## CIVL3140 Introduction to Open Channel Hydraulics - TUTORIAL 3

The course is a professional subject in which the students are expected to have a sound knowledge of the basic principles of continuity, energy and momentum, and understand the principles of fluid flow motion. The students should have completed successfully the core course Introduction to Fluid mechanics in semester 1 (CIVL3130).

Past course results demonstrated a very strong correlation between the attendance of tutorials during the semester, the performances at the end-of-semester examination, and the overall course result.

More exercises in textbook pp. 87-93, 108-110, 111-118, 288-289, 533-572.
"The Hydraulics of Open Channel Flow: An Introduction", Butterworth-Heinemann Publ., Oxford, UK, 2004.

## Revision problems

Each and every student is strongly encouraged to work on the Revision exercises and Problems in the textbook, pages 111-118 \& 533-572. The latter section includes also some hydrology questions.

## 1. Uniform equilibrium flows

1.1 A concrete trapezoidal channel ( $1 \mathrm{~V}: 3 \mathrm{H}$ sideslopes, 1.2 m bottom width) is designed to carry $52 \mathrm{~m}^{3} / \mathrm{s}$. Calculate the uniform equilibrium depth for a bed slope of 0.0012 . Is the invert slope a mild or steep slope ?

Numerical solution: $\mathrm{d}_{\mathrm{O}}=2.25 \mathrm{~m}\left(\mathrm{k}_{\mathrm{S}}=1 \mathrm{~mm}\right)$, Mild slope (but almost critical slope $\left(\mathrm{Fr}_{\mathrm{O}}=0.84\right)$ )
1.2 An open channel must be lined with riprap to satisfy environmental regulations. The channel has a trapezoidal cross-section with an invert slope of 0.0009 . The bottom width is $3-\mathrm{m}$ and the side slopes are $1 \mathrm{~V}: 2 \mathrm{H}$. Assuming a Gauckler-Manning coefficient of 0.03 , calculate the normal depth for $\mathrm{Q}=28 \mathrm{~m}^{3} / \mathrm{s}$.

Numerical solution: $\mathrm{d}_{\mathrm{O}}=2.63 \mathrm{~m}$
1.3 Considering a rectangular channel $(B=12 \mathrm{~m})$, the uniform equilibrium depth is 1.1 m and the bed slope is 0.02 . The channel is rough concrete $\left(\mathrm{k}_{\mathrm{S}}=5 \mathrm{~mm}\right)$. Calculate the flow rate.

Numerical solution: $\mathrm{Q}=110 \mathrm{~m}^{3} / \mathrm{s}, \mathrm{f}=0.021, \mathrm{Fr}=2.5, \tau_{\mathrm{O}}=182 \mathrm{~Pa}$
1.4 A storm waterway channel consists of a 1.5 m deep concrete lined rectangular channel ( $\mathrm{B}=2.3 \mathrm{~m}$ ) surrounded by two grass-lined flood plains ( $B=25 \mathrm{~m}$ each, $\mathrm{k}_{\mathrm{S}}=100 \mathrm{~mm}$ ). The beds of the flood plain are at the same elevation: i.e., 1.5 m above the deep channel invert. The bed slope is $0.001\left(\mathrm{~S}_{\mathrm{O}}=0.1 \%\right)$.

During a flood event, the observed water depth in the deep channel is 2.1 m . Assuming uniform equilibrium flow conditions, calculate the total flow rate conveyed by the storm waterway. Estimate the discharges in the concrete-lined and in each flood plain.

Numerical solution: $\mathrm{Q}=35.6 \mathrm{~m}^{3} / \mathrm{s}\left(10.6 \mathrm{~m}^{3} / \mathrm{s}\right.$ in the deep channel and $12.5 \mathrm{~m}^{3} / \mathrm{s}$ in each flood plain)

## 2. Gradually-varied flows

2.1 A concrete-lined rectangular channel $(B=21 \mathrm{~m})$ supplies a discharge of $33 \mathrm{~m}^{3} / \mathrm{s}$ to a hydropower plant. The longitudinal bed slope is 5 m per km . At a gauging station, the measured flow depth is 1.85 m .
(a) Predict the type of free-surface profile (as per course handout, textbook pages 104-105, or Henderson's (1966) textbook pages 107-111).
(b) Will the downstream flow depth be larger or smaller than at the gauging station?
(c) From where is the flow (at the gauging station) best controlled?

Numerical solution:
(a) $\mathrm{d}_{\mathrm{O}}=0.483 \mathrm{~m} \mathrm{\&}_{\mathrm{C}}=0.63 \mathrm{~m} \Rightarrow \mathrm{~d}_{\mathrm{O}}<\mathrm{d}_{\mathrm{C}}<\mathrm{d}$

This is a steep slope with a S1 free-surface profile: e.g., a subcritical flow downstream of a hydraulic jump on a steep slope.
(b) The flow depth will increase with increasing downstream distance (Profile S1).
(c) The flow is subcritical and it is best controlled from downstream as for any Profile S1.
2.2 An artificial canal carries a discharge of $20 \mathrm{~m}^{3} / \mathrm{s}$. The channel cross-section is trapezoidal and symmetrical, with a $1.1-\mathrm{m}$ bottom width and $1 \mathrm{~V}: 1.5 \mathrm{H}$ sidewall slopes. The longitudinal bed slope is 12.5 m per km . The channel bottom and sidewalls consist of a mixture of fine sands $\left(\mathrm{d}_{50}=0.8 \mathrm{~mm}\right)$. At a bridge, crossing the waterway, the observed flow depth, at the gauging station, is 2.1 m .
(a) Predict the type of free-surface profile (as per textbook pages 104-105, or Henderson pages 107-111).

