partial v}{\partial y}$$

where τ is the shear stress, μ is the dynamic viscosity of the flowing fluid, v is the velocity and y is the direction normal to the flow direction. For large shear stresses, the fluid can no longer sustain the viscous shear stress and turbulence spots develop. After apparition of turbulence spots, the turbulence expands rapidly to the entire shear flow. The apparent shear stress in turbulent flow is expressed as :

$$\tau = \rho * (v + v_T) * \frac{\partial v}{\partial y}$$

where ρ is the fluid density, ν is the kinematic viscosity (i.e. $\nu = \mu/\rho$), and ν_T is a factor depending upon the fluid motion and called the eddy viscosity or momentum exchange coefficient in turbulent flow.

Similitude : correspondence between the behaviour of a model and that of its prototype, with or without geometric similarity. The correspondence is usually limited by scale effects.

Storm water : excess water running off the surface of a drainage area during and immediately following a period of rain. In urban areas, waters drained off a catchment area during or after a heavy rainfall are usually conveyed in man-made storm waterways.

Storm waterway : channel built for carrying storm waters.

Streamline : is the line drawn so that the velocity vector is always tangential to it (i.e. no flow across a streamline). When the streamlines converge the velocity increases. The concept of streamline was first introduced by the Frenchman J.C. de BORDA.

Subcritical flow : In open channel the flow is defined as subcritical if the flow depth is larger than the critical flow depth. In practice, subcritical flows are controlled by the downstream flow conditions.

Supercritical flow : In open channel, when the flow depth is less than the critical flow depth, the flow is supercritical and the Froude number is larger than one. Supercritical flows are controlled from upstream.

Surfactant (or surface active agent) : substance (e.g. detergent) that, when added to a liquid, reduces its surface tension thereby increasing its wetting property.

Surge : A surge in an open channel is a sudden change of flow depth (i.e. abrupt increase or decrease in depth). An abrupt increase in flow depth is called a positive surge while a sudden decrease in depth is termed a negative surge. A positive surge is also called (improperly) a 'moving hydraulic jump' or a 'hydraulic bore'.

Système international d'unités : international system of units adopted in 1960 based on the metre-kilogram-second (MKS) system. It is commonly called SI unit system. The basic seven units are : for length, the metre; for mass, the kilogram; for time, the second; for electric current, the ampere; for luminous intensity, the candela; for amount of substance, the mole; for thermodynamic temperature, the Kelvin.

Système métrique : international decimal system of weights and measures which was adopted in 1795 during the French Révolution. Between 1791 and 1795, the Académie des Sciences de Paris prepared a logical system of units based on the metre for length and the kilogram for mass. The standard metre was defined as $1/10^7$ times a meridional quadrant of earth. The gram was equal to the mass of 1 cm^3 of pure water at the temperature of its maximum density (i.e. 4 Celsius) and 1 kilogram equalled 1,000 grams. The litre was defined as the volume occupied by a cube of 10^3 cm^3 .

TAYLOR : Geoffrey Ingram TAYLOR was a British fluid dynamicist based in Cambridge. He established the basic developments of shear dispersion (TAYLOR 1953,1954). He was the great-son of the British mathematician George BOOLE (1815-1864) who established modern symbolic logic and Boolean algebra.

Turbulence : Flow motion characterised by its unpredictable behaviour, strong mixing properties and a broad spectrum of length scales.

Turbulent flow : In turbulent flows the fluid particles move in very irregular paths, causing an exchange of momentum from one portion of the fluid to another. Turbulent flows have great mixing potential and involve a wide range of eddy length scales.

Uniform equilibrium flow : occurs when the velocity is identically the same at every point, in magnitude and direction, for a given instant :

$$\frac{\partial V}{\partial s} = 0$$

in which time is held constant and ∂s is a displacement in any direction. That is, steady uniform flow (e.g. liquid flow through a long pipe at a constant rate) and unsteady uniform flow (e.g. liquid flow through a long pipe at a decreasing rate).