Assume that the equivalent roughness height equals the median sediment size. See discussions in textbook pages 84 \& 234-235.
(b) Using the software HydroChan ${ }^{\mathrm{TM}}$, compute the water depth 80 m upstream of the bridge.

Assume a Keulegan coefficient $k=d_{50}$. See discussion in textbook p. 78.
(c) Assuming that the observed water depth at the bridge is 3.2 m , use the software HydroChan ${ }^{\mathrm{TM}}$ to compute the water depth 80 m upstream of the bridge.

Numerical solution:
(a) $\mathrm{d}_{\mathrm{O}}<\mathrm{d}_{\mathrm{C}}<\mathrm{d} \Rightarrow$ profile S 1

This situation corresponds to a steep slope ( $\mathrm{d}_{\mathrm{o}}<\mathrm{d}_{\mathrm{C}}$ )
(b) The problem cannot be solved because the observed flow depth is smaller than the subcritical conjugate depth and greater than the supercritical conjugate depth.
Physically, this flow situation may occur downstream of a break in slope, or downstream of a control structure, but it cannot take place in a long prismatic channel.
(c) The problem has a solution because the observed flow depth is greater than the subcritical conjugate flow depth : d
$=1.15 \mathrm{~m}$ at $\mathrm{x}=-80 \mathrm{~m}$. Note the presence of a hydraulic jump at $\mathrm{x} \sim-62 \mathrm{~m}$
2.3 A $25-\mathrm{m}$ wide rectangular spillway is designed to carry $1000 \mathrm{~m}^{3} / \mathrm{s}$. The spillway system comprises a broad-crest followed by the spillway chute built on the downstream slope of the embankment dam. The chute invert slope is $25^{\circ}$. (The channel is made of rough concrete ( $\mathrm{k}_{\mathrm{S}}=10 \mathrm{~mm}$ ).)
Calculate the water depth and flow velocity at an elevation $10-\mathrm{m}$ below the crest elevation (i.e. $\mathrm{z}_{\mathrm{O}}=\mathrm{z}_{\text {crest }}-10 \mathrm{~m}$ ).
Design your own spreadsheet calculations using a standard step method (distance calculated from depth). (See textbook pages 106-108 \& 275-288.)

Numerical solution: $\mathrm{d}=2.335 \mathrm{~m}, \mathrm{~V}=17.13 \mathrm{~m} / \mathrm{s}$
2.4 The outflow from a reservoir of constant free-surface elevation discharges into a long channel ( 120 m length). The channel ends into another reservoir of constant free-surface elevation. The channel has a rectangular cross-section (width: 4.2 m ). The channel bed is made of smooth concrete and the equivalent roughness height of bed and sidewalls is $\mathrm{k}_{\mathrm{s}}=3 \mathrm{~mm}$.
The downstream reservoir level is 1.14 m above the channel outlet invert and the flow rate is $2.3 \mathrm{~m}^{3} / \mathrm{s}$. Sketch the composite profile and computer the free-surface elevation for a channel slope $S_{o}=0.10 \%$ and determine the upstream reservoir elevation.

Numerical solution
First we must ascertain whether the channel slope is steep or mild. For the discharge, the normal depth is $\mathrm{d}_{0}=0.48$ and $\mathrm{Fr}_{0}=0.5$. Hence the slope is mild and the flow is controlled from downstream: i.e., by the downstream reservoir.
Second, backwater calculations must be performed from the downstream reservoir. Note that the downstream boundary conditions include the specific energy rather than the water depth.
Neglecting singular losses, the calculations yield an upstream reservoir level at 1.03 m above the inlet invert : i.e., 0.012 m above the downstream reservoir elevation.
2.5 Let us consider the outflow from a reservoir of constant free-surface elevation into a flat channel cut through the embankment and followed by a steep chute over the embankment downstream slope, ending with a hydraulic stilling basin. (This configuration could be an emergency spillway for an old embankment structure.)
The flat channel section is 25 m long with a rectangular cross-section (width: 4 m ). The bed slope is $0.15 \%$. The channel bed is made of smooth concrete and the equivalent roughness height of bed and sidewalls is $\mathrm{k}_{\mathrm{s}}=2 \mathrm{~mm}$. The steep channel has a trapezoidal cross-section with a 3 m bottom width and $1 \mathrm{H}: 3.5 \mathrm{~V}$ slopes. The bed slope is $1 \mathrm{~V}: 2.5 \mathrm{H}$ and the chute is 40 m long. The chute bed and walls are made of rocks embedded in mortar with an equivalent roughness height $\mathrm{k}_{\mathrm{s}}=150 \mathrm{~mm}$.
(a) Assuming that the flow rate in the channel is $25 \mathrm{~m}^{3} / \mathrm{s}$, sketch the composite profile in each channel.
(b) For that flow rate, compute the backwater profile in both channels. Predict the flow velocity at the downstream end of the steep chute.

## Solution

(a) The correct composite profile shows critical flow conditions at the downstream end of the flat channel cut through the embankment, and a supercritical flow at the upstream end of the steep chute over the embankment downstream slope.
(b) The correct solution yields a residual velocity at the downstream end of the steep chute: $\mathrm{V}=10.2 \mathrm{~m} / \mathrm{s}$.