Universal gas constant (also called *molar gas constant* or *perfect gas constant*) : fundamental constant equal to the pressure times the volume of gas divided by the absolute temperature for one mole of perfect gas. The value of the universal gas constant is $8.31441 \text{ J.K}^{-1}.\text{mole}^{-1}$.

Viscosity : fluid property which characterises the fluid resistance to shear : i.e. resistance to a change in shape or movement of the surroundings.

VOC : Volatile Organic Compound.

Wake region : The separation region downstream of the streamline that separates from a boundary is called a wake or wake region.

Water : common name applied to the liquid state of the hydrogen-oxygen combination H_2O . Although the molecular structure of water is simple, the physical and chemical properties of H_2O are unusually complicated. Water is a colourless, tasteless, and odourless liquid at room temperature. One most important property of water is its ability to dissolve many other substances : H_2O is frequently called the universal solvent. Under standard atmospheric pressure, the freezing point of water is 0 Celsius (273.16 K) and its boiling point is 100 Celsius (373.16 K).

WOOD : I.R. WOOD is an Emeritus Professor in civil engineering at the University of Canterbury (New Zealand).

About the author

Hubert Chanson received a degree of 'Ingénieur Hydraulicien' from the Ecole Nationale Supérieure d'Hydraulique et de Mécanique de Grenoble (France) in 1983 and a degree of 'Ingénieur Génie Atomique' from the 'Institut National des Sciences et Techniques Nucléaires' in 1984. He worked for the industry in France as a R&D engineer at the Atomic Energy Commission from 1984 to 1986, and as a computer professional in fluid mechanics for Thomson-CSF between 1989 and 1990. From 1986 to 1988, he studied at the University of Canterbury (New Zealand) as part of a Ph.D. project. He was awarded a Doctor of Engineering from the University of Queensland in 1999 for outstanding research achievements in gas-liquid bubbly flows.

Hubert Chanson is a reader in environmental fluid mechanics and water engineering at the University of Queensland since 1990. His research interests include design of hydraulic structures, experimental investigations of two-phase flows, coastal hydrodynamics, water quality modelling, environmental management and natural resources. He is the author of four books : "[Hydraulic Design of Stepped Cascades, Channels, Weirs and Spillways](#)" (*Pergamon*, 1995), "[Air Bubble Entrainment in Free-Surface Turbulent Shear Flows](#)" (*Academic Press*, 1997), "[The Hydraulics of Open Channel Flows : An Introduction](#)" (*Butterworth-Heinemann*, 1999) and "The Hydraulics of Stepped Chutes and Spillways" (*Balkema*, 2001). His publication record includes over 180 international refereed papers and his work was cited over 600 times since 1990. Hubert Chanson has been active also as consultant for both governmental agencies and private organisations.

He has been awarded five fellowships from the Australian Academy of Science. In 1995 he was a Visiting Associate Professor at National Cheng Kung University (Taiwan R.O.C.) and he was Visiting Research Fellow at Toyohashi University of Technology (Japan) in 1999.

Hubert Chanson was the keynote lecturer at the 1998 ASME Fluids Engineering Symposium on Flow Aeration (Washington DC), at the Workshop on Flow Characteristics around Hydraulic Structures (Nihon University, Japan 1998) and at the first International Conference of the International Federation for Environmental Management System [IFEMS'01](#) (Tsurugi, Japan 2001). He gave an invited lecture at the International Workshop on [Hydraulics of Stepped Spillways](#) (ETH-Zürich, 2000). He lectured several short courses in Australia and overseas (e.g. Taiwan).

His Internet home page is <http://www.uq.edu.au/~e2hchans>. He developed a gallery of photographs website {<http://www.uq.edu.au/~e2hchans/photo.html>}, that received more than 1,300 hits per month since inception, and a series of world-known technical Internet resources {http://www.uq.edu.au/~e2hchans/url_menu.html}.